If $f(t) = t$, the original differential eqn. is

$\frac{d^2y}{dt^2} + 4y = t$, which has the particular

soln $y = \frac{t}{4}$

$\therefore$ complete soln in this case is

$y = A\cos 2t + B\sin 2t + \frac{t}{4}$

to find A, B sub initial conditions in

$3 = A + 0 + 0$ ie $3 = A$

$-1 = -2A \times 0 + 2B \times 1 + 1/4$

$B = (-1 - 1/4)/2 = -5/8$

This gives

$\therefore y = 3\cos 2t - 5/8 \sin 2t + t/4$

$\therefore 3\cos 2t - \frac{5}{8}\sin 2t + \frac{t}{4} = 3\cos 2t - \frac{1}{2}\sin 2t + \frac{1}{2}\int_0^t \tau \sin 2(t-\tau)d\tau$

$-\frac{1}{8}\sin 2t + \frac{t}{4} = \frac{1}{2}\int_0^t \tau \sin 2(t-\tau)d\tau$

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$= \int_0^t \tau \sin 2(t-\tau)d\tau = \frac{t}{2} - \frac{1}{4}\sin 2t$

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4) i) $\int_0^t \frac{y(s)}{(t-s)^{1/2}} ds = t + t^2$

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$L\left(\int_0^t \frac{y(s)}{(t-s)^{1/2}} ds\right) = L(t + t^2) = L(t) + L(t^2)$

$L(y(t) * t^{-1/2})$ $= \frac{\Gamma(2)}{u^2} + \frac{\Gamma(3)}{u^3}$

$L(y(t))\, L(t^{-1/2})$