②

we get

$$\lim_{m\to\infty}\int_{\pi/4}^{\pi/2}\frac{\sin(2m+1)\theta}{\sin\theta}\,d\theta = 0$$

$$= \int_{\pi/4}^{\pi/2}\left[1 + 2\sum_{n=1}^{\infty}\cos 2n\theta\right]d\theta$$

$$= \frac{\pi}{4} + 2\sum_{n=1}^{\infty}\left[\frac{\sin 2n\theta}{2n}\right]_{\pi/4}^{\pi/2} = 0$$

$$\frac{\pi}{4} = -2\sum_{n=1}^{\infty}\frac{1}{2n}\left(\sin\pi n - \sin\pi n/2\right)$$

$$= 2\sum_{n=1}^{\infty}\frac{1}{2n}\sin\pi n/2$$

$$= 2\left(\frac{1}{2}\sin\pi/2 + \frac{1}{4}\sin\pi + \frac{1}{6}\sin 3\pi/2 + \frac{1}{8}\sin 2\pi + \frac{1}{10}\sin 5\pi/2 + \frac{1}{12}\sin 6\pi \dots\right)$$

$$= 2\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + \frac{1}{18} \dots\right)$$

$$= \cancel{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots\right)$$

$$\frac{\pi}{4} = \sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1}$$

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$$\therefore \pi = 4\sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1}$$

ii) $$L\left(t\frac{d^2x}{dt^2}\right) = -\frac{d}{ds}\left(s^2F(s) - sF(0) - F'(0)\right)$$

$$= -2sF - s^2F'(s) - \cancel{F}F(0)$$

$$L\left(\frac{dx}{dt}\right) = sF(s) - F(0)$$

$$L(tx) = -\frac{d}{ds}(F(s)) = -F'(s)$$