M828 TMA01

23/25

1) i) Put $\omega = e^{i\theta}$ ✓

$$f(\omega) = \frac{1-e^{i\theta}}{1+e^{i\theta}} \times$$

$$= \frac{1-e^{i\theta}}{1+e^{i\theta}} \times \frac{1+e^{-i\theta}}{1+e^{-i\theta}}$$

$$= \frac{1+e^{-i\theta}-e^{+i\theta}-1}{1+e^{-i\theta}+e^{i\theta}+1}$$

$$= \frac{-2i\sin\theta}{2+2\cos\theta} = -i\left(\frac{\sin\theta}{1+\cos\theta}\right)$$ ✓ 5/5

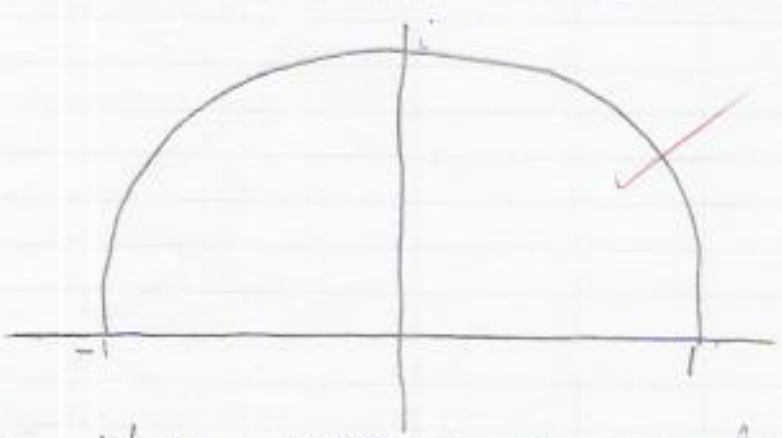
The form in brackets is real, $f(\omega)$ is imaginary on $e^{i\theta}$.

ii) If $z = re^{i\theta}$ $\quad 0 < \theta < \pi/3$, $0 \leq r < 1$

then $\omega = z^3 = r^3 e^{3i\theta}$ ✓ has $\quad 0 < 3\theta < \pi$ ✓, $0 < r < 1$ ✓

So the map takes D from the sector given to the upper half circular region D'.

The boundary of D' consists of three parts: the upper circular boundary, the +ve x axis with $0 < x < 1$ and the -ve x axis with $-1 < x < 0$



On the upper circular boundary, $\arg(f(\omega)) = \arg\left(-i \times \frac{\sin\theta}{1+\cos\theta}\right) = -\frac{\pi}{2}$, since $\sin\theta$ +ve

On +ve axis $\arg(f(\omega)) = \arg\left(\frac{1-x}{1+x}\right) = 0$ since $\frac{1-x}{1+x}$ +ve for $-1 < x < 1$

10/10 On -ve axis $\arg(f(\omega)) = \arg\left(\frac{1-x}{1+x}\right) = 0$ ✓