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The equation becomes

$$-2sF - s^2F'(s) + F(0) + sF(s) - F(0) - F'(s) = 0$$

$$F'(s)(-s^2-1) - sF(s) + 1 - 1 = 0$$

$$F'(s)(1+s^2) + sF(s) = 0$$

$$F'(s) + \frac{s}{1+s^2}F(s) = 0$$

$$\frac{d}{ds}\left(F(s)\exp\int\frac{s}{1+s^2}ds\right) = 0$$

$$\frac{d}{ds}\left(F(s)\exp\frac{1}{2}\ln(1+s^2)\right) = 0$$

$$\frac{d}{ds}\left(F(s)\sqrt{1+s^2}\right) = 0$$

$$\Rightarrow F(s)\sqrt{1+s^2} = C$$

$$F(s) = \frac{C}{\sqrt{1+s^2}}$$ where $F(s) = L(J_0(t))$

Applying the initial value theorem

$$\lim_{t\to 0} x(t) = 1 = \lim_{s\to\infty} s \cdot \frac{C}{\sqrt{1+s^2}} = C$$

$\therefore C = 1$

$$\therefore F(s) = \frac{1}{\sqrt{1+s^2}}$$

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iii) $$\int_0^\infty J_0(t)dt = \int_0^\infty \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2}\left(\frac{t}{2}\right)^{2n} dt$$

extraneous inform. but quoted in Qu. 2

You have missed the connection with Part (ii)

$$= \sum_{n=0}^{\infty}\int_0^\infty \frac{(-1)^n}{(n!)^2} t^{2n}/2^{2n} dt$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(n!)^2 2^{2n}}\int_0^\infty t^{2n} dt$$