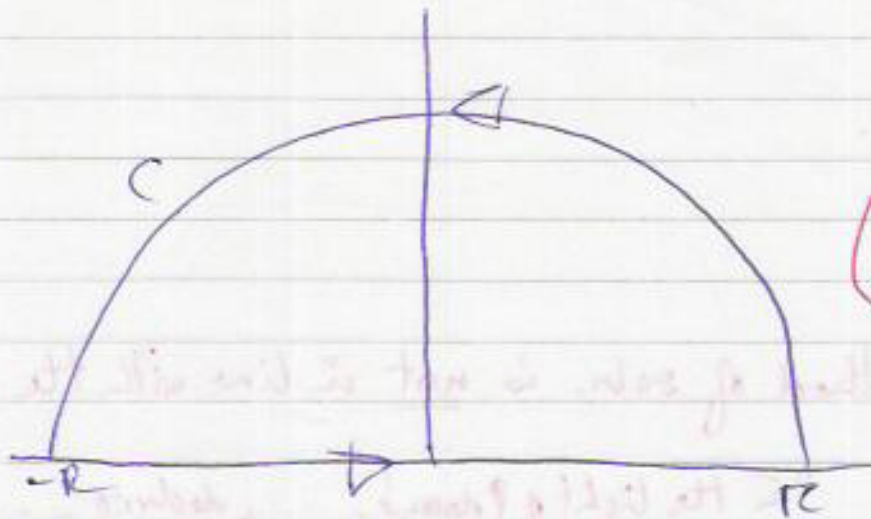


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iii) We take the contour integral around the curve C



$7\pi$

to give

$$\begin{aligned}
 I &= \operatorname{Re} \left( \int_C \frac{e^{iz}}{(z-x_0)^2 + y_0^2} dz \right) \\
 &= \operatorname{Re} \left( \int_C \frac{e^{iz}}{(z-x_0-iy_0)(z-x_0+iy_0)} dz \right) \\
 &= \operatorname{Re} \left( 2\pi i \sum \operatorname{Residues} \left( \frac{e^{iz}}{(z-x_0-iy_0)(z-x_0+iy_0)} \right) \right) \\
 &= \operatorname{Re} \left( 2\pi i \frac{e^{i(x_0+iy_0)}}{x_0+iy_0 - x_0+iy_0} \right) = \operatorname{Re} \left( \pi e^{i(x_0+iy_0)} \right)
 \end{aligned}$$

Since only residue at  $x_0+iy_0, y_0 > 0$ .

We split the integral up

$$\begin{aligned}
 \int_C &= \int_{-R}^R + \int_{|z|=R} \\
 \int_{|z|=R} \frac{e^{iz}}{(z-x_0)^2 + y_0^2} dz &= \int_0^\pi \frac{e^{iRe^{i\theta}}}{(Re^{i\theta} - x_0)^2 + y_0^2} d(Re^{i\theta}) \\
 &= iR \int_0^\pi \frac{e^{i(R\cos\theta + iR\sin\theta)}}{(Re^{i\theta} - x_0)^2 + y_0^2} e^{i\theta} d\theta
 \end{aligned}$$