We have

$$\frac{y}{\pi}\left(\frac{-1}{y}\left(\tan^{-1}\left(\frac{x}{y}\right)-\frac{\pi}{2}\right)+\frac{2}{y}\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{x}{y}\right)\right)\right)$$

$$=\frac{1}{\pi}\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{x}{y}\right)\right)+\frac{2}{\pi}\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{x}{y}\right)\right)$$

10/10 $$=\frac{3}{2}+\frac{1}{\pi}\tan^{-1}\left(\frac{x}{y}\right) \checkmark \quad (*)$$

Note Poisson's formula applies since if $u=\frac{3}{2}+\frac{1}{\pi}\tan^{-1}\left(\frac{x}{y}\right)=\frac{3}{2}+\frac{\theta}{\pi}$

$$\frac{\partial u}{\partial r}=\frac{\partial}{\partial r}\left(\frac{3}{2}+\frac{\theta}{\pi}\right)=0 \checkmark$$

So $\int_0^{\pi} u\frac{\partial u}{\partial r}R\,d\theta=0$ (I hadn't asked for this)

and u is harmonic and continuous (except at the origin).

You have omitted the direct verification of the answer (*) and that its harmonic and that it satisfies the b.c. (as given in x, y).

0/6

It's easy to show from $u=\frac{3}{2}+\frac{1}{\pi}\tan^{-1}(x/y)$

that $\frac{\partial u}{\partial x}=\frac{y}{x^2+y^2}$, $\frac{\partial^2 u}{\partial x^2}=\frac{-2xy}{(x^2+y^2)^2}$, etc.

so $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

Also, as $y\to 0$, we have $\tan^{-1}(x/y)\to\begin{cases}+\pi/2, & x>0\\ -\pi/2, & x<0\end{cases}$

so $u\to\begin{cases}2, & x>0.\\ 1, & x<0\end{cases}$