There are two integrals here, but each integral has the form

$$I_\lambda(t) = \int_0^t s^\lambda (t-s)^{-1/2} ds$$

and may be evaluated by setting $x = s/t$, $dx = ds/t$ to give

$$I_\lambda = t^{\lambda+\frac{1}{2}} \int_0^1 x^\lambda (1-x)^{-1/2} dx = \frac{\Gamma(1+\lambda)\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{2}+\lambda)},$$

on using the quoted formula in the Note. The verification is then straightforward.

Incidentally, several of you ignored my Note and preferred to evaluate the integral directly, using an elementary substitution. This is fine but you should be aware of the gamma functions approach too.

Good luck with the remainder of the course, dealing with **asymptotic analysis**. This is an important topic in modern applied mathematics, though Dettman's treatment is unfortunately very brief. Accordingly I have placed emphasis on it by devoting the whole of TMA 04 to this topic, and making some of my questions exploratory in form. You should read the relevant section of my Course Notes for guidance on asymptotic analysis, and if possible consult one of the books I've mentioned elsewhere that deals with the topic (eg, Ablowitz and Fokas, or Bender and Orszag - see CNs for details). Also, access to a computer would be helpful in Qu 4, but don't worry if that is not possible; the important thing is a good clear sketch (by hand or computer) that faithfully represents the function under discussion.

The M828 Exam. My understanding is that the exam is fixed for the morning of **Thursday 16 October.**

The exam consists of 8 questions and you will be assessed on your FIVE best answers. All questions carry equal marks. The main guide as to what to expect is the **TMAs** and then the **Specimen Paper.** The Specimen paper, with brief solutions, will be sent to you soon.

Very few people fail the exam, but I know some do find it quite difficult simply because one has to tackle everything in 3 hours instead of the possibly 3 weeks spent on a TMA. There is no time to spend the hours (days?) that I am given to believe some folk spend on certain bits of the TMA questions!

Away!

I will be away from St Andrews 27 June - 13 July (on a Baltic cruise) and again 27 September - 1 October (at a conference in Palma, Majorca).

Some of you may be interested in what is my research area: I look at mathematical aspects of magnetic waves in the Sun. This has become quite exciting for me recently because space observations by an American telescope called TRACE has detected oscillations in the solar corona of the type predicted by some theory I did almost 2 decades ago. No complex analysis of the type in this course was involved (if you discount Fourier representations of waves) but it was an example of theory and observations (finally) coming together. Of course, nothing is completely satisfactory; it turns out that while the periods and nature of the observed waves match the predictions of the theory, the observations show that the waves are also strongly damped. So that now remains to be explained. I am giving an invited review on this topic in Majorca.

Incidentally, while only a small amount of this course finds direct application in my research field, it is interesting that the first determination of the run of sound speed in the *interior* of the Sun was done in part by solving an Abel integral equation (combined with data about oscillations at the observable solar surface). I have been tempted to make up a TMA question on this topic!

Enjoy your summer!

Bernard Roberts

19 June 2003