

(13)

$$\begin{aligned}
 &= \int \left[ x - \frac{8}{3\pi^2} (x - \pi/2)^3 + \frac{16}{5\pi^4} (x - \pi/2)^5 \right] \pi \\
 &= \pi - 0 - \frac{8}{3\pi^2} \left( \frac{\pi^3}{3} + \frac{\pi^3}{3} \right) + \frac{16}{5\pi^4} \left( \frac{\pi^5}{32} + \frac{\pi^5}{32} \right) \\
 &= \pi - \frac{2\pi}{3} + \frac{\pi}{5} = \frac{8\pi}{15}
 \end{aligned}$$

$$\text{ie } \frac{8\pi}{15} = \frac{512}{\pi^5} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^6}$$

$$\therefore \sum_{k=0}^{\infty} \frac{1}{(2k+1)^6} = \frac{\pi^5}{512} \times \frac{8\pi}{15} = \frac{\pi^6}{960}$$

As required. ✓

2.