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$$\int_{-1}^{1} \frac{l_i(x)}{\sqrt{1-x^2}}\,dx = \int_{-1}^{1} \frac{T_{k+1}(x)}{\sqrt{1-x^2}\,(x-x_i)\,T'_{k+1}(x_i)}\,dx$$

$$= \frac{1}{T'_{k+1}(x_i)} \int_{-1}^{1} \frac{T_{k+1}(x)}{\sqrt{1-x^2}\,(x-x_i)}\,dx$$

$$= \frac{1}{T'_{k+1}(x_i)} \int_{-1}^{1} \frac{T_{k+1}(x) - T_{k+1}(x_i)}{\sqrt{1-x^2}\,(x-x_i)}\,dx$$

(we can write this since $T_{k+1}(x_i) = 0$.)

$$= \frac{\varphi_{k+1}(x_i)}{T'_{k+1}(x_i)} \text{ by defn of } \varphi_{k+1}(x_i)$$

$$p(x) = \sum_{i=0}^{k} l_i(x) f(x_i)$$

Then $p(x_i) = f(x_i)$ $i = 0, \ldots, k$

$$\therefore \int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}}\,dx = \int_{-1}^{1} \frac{p(x)}{\sqrt{1-x^2}}\,dx$$

$$= \int_{-1}^{1} \sum_{i=0}^{k} \frac{l_i(x) f(x_i)}{\sqrt{1-x^2}}\,dx$$

$$= \sum_{i=0}^{k} \int_{-1}^{1} \frac{l_i(x) f(x_i)}{\sqrt{1-x^2}}\,dx$$

$$= \sum_{i=0}^{k} f(x_i) \int_{-1}^{1} \frac{l_i(x)}{\sqrt{1-x^2}}\,dx$$

$$= \sum_{i=0}^{k} \frac{\varphi_{k+1}(x_i)}{T'_{k+1}(x_i)} f(x_i)$$

$$= \sum_{i=0}^{k} \frac{\pi\, U_k(x_i)}{(k+1)\, U_k(x_i)} f(x_i) \text{ from a), b)}$$