

(13)

$$\begin{aligned}
y(t) &= \frac{1}{\pi} \int_0^t \frac{1+2s}{(t-s)^{1/2}} ds \\
&= \frac{1}{\pi} \int_0^t \frac{ds}{(t-s)^{1/2}} + \frac{1}{\pi} \int_0^t \frac{2s}{(t-s)^{1/2}} ds \\
&\text{Sub } s = t \sin^2 \theta, ds = 2t \sin \theta \cos \theta d\theta \\
&= \frac{1}{\pi} \int_0^{\pi/2} \frac{2t \sin \theta \cos \theta d\theta}{t^{1/2} \sin \theta} + \frac{1}{\pi} \int_0^{\pi/2} \frac{4t^2 \sin^3 \theta \cos \theta d\theta}{t^{1/2} \cos \theta} \\
&= \frac{1}{\pi} \int_0^{\pi/2} 2t^{1/2} \cos \theta d\theta + \frac{1}{\pi} \int_0^{\pi/2} 4t^{3/2} \sin^3 \theta d\theta \\
&= \frac{2t^{1/2}}{\pi} + \frac{4t^{3/2}}{\pi} \int_0^{\pi/2} \sin^3 \theta d\theta \\
&= \frac{2t^{1/2}}{\pi} + \frac{4t^{3/2}}{\pi} \int_0^{\pi/2} \frac{3 \sin \theta}{4} - \frac{1 \sin 3\theta}{4} d\theta \\
&= \frac{2t^{1/2}}{\pi} + \frac{4t^{3/2}}{\pi} \left[ \frac{-3 \cos \theta}{4} + \frac{1 \cos 3\theta}{12} \right]_0^{\pi/2} \\
&= \frac{2t^{1/2}}{\pi} + \frac{4t^{3/2}}{\pi} \left[ 0 + 3 + 0 - 1 \right] \\
&= \frac{2t^{1/2}}{\pi} + \frac{8t^{3/2}}{3\pi} \quad \checkmark \text{ as req.}
\end{aligned}$$

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Again, using  $s = tx$ 

and the Note saves having to work out these integrals in detail.