$$= \frac{\pi}{k+1} \sum_{i=0}^{k} f(x_i)$$

With $k=2$ zeros of T_2 are

$$x_i = \cos\left(\frac{(2(2-i)+1)\pi}{2(2+1)}\right) \quad i=0,1,2.$$

$$= \cos\left(\frac{5-2i}{6}\right)\pi$$

$$x_0 = \cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$$

$$x_1 = \cos\frac{\pi}{2} = 0$$

$$x_2 = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^2}}\,dx \simeq \frac{\pi}{3}\left(\left(\frac{-\sqrt{3}}{2}\right)^2 + 0 + \left(\frac{\sqrt{3}}{2}\right)^2\right)$$

$$= \frac{\pi}{3} \times \frac{3}{2} = \frac{\pi}{2}$$

Sub $x = \cos\theta \quad dx = -\sin\theta\, d\theta$

$$\int_{\pi}^{0} \frac{\cos^2\theta}{\sin\theta} \cdot -\sin\theta\, d\theta = \int_{0}^{\pi} \cos^2\theta\, d\theta$$

$$= \int_{0}^{\pi} \frac{1}{2}(1+\cos 2\theta)\, d\theta = \left[\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$= \frac{\pi}{2}$ so this value is exact. Some integrals cannot be found analytically and numerical methods are required.

$$\int_{-1}^{1} \frac{e^x}{\sqrt{1-x^2}}\,dx = \frac{\pi}{3}\left(e^{-\sqrt{3}/2} + e^0 + e^{\sqrt{3}/2}\right)$$

$$= \frac{\pi}{3}\left(2\cos\sqrt{3}/2 + 1\right)$$

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$$\int_{-1}^{1} \frac{x e^x}{\sqrt{1-x^2}}\,dx = \frac{\pi}{3}\left(\frac{-\sqrt{3}}{2} e^{-\sqrt{3}/2} + \frac{\sqrt{3}}{2} e^{\sqrt{3}/2}\right)$$

$$\frac{2\pi\sqrt{3}}{6} \sinh\sqrt{3}/2 = \frac{\pi\sqrt{3}}{3} \sinh\sqrt{3}/2.$$