TMA 03 M828

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1) i) $\sin(2m+1)\theta = \sin\theta + 2\sum_{n=1}^{m} \sin\theta\cos(2n\theta)$

Concentrate on RHS

$2\sin\theta\cos 2n\theta = \sin(2n\theta+\theta) - \sin(2n\theta-\theta)$ ✓

$\therefore \sin\theta + 2\sum_{n=1}^{m} \sin\cos(2n\theta)$

$= \sin\theta + \sum_{n=1}^{m} \sin(2n+1)\theta - \sin(2n-1)\theta$

$= \sin\theta + \sin(2m+1)\theta - \sin(2m-1)\theta + \sin(2m-1)\theta + \sin(2m-3)\theta + \ldots + \sin 5\theta - \sin 3\theta + \sin 3\theta - \sin\theta$

Everything cancels ✓ apart from $\sin(2m+1)\theta$

$\therefore \frac{\sin(2m+1)\theta}{\sin\theta} = 1 + 2\sum_{n=1}^{m}\cos 2n\theta$ ✓

$$\int_0^{\pi/2} \frac{\sin(2m+1)\theta}{\sin\theta}\,d\theta = \int_0^{\pi/2}\left[1 + 2\sum_{n=1}^{m}\cos 2n\theta\right]d\theta$$

$$= \frac{\pi}{2} + 2\left[\frac{\sin 2n\theta}{2n}\right]_0^{\pi/2} = \frac{\pi}{2}$$ ✓

This is not contrary to Riemann Lebesgue Lemma, since $1/\sin\theta$ ✓ is not ✓ piecewise continuous on $[0, \pi/2]$

✓ But $\frac{1}{\sin\theta}$ is piecewise cts on $[\frac{\pi}{4}, \frac{\pi}{2}]$, so

When we apply ✓ the Riemann-Lebesgue Lemma to $\int_{\pi/4}^{\pi/2} \frac{\sin(2m+1)\theta}{\sin\theta}\,d\theta$