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$$\int^{C_2} = \int_{\Gamma_+} = \int_{1}^{-1} \frac{1}{(2i)^{2n}} \left(z - \frac{1}{z}\right)^{2n} \frac{dz}{iz}$$

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$$= a_n = \frac{(2n)!}{2^{2n}(n!)^2}\pi$$

$$\cos(t\sin\theta) = \sum \frac{(-1)^n}{(2n)!}(t\sin\theta)^{2n}$$

$$= \sum \frac{(-1)^n}{(2n)!} t^{2n} \sin^{2n}\theta$$

$$\frac{1}{\pi}\int_0^\pi \cos(t\sin\theta)\,d\theta = \frac{1}{\pi}\sum \frac{(-1)^n}{(2n)!} t^{2n} \int_0^\pi (\sin\theta)^{2n}\,d\theta$$

$$= \frac{1}{\pi}\sum \frac{(-1)^n}{(2n)!} t^{2n} \cdot \frac{(2n)!}{2^{2n}(n!)^2}\pi$$

$$= \sum \frac{(-1)^n}{2^{2n}(n!)^2} t^{2n} = J_0(t)$$

ie $J_0(t) = \frac{1}{\pi}\int_0^\pi \cos(t\sin\theta)\,d\theta.$

$$L(J_0(t)) = L\left(\frac{1}{\pi}\int_0^\pi \cos(t\sin\theta)\,d\theta\right)$$

$$= \frac{1}{\pi} L\left(\int_0^\pi \cos(t\sin\theta)\,d\theta\right)$$

$$= \frac{1}{\pi}\int_0^\infty e^{-st}\int_0^\pi \cos(t\sin\theta)\,d\theta\,dt$$

$$= \frac{1}{\pi}\int_0^\pi \int_0^\infty e^{-st}\cos(t\sin\theta)\,dt\,d\theta$$

$$= \frac{1}{\pi}\int_0^\pi \left[L(\cos t(\sin\theta))\right] d\theta$$

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$$= \frac{1}{\pi}\int_0^\pi \frac{s\,d\theta}{s^2+\sin^2\theta} = \frac{s}{\pi}\int_0^\pi \frac{d\theta}{s^2+\sin^2\theta}$$