(11)

$$\mathcal{L}(y(t))\cdot\frac{\Gamma(\frac{1}{2})}{\sqrt{u}} = \frac{1}{u^2} + \frac{2}{u^3}$$

$$\mathcal{L}(y(t))\frac{\sqrt{\pi}}{\sqrt{u}} = \frac{1}{u^2} + \frac{2}{u^3}$$

$$\mathcal{L}(y(t)) = \frac{1}{\sqrt{\pi}u^{3/2}} + \frac{2}{\sqrt{\pi}\,u^{5/2}}$$

ii) $$y(t) = \frac{1}{\sqrt{\pi}}\mathcal{L}^{-1}(u^{-3/2}) + \frac{2}{\sqrt{\pi}}\mathcal{L}^{-1}(u^{-5/2})$$

$$= \frac{1}{\sqrt{\pi}}\frac{t^{+1/2}}{\Gamma(3/2)} + \frac{2}{\sqrt{\pi}}\frac{t^{3/2}}{\Gamma(5/2)}$$

$$= \frac{2t^{1/2}}{\pi} + \frac{2}{\sqrt{\pi}}\frac{t^{3/2}}{\frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2})}$$

$$= \frac{2t^{1/2}}{\pi} + \frac{8\sqrt{t^3}}{3\pi} = \frac{2t^{1/2}(3+4t)}{3\pi}$$

$$y(t) = \frac{2t^{1/2}(3+4t)}{3\pi}$$

$$\int_0^t \frac{2s^{1/2}(3+4s)}{3\pi\,(t-s)^{1/2}}\,ds = \frac{2}{\pi}\int_0^t\left(\frac{s^{1/2}}{(t-s)^{1/2}} + \frac{4s^{3/2}}{(t-s)^{1/2}}\right)ds$$

s=tx works too, w:j the Note.

Sub $s = t\sin^2\theta$, $ds = 2t\sin\theta\cos\theta$

Integrals become

$$\frac{2}{\pi}\int_0^{\pi/2}\tan\theta\cdot 2t\sin\theta\cos\theta\,d\theta + \frac{8}{3\pi}\int_0^{\pi/2}\frac{t^{3/2}\sin^3\theta\cdot 2t\sin\theta\cos\theta\,d\theta}{t^{1/2}\cos\theta}$$

$$= \frac{2}{\pi}\int_0^{\pi/2} 2t\sin^2\theta\,d\theta + \frac{16}{3\pi}\int_0^{\pi/2} t^2\sin^4\theta\,d\theta$$

$$= \frac{4t}{\pi}\int_0^{\pi/2}\frac{(1+\cos 2\theta)}{2}d\theta + \frac{16t^2}{3\pi}\int_0^{\pi/2}\frac{(1-\cos 2\theta)^2}{4}d\theta$$

$$= \frac{4t}{\pi}\cdot\frac{\pi}{4} + \frac{16t^2}{3\pi 4}\int_0^{\pi/2}(1 - 2\cos 2\theta + \cos^2 2\theta)\,d\theta$$

$$= t + \frac{4t^2}{3\pi}\int_0^{\pi/2}\left(1 - 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right)d\theta$$

$$= t + \frac{4t^2}{3\pi}\left[\theta - \sin 2\theta + \frac{\theta}{2} + \frac{\sin 4\theta}{8}\right]_0^{\pi/2}$$