(8)

Put $\sin\theta = \frac{(z - z^{-1})}{2i}$ $\quad dz = \frac{d\theta}{iz}$

Then $s^2 + \sin^2\theta = s^2 + \left(\frac{z - z^{-1}}{2i}\right)^2$

$= \frac{1}{4}\left(4s^2 - (z - z^{-1})^2\right)$

$= \frac{1}{4z^2}\left(4s^2 z - (z^2 - 1)^2\right)$ (diff. of squares)

$= \frac{1}{4z^2}(2sz - z^2 + 1)(2sz + z^2 - 1)$

$= \frac{-1}{4z^2}(z^2 - 2sz - 1)(z^2 + 2sz - 1)$

The first term has zeros at

$z = \frac{2s \pm \sqrt{4s^2 + 4}}{2} = s + \sqrt{s^2+1}, s - \sqrt{s^2+1}$

Only $s - \sqrt{s^2+1}$ is inside $|z| < 1$ for $s > 0$

The second term has zeros at

$z = \frac{-2s \pm \sqrt{4s^2 + 4}}{2} = -s + \sqrt{s^2+1}, -s - \sqrt{s^2+1}$

Only $-s + \sqrt{s^2+1}$ is inside $|z| < 1$ for $s > 0$

$L(J_0) = \frac{s}{\pi} \int \frac{-4z^2}{(z - \alpha_1)(z - \alpha_2)(z - \beta_1)(z - \beta_2)} \frac{dz}{iz}$

$= \frac{-4s}{i\pi} \int \frac{z\,dz}{(z - \alpha_1)(z - \alpha_2)(z - \beta_1)(z - \beta_2)}$

where $\alpha_1 = +s - \sqrt{s^2+1}$ $\quad \alpha_2 = +s + \sqrt{s^2+1}$

$\beta_1 = -s - \sqrt{s^2+1}$ $\quad \beta_2 = -s + \sqrt{s^2+1}$.

$L(J_0) = \frac{-4s}{\pi i} \cdot \pi i(\text{Res at } \alpha_1 + \text{Res at } \beta_2)$

$= -4s\left(\frac{s - \sqrt{s^2+1}}{(-2\sqrt{s^2+1})(+2s)(2s - 2\sqrt{s^2+1})} + \frac{-s + \sqrt{s^2+1}}{(-2s + 2\sqrt{s^2+1})(-2s)(2\sqrt{s^2+1})}\right)$