

Prove that

$$\ell_i(x) = \frac{T_{k+1}(x)}{(x - x_i) T'_{k+1}(x_i)},$$

and deduce that

$$\int_{-1}^1 \frac{\ell_i(x)}{\sqrt{1-x^2}} dx = \frac{\psi_{k+1}(x_i)}{T'_{k+1}(x_i)}, \quad i = 0, 1, \dots, k.$$

Hence obtain the Gaussian quadrature formula (*).

- (iv) Use the quadrature formula (*) with $k = 2$ to obtain values for the following integrals:

$$\int_{-1}^1 \frac{x^2}{\sqrt{1-x^2}} dx, \quad \int_{-1}^1 \frac{e^x}{\sqrt{1-x^2}} dx, \quad \int_{-1}^1 \frac{xe^x}{\sqrt{1-x^2}} dx.$$

Which of these values is exact, and why might we want to obtain numerical values for these integrals?

Question 3 (Chapter 13) - 20 marks

Powell, Exercise 13.2.

Question 4 (Chapter 13) - 20 marks

Apply the FFT method to calculate the approximation in Q_3 to the data

$$\begin{aligned} f(0) &= -0.3, & f(\pi) &= -0.1, \\ f(\pi/4) &= -0.5, & f(5\pi/4) &= 1.1, \\ f(\pi/2) &= -0.6, & f(3\pi/2) &= 2.2, \\ f(3\pi/4) &= -0.5, & f(7\pi/4) &= 0.4, \end{aligned}$$

which minimizes

$$\sum_{k=0}^7 \left[f\left(\frac{2\pi k}{8}\right) - q\left(\frac{2\pi k}{8}\right) \right]^2, \quad q \in Q_3,$$

and determine the value of this minimum.

TMA M832 04

Cut-off date 12 September 2000

Please make sure that the assignment number is correctly entered on your PT3 form as

M832 04.

Give any numerical answers correct to 3 significant figures.

Question 1 (Chapter 18) - 20 marks

In this question you are required to calculate the following two splines on $[-1, 1]$, both of which interpolate the data $f(-1) = 0$, $f(-\frac{1}{2}) = -\frac{1}{2}$, $f(0) = 1$, $f(\frac{1}{2}) = -\frac{1}{2}$, $f(1) = 0$:

- the natural cubic spline with knots at the data points;
- the quadratic spline with knots at $-\frac{1}{4}$, $\frac{1}{4}$.

Give each of your answers in piecewise polynomial form.