

You need to explicitly check that your soln. does indeed satisfy the ode.

$$y = 3 \cos 2t - \frac{1}{2} \sin 2t + \frac{1}{2} \int_0^t f(\tau) \sin(2t - 2\tau) d\tau$$

$$\text{So } \dot{y} = -6 \sin 2t - \cos 2t + \frac{1}{2} f(\tau=t) \sin(2t - 2\tau) \Big|_{\tau=t} + \frac{1}{2} \int_0^t \frac{\partial}{\partial t} [f(\tau) \sin(2t - 2\tau)] d\tau,$$

on using Note.

$$\text{Hence } \dot{y} = -6 \sin 2t - \cos 2t + \int_0^t f(\tau) \cos(2t - 2\tau) d\tau$$

$$\text{Similarly, } \ddot{y} = -12 \cos 2t + 2 \sin 2t + f(t) - 2 \int_0^t f(\tau) \sin(2t - 2\tau) d\tau$$

on using Note again.