

$$1) i) \phi_0 = 1 \quad \checkmark$$

$$\begin{aligned} \phi_1(x) &= (x - \alpha_0) \phi_0 = x - \alpha_0 \\ \alpha_0 &= \frac{(\phi_0, x \phi_0)}{(\phi_0, \phi_0)} = \frac{\int_0^1 x^3 dx}{\int_0^1 x^2 dx} = \frac{[\frac{1}{4} x^4]_0^1}{[\frac{1}{3} x^3]_0^1} \\ &= \frac{1/4}{1/3} = \frac{3}{4} \end{aligned}$$

$$\phi_1(x) = x - 3/4 \quad \checkmark$$

$$\phi_2(x) = (x - \alpha_1) \phi_1 - \beta_1 \phi_0$$

$$\alpha_1 = \frac{(\phi_1, x \phi_1)}{(\phi_1, \phi_1)} = \frac{\int_0^1 x^3 (x - 3/4)^2 dx}{\int_0^1 x^2 (x - 3/4)^2 dx}$$

$$= \frac{\int_0^1 x^5 - \frac{3}{2} x^4 + \frac{9}{16} x^3 dx}{\int_0^1 x^4 - \frac{3}{2} x^3 + \frac{9}{16} x^2 dx}$$

$$= \frac{[\frac{1}{6} x^6 - \frac{3}{10} x^5 + \frac{9}{64} x^4]_0^1}{[\frac{1}{5} x^5 - \frac{3}{8} x^4 + \frac{3}{16} x^3]_0^1} = \frac{1/6 - 3/10 + 9/64}{1/5 - 3/8 + 3/16} = \frac{7}{12}$$

$$\beta_1 = \frac{(\phi_1, \phi_1)}{(\phi_0, \phi_0)} = \frac{\int_0^1 x^2 (x - 3/4)^2 dx}{\int_0^1 x^2 dx}$$

$$= \frac{\int_0^1 x^4 - \frac{3}{2} x^3 + \frac{9}{16} x^2 dx}{\int_0^1 x^2 dx} = \frac{[\frac{1}{5} x^5 - \frac{3}{8} x^4 + \frac{3}{16} x^3]_0^1}{[\frac{1}{3} x^3]_0^1}$$

$$= \frac{1/5 - 3/8 + 3/16}{1/3} = \frac{3}{80}$$

$$\phi_2(x) = (x - 7/12)(x - 3/4) - 3/80$$

$$= x^2 - \frac{4x}{3} + \frac{2}{5}$$

Please  
don't miss  
out the  $-\frac{3}{80}$   
line as fractions.