ii) In polar coordinates $x = r\cos\theta$, $y = r\sin\theta$

$$\therefore u(r,\theta) = \frac{3}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{r\cos\theta}{r\sin\theta}\right)$$

$$= \frac{3}{2} + \frac{1}{\pi}\cdot\tan^{-1}(\cot\theta)$$

$\cot\theta = \pi/2 - \theta$

$$\therefore u(r,\theta) = \frac{3}{2} + \frac{1}{\pi}\tan^{-1}(\pi/2 - \theta)$$

$$= \frac{3}{2} + \frac{1}{\pi}\left(\frac{\pi}{2} - \theta\right)$$

$$= 2 - \frac{\theta}{\pi}$$

$$\nabla^2 u = \nabla^2\left(2 - \frac{\theta}{\pi}\right)$$

$$= \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

$$= 0 + 0 + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(2 - \frac{\theta}{\pi}\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{-1}{\pi}\right) = 0$$

$\therefore u$ is harmonic

4/4 Also if $x > 0$, $\theta = 0$, $u = 2 - 0 = 2$

$x < 0$, $\theta = +\pi$, $u = 2 - \pi/\pi = 1$

$\therefore$ boundary conditions are satisfied.

iii) Let $u = e^{iz} = e$

then on $|z| = r$, $z = re^{i\theta}$

$u = e^{ire^{i\theta}}$

$$\frac{\partial u}{\partial r} = e^{ire^{i\theta}} \cdot e^{i\theta}$$

$$\int_0^\pi u \left.\frac{\partial u}{\partial r}\right|_{(r=R)} \cdot R\,d\theta = \int_0^\pi e^{iRe^{i\theta}} \cdot e^{i\theta} \cdot e^{iRe^{i\theta}} \cdot R\,d\theta$$

$$= R\int_0^\pi e^{2iRe^{i\theta}} e^{i\theta}\,d\theta =$$

$$\geq 0 \quad \left|\int_0^\pi u \left.\frac{\partial u}{\partial r}\right|_{(r=R)} R\,d\theta\right| \leq R\int_0^\pi \left|e^{2iRe^{i\theta}}\right| d\theta$$