13) $f(x) = 1 - \frac{4}{\pi^2}(x - \pi/2)^2$

$1 - \frac{4}{\pi^2}(x-\pi/2)^2 = \sum c_i \sin \frac{\pi n x}{\pi}$

$c_i = \frac{2}{\pi}\int_0^{\pi}\left(1 - \frac{4}{\pi^2}(x-\pi/2)^2\right)\sin nx \, dx$

Integrate by parts

$u = \left(1 - \frac{4}{\pi^2}(x-\pi/2)^2\right) \quad du = -\frac{8}{\pi^2}(x-\pi/2)$

$dv = \sin nx \Rightarrow v = -\frac{1}{n}\cos nx$

$c_i = \frac{2}{\pi}\Big[\left[\frac{-8}{\pi^2}\left(1 - \frac{4}{\pi^2}(x-\pi/2)^2\right)\cos nx\right]_0^{\pi}$

$- \int_0^{\pi}\frac{8}{\pi^2 n}(x-\pi/2)\cos nx \, dx\Big]$

$= \frac{2}{\pi}\left[-0 + 0 - \int_0^{\pi}\frac{8}{\pi^2 n}(x-\pi/2)\cos nx \, dx\right]$

Again integrate by parts

$c_i = \frac{2}{\pi}\left[-\left(\left[\frac{8}{\pi^2 n^2}(x-\pi/2)\sin nx\right]_0^{\pi} - \int_0^{\pi}\frac{8}{\pi^2 n^2}\sin nx \, dx\right)\right]$

$= \frac{2}{\pi}\int_0^{\pi}\frac{8}{\pi^2 n^2}\sin nx \, dx$

$= \frac{16}{\pi^3 n^3}\left[-\cos nx\right]_0^{\pi}$

$= \frac{16}{\pi^3 n^3}(1 - (-1)^n) = 0 \quad n \text{ even}$

$= \frac{32}{\pi^3 n^3} \quad n \text{ odd.}$