

(4)

$$\text{so } W = \frac{a}{I} \int_0^z (t^2 - 1)^{-3/4} dt$$

$$i) B(1/4, 1/4) = \frac{\Gamma(1/4)\Gamma(1/4)}{\Gamma(1/2)} = \frac{\Gamma^2(1/4)}{\sqrt{\pi}}$$

Also, using the defining formula for $B(z, w)$

$$B(1/4, 1/4) = \int_0^1 t^{(1/4)-1} (1-t)^{(1/4)-1} dt$$

$$= \int_0^1 t^{-3/4} (1-t)^{-3/4} dt$$

$$= \int_0^1 (t - t^2)^{-3/4} dt$$

$$= \int_0^1 (1/4 - (1/2 - t)^2)^{-3/4} dt, \text{ by completing the square}$$

$$= \int_0^1 \frac{dt}{(1/4 - (1/2 - t)^2)^{3/4}} = \frac{1}{(1/4)^{3/4}} \int_0^1 \frac{dt}{(1 - (1-2t)^2)^{3/4}}$$

$$= 2^{3/2} \int_0^1 \frac{dt}{(1 - (1-2t)^2)^{3/4}}$$

Put \checkmark $1-2t = u$ then $dt = -\frac{du}{2}$

when $t=1$ $u=-1$
 $t=0$ $u=1$

$$B(1/4, 1/4) = 2^{3/2} \int_1^{-1} \frac{-du/2}{(1-u^2)^{3/4}} = -\frac{2^{3/2}}{2} \int_1^{-1} \frac{du}{(1-u^2)^{3/4}}$$

Integral in u is even about 0 so can write

$$B(1/4, 1/4) = \cancel{2} \times \frac{2^{3/2}}{2} \int_0^1 \frac{du}{(1-u^2)^{3/4}} = -2^{3/2} \int_0^{-1} \frac{du}{(1-u^2)^{3/4}}$$

\checkmark Now write $u = -v$ then $u=-1$ $v=1$, $du = -dv$
 $u=0$ $v=0$

$$B(1/4, 1/4) = 2^{3/2} \int_0^1 \frac{dv}{(1-v^2)^{3/4}}$$