④

Let $a_n = \frac{(-1)^n}{(n!)^2 2^{2n}} \int_0^\infty t^{2n}\,dt$ These integrals are divergent.

$$= \frac{(-1)^n}{(n!)^2 2^{2n}} \left[\lim_{t\to\infty} \frac{t^{2n+2}}{2n+2} - \lim_{t\to 0} \frac{t^{2n+2}}{2n+2} \right]$$

$$\frac{a_{n+1}}{a_n} = \frac{-1}{(n+1)^2 2^2} \frac{\left(\lim_{t\to\infty} t^{2n+2}/2n+2 - \lim_{t\to 0} t^{2n+2}/2n+2\right)}{\left(\lim_{t\to\infty} t^{2n}/2n - \lim_{t\to 0} t^{2n}/2n\right)}$$

$$= \frac{-1}{4(n+1)^2} \frac{2n}{2n+2} \lim_{t\to\infty} t^2$$

?

as $n \to \infty$ $\frac{1}{4(n+1)^2} \frac{2n}{2n+2} \to 0$

$\therefore$ radius of convergence of integral is ∞ and $\int_0^\infty J_0(t)\,dt$ converges ✓ Flawed proof.

$$\int_0^\infty t^2 J_0(t)\,dt = \sum \frac{(-1)^n}{(n!)^2 2^{2n}} \int_0^\infty t^{2n+2}\,dt$$

divergent integrals again

$$= \sum \frac{(-1)^n}{(n!)^2 2^{2n}} \left[\lim_{t\to\infty} t^{2n+3}/2n+3 \right]$$

$$a_n = \frac{(-1)^n}{(n!)^2 2^{2n}} \lim_{t\to\infty} \frac{t^{2n+3}}{2n+3}$$

$$\frac{a_{n+1}}{a_n} = \frac{-1}{(n+1)^2 2^2} \frac{\lim_{t\to\infty} t^{2n+5}/2n+5}{\lim_{t\to\infty} t^{2n+3}/2n+3}$$

$$= \frac{-1}{4(n+1)^2} \frac{2n+3}{2n+5} t^2$$

as $n \to \infty$ $\frac{1}{4(n+1)^2} \frac{2n+3}{2n+5} \to 0$ $\therefore$

radius of convergence of integral is ∞. I don't regard the above as establishing anything.

We can evaluate the integrals from the Laplace transforms ✓