(5)

$$\int_0^1 \frac{dv}{(1-v^4)^{3/2}} = \frac{1}{2^{3/2}} B(1/4, 1/4)$$
$$= \frac{1}{2\sqrt{2}} \frac{\Gamma^2(1/4)}{\sqrt{\pi}}$$
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$$= \frac{\Gamma^2(1/4)}{2\sqrt{2\pi}}$$

I just put $s = 1 - t^2$ and the result comes out quickly.

3) i) Poisson's formula states

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$$u(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{g(s)}{(x-s)^2+y^2}$$

where $g(s) = u(s,0)$

$$\therefore u(x,y) = \frac{y}{\pi}\left[\int_{-\infty}^{0} \frac{ds_1}{(x-s_1)^2+y^2} + 2\int_0^{\infty} \frac{ds}{(x-s)^2+y^2}\right]$$

In 1st integral sub $x-s = y\tan\theta$

Then $-ds = y\sec^2\theta\, d\theta$

when $s=0$ $\theta = \tan^{-1}(x/y)$

$s = -\infty$ $\theta = +\pi/2$

1st integral becomes $\int_{-\pi/2}^{\tan^{-1}(x/y)} \frac{-y\sec^2\theta\, d\theta}{y^2\sec^2\theta}$

$$= \frac{1}{y}\int_{+\pi/2}^{\tan^{-1}(x/y)} -d\theta$$
$$= \frac{-1}{y}\left[\tan^{-1}(x/y) - +\frac{\pi}{2}\right]$$
$$= -\frac{1}{y}\left(\tan^{-1}(x/y) - \frac{\pi}{2}\right)$$

In 2nd integral sub $x-s = y\tan\theta$

Then $-ds = y\sec^2\theta\, d\theta$

when $s=0$ $\theta = \tan^{-1}(x/y)$

when $s=\infty$ $\theta = -\pi/2$

2nd Integral becomes

$$2\int_{\tan^{-1}(x/y)}^{-\pi/2} \frac{-y\sec^2\theta}{y^2\sec^2\theta} d\theta = \frac{-2}{y}\left[\theta\right]_{\tan^{-1}(x/y)}^{-\pi/2}$$
$$= -\frac{2}{y}\left[-\frac{\pi}{2} - \tan^{-1}\left(\frac{x}{y}\right)\right]$$
$$= \frac{2}{y}\left(\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{y}\right)\right)$$