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$$\int_0^\infty J_0(t)\,dt = \lim_{s\to 0}\int_0^\infty e^{-st} J_0(t)\,dt$$

$$= \lim_{s\to 0} L(J_0(t)) = \lim_{s\to 0} \frac{1}{\sqrt{1+s^2}} = 1$$

$$\int_0^\infty t^2 J_0(t)\,dt = \lim_{s\to 0}\int e^{-st}\, t^2 J_0(t)\,dt$$

$$= \lim_{s\to 0} L(t^2 J_0(t)) =$$

$$= \lim_{s\to 0} -\frac{d}{ds}\left(L(\cancel{t}\, J_0(t)\right)$$

$$= \lim_{s\to 0} -\frac{d}{ds}\left(-\frac{d}{ds}(L(J_0(t)))\right)$$

$$= \lim_{s\to 0} \frac{d^2}{ds^2}\left(\frac{1}{\sqrt{1+s^2}}\right)$$

$$= \lim_{s\to 0} \frac{d}{ds}\left(\frac{-2s}{2}(1+s^2)^{-3/2}\right)$$

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$$= \lim_{s\to 0}\left(-(1+s^2)^{-3/2} + 3s^2(1+s^2)^{-5/2}\right)$$

$$= -1$$

You should also comment on the convergence of these integrals. I guess your work Pro

2) $a_n = \int_0^\pi (\sin\theta)^{2n}\,d\theta$

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put $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$$= \frac{z - z^{-1}}{2i}, \quad z = e^{i\theta}$$

then $dz = ie^{i\theta}\,d\theta \Rightarrow d\theta = \dfrac{dz}{iz}$

$$a_n = \frac{1}{(2i)^{2n}}\oint \left(z - \frac{1}{z}\right)^{2n} \frac{1}{iz}\,dz$$

only residue is at $z=0$. Coefficient of $1/z$ is