

(12)

$$\therefore f(x) = \sum_{n=0}^{\infty} \frac{32}{\pi^3 (2n+1)^3} \sin(2n+1)x$$

$$f(x) = \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin(2n+1)x$$

Hence

$$\int_0^{\pi} f^2(x) dx = \frac{1024}{\pi^6} \int_0^{\pi} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{\sin(2n+1)x \sin(2k+1)x}{(2n+1)^3 (2k+1)^3} dx$$

$$= \frac{1024}{\pi^6} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{(2n+1)^3 (2k+1)^3} \int_0^{\pi} \sin(2n+1)x \sin(2k+1)x dx$$

$\sin(2n+1)x$ & $\sin(2k+1)x$ are orthogonal on $[0, \pi]$ so the integral is zero unless $k=n$

$$\int_0^{\pi} \sin^2(2n+1)x dx = \int_0^{\pi} \frac{1 + \cos 2(2n+1)x}{2} dx$$

$$= \left[\frac{x}{2} + \frac{\sin 2(2n+1)x}{4(2n+1)} \right]_0^{\pi}$$

$$= \frac{\pi}{2}$$

$$\therefore \int_0^{\pi} f^2(x) dx = \frac{1024}{\pi^6} \sum_{k=0}^{\infty} \frac{\pi}{2(2k+1)^6}$$

$$= \frac{512}{\pi^5} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^6}$$

$$\int_0^{\pi} f^2(x) dx = \int_0^{\pi} \left(1 - \frac{4}{\pi^2} (x - \pi/2)^2 \right)^2 dx$$

$$= \int_0^{\pi} \left(1 - \frac{8}{\pi^2} (x - \pi/2)^2 + \frac{16}{\pi^4} (x - \pi/2)^4 \right) dx$$