$7\frac{3}{3}$

taking magnitudes we obtain

$$\left|R\int_{0}^{\pi}\frac{e^{iR(\cos\theta+i\sin\theta)}}{(Re^{i\theta}-x_0)^2+y_0^2}e^{i\theta}\,d\theta\right|$$

$$\leq R\int_{0}^{\pi}\frac{e^{-\sin\theta}}{(R-x_0)^2+y_0^2}d\theta \quad \left(\int_C=\int_{-R}^{R}\right)$$

$$\leq \frac{R\pi}{(R-x_0)^2+y_0^2} \to 0 \text{ as } R\to\infty$$

for fixed x_0, y_0

$$\therefore \frac{y_0}{\pi}\int_{-R}^{R}\frac{\cos x}{(x-x_0)^2+y_0^2}dx = \frac{y_0}{\pi}\mathrm{Re}\left(\pi\frac{e^{i(x+iy)}}{y_0}\right)$$

2/5 $$= \frac{y_0}{\pi}\frac{e^{-y}\cos x}{y} = e^{-y}\cos x$$ ANSWER

~~Alternatively we could put $y=e^{iz}$ and because~~

$$\cancel{\int_{|z|=R,\,y>0} e^{iz}\frac{\partial}{\partial R}(e^{iRe^{i\theta}})\,dz}$$

$$\cancel{=i\int_0^{\pi}Re^{iRe^{i\theta}}e^{i\theta}d\theta \to 0 \text{ as } R\to\infty}$$

$$\cancel{\left(\text{Since }\left|\int_0^{\pi}Re^{2iRe^{i\theta}}e^{i\theta}d\theta\right| \leq R\int_0^{\pi}e^{-2R\sin\theta}d\theta\right.}$$

~~which tends to 0 as $R\to\infty$.~~

~~then $\cos s=\mathrm{Re}(e^{is})$~~

$$\cancel{\text{So we take }\mathrm{Re}\left(\frac{y}{\pi}\int_{-\infty}^{\infty}\frac{e^{is}}{(x-s)^2+y^2}ds\right)=\mathrm{Re}(e^{i(x+iy)})}$$

$$\cancel{=\cos x\, e^{-y}}$$