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$$\begin{aligned} \int_{C_2} z^n dz &= \int_{\Gamma_+} z^n dz = \int_{-1}^1 \frac{1}{2i} z^n (z-1) dz \\ &= a_n = \frac{(2n)!}{2^{2n} (n!)^2} z^n \end{aligned} \quad (7)$$

$$\begin{aligned} \cos(t \sin \theta) &= \sum \frac{(-1)^n}{(2n)!} (t \sin \theta)^{2n} \\ &= \sum \frac{(-1)^n}{(2n)!} t^{2n} \sin^{2n} \theta \end{aligned}$$

$$\begin{aligned} \frac{1}{\pi} \int_0^\pi \cos(t \sin \theta) d\theta &= \frac{1}{\pi} \sum \frac{(-1)^n}{(2n)!} t^{2n} \int_0^\pi \sin^{2n} \theta d\theta \\ &= \frac{1}{\pi} \sum \frac{(-1)^n}{(2n)!} t^{2n} \cdot \frac{(2n)!}{2^{2n} (n!)^2} \pi \\ &= \sum \frac{(-1)^n}{2^{2n} (n!)^2} t^{2n} = J_0(t) \end{aligned}$$

$$\text{ie } J_0(t) = \frac{1}{\pi} \int_0^\pi \cos(t \sin \theta) d\theta$$

$$L(J_0(t)) = L\left(\frac{1}{\pi} \int_0^\pi \cos(t \sin \theta) d\theta\right)$$

$$= \frac{1}{\pi} L\left(\int_0^\pi \cos(t \sin \theta) d\theta\right)$$

$$= \frac{1}{\pi} \int_0^\infty e^{-st} \int_0^\pi \cos(t \sin \theta) d\theta dt$$

$$= \frac{1}{\pi} \int_0^\pi \int_0^\infty e^{-st} \cos(t \sin \theta) dt d\theta$$

$$= \frac{1}{\pi} \int_0^\pi \left[L(\cos(t \sin \theta)) \right] d\theta$$

$$= \frac{1}{\pi} \int_0^\pi \frac{s d\theta}{s^2 + \sin^2 \theta} = \frac{s}{\pi} \int_0^\pi \frac{d\theta}{s^2 + \sin^2 \theta}$$

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