iii) $l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{k} \frac{(x-x_j)}{(x_i-x_j)} = \frac{1}{(x-x_i)} \prod_{\substack{j=0 \\ j\neq i}}^{k} \frac{(x-x_i)(x-x_j)}{(x_i-x_j)}$

The numerator in this expression is a polynomial of degree $k+1$ whose zeros $x_j, j=0,1,\ldots,i,\ldots,k$ are the zeros of T_{k+1}, of degree $k+1$ $\therefore$ the numerator is $\frac{1}{2^k} T_{k+1}$

where we have taken account of that the cofficient of x^{k+1} in T_{k+1} is 2^k.

Suppose p is any other polynomial with $k+1$ zeros as given above, with coefficient of $x^{k+1} = 1$

$p_1 = \frac{T_{k+1}}{2^k} - p$ is a polynomial of degree k and has $k+1$ zeros $\therefore$ p_1 is identically zero $\therefore$ numerator is $T_{k+1}(x)$

$$T_{k+1} = \prod_{j=0}^{k} (x-x_j)$$

$$T'_{k+1} = \sum_{i=0}^{k} \prod_{\substack{j=0 \\ j\neq i}}^{k} (x-x_j)$$

Every term in this sum is zero at $x=x_i$ except the one corresponding to i

$$\therefore T'_{k+1}(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{k} (x_i-x_j)$$

$$\therefore l_i(x) = \frac{1}{(x-x_i)} \frac{T_{k+1}(x)}{T'_{k+1}(x_i)}$$