$$= t + \frac{4t^2}{3\pi}\left(\frac{\pi}{2} - 0 + \frac{\pi}{4} + 0\right)$$

$$= t + \frac{4t^2}{3\pi} \times \frac{3\pi}{4} = t + t^2$$

15/15 ie $\int_0^t \frac{y(s)}{(t-s)^{\frac{1}{2}}} ds = t + t^2$ ✓

iii) $y(t) * \frac{1}{t^{\frac{1}{2}}} = \int_0^t \phi(s) ds$ (1)

$$L\left(y * \frac{1}{t^{\frac{1}{2}}}\right) = L\left(\int_0^t \phi(s) ds\right)$$ ✓

$$L(y)\, L(t^{-\frac{1}{2}}) = \frac{1}{u} L(\phi)$$

$$L(y) \cdot \frac{\Gamma(\frac{1}{2})}{t^{\frac{1}{2}}} = \frac{1}{u} L(\phi)$$ ✓

$$L(y) = \frac{1}{u^{\frac{1}{2}}\,\Gamma(\frac{1}{2})} L(\phi)$$

$$y = \frac{1}{\Gamma(\frac{1}{2})} L^{-1}\left(u^{-\frac{1}{2}} L(\phi)\right)$$

$$= \frac{1}{\sqrt{\pi}} L^{-1}\left(\frac{1}{\sqrt{\pi}} \cdot \sqrt{\pi}\, u^{-\frac{1}{2}} L(\phi)\right)$$

$$= \frac{1}{\pi} L^{-1}\left(L(t^{-\frac{1}{2}}) L(\phi)\right)$$

$$= \frac{1}{\pi} t^{-\frac{1}{2}} * \phi(t)$$

$$= \frac{1}{\pi} \int_0^t \frac{\phi(s)}{(t-s)^{\frac{1}{2}}} ds$$ as req. ✓

Comparing (1) with the integral equation gives $\phi = 1 + 2s$ ✓