$$= \frac{1}{2\pi}\int_0^{2\pi} u(\phi)\,d\phi + \frac{r}{\pi}\int_0^{2\pi} (\cos\theta\cos\phi + \sin\theta\sin\phi)\,u(\phi)\,d\phi$$

$$+ \dots + \frac{r^n}{\pi}\int_0^{2\pi} (\cos n\theta\cos n\phi + \sin n\theta\sin n\phi)\,u(\phi)\,d\phi$$

$$+ \dots$$

$$= \frac{a_0}{2} + \frac{r\cos\theta}{\pi}\int_0^{2\pi} \cos\phi\, u(\phi)\,d\phi + \frac{r\sin\theta}{\pi}\int_0^{2\pi} \sin\phi\, u(\phi)\,d\phi$$

$$+ \dots + \frac{r^n\cos n\theta}{\pi}\int_0^{2\pi} \cos n\phi\, u(\phi)\,d\phi$$

$$+ \frac{r^n \sin n\theta}{\pi}\int_0^{2\pi} \sin n\phi\, u(\phi)\,d\phi$$

$$= \frac{a_0}{2} + \frac{r}{\pi}\cos\theta\, a_1 + \frac{r}{\pi}\sin\theta\, b_1 + \dots + \frac{a_n r^n \cos n\theta}{\pi}$$

$$+ \frac{b_n r^n \sin n\theta}{\pi} + \dots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} r^n (a_n\cos n\theta + b_n \sin n\theta)$$

where $a_n = \int_0^{2\pi} \cos n\phi\, u(\phi)\,d\phi$

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$$b_n = \int_0^{2\pi} \sin n\phi\, u(\phi)\,d\phi.$$

(iii) $u(\theta) = \cos^2\theta = \frac{1+\cos 2\theta}{2}$, $b_n = \frac{1}{\pi}\int_0^{2\pi} \sin n\phi \left(\frac{1+\cos 2\phi}{2}\right) d\phi$

$a_n = \frac{1}{\pi}\int_0^{2\pi} \cos n\phi \left(\frac{1+\cos 2\phi}{2}\right) d\phi$, $b_n = 0$

Not clear.

$= 0$ unless $n = 0, 2$ by orthogonality of $\cos mx$, $\cos nx$, $n \neq m$ and ~~cos~~ $\sin n\phi$ with 1, $\cos n\phi$, $n \neq m$.

$$a_2 = \frac{1}{\pi}\int_0^{2\pi} \frac{\cos 2\phi}{2} + \frac{\cos^2 2\phi}{2}\,d\phi$$

$$= 0 + \frac{1}{2\pi}\int_0^{2\pi} \frac{(1+\cos 4\phi)}{2}\,d\phi$$

$$= \frac{1}{4\pi}\left[\phi + \frac{\sin 4\phi}{4}\right]_0^{2\pi} = \frac{1}{2}.$$