(4)

$2a^2+2b^2+3c^2+2ab+4ac+4bc=1$

$2a^2+2(2a)^2+3(-2a)^2+2a(2a)+4a(-2a)+4(2a)(-2a)=1$

$2a^2=1 \Rightarrow a=1/\sqrt{2}$

$\Rightarrow \hat{e}_2=(1/\sqrt{2}, \sqrt{2}, -\sqrt{2})$.

the basis is $\left\{ \begin{pmatrix} -\sqrt{3/2} \\ 0 \\ \sqrt{2/3} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ \sqrt{2} \\ -\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1/\sqrt{3} \end{pmatrix} \right\}$ ✓ 6

iv) $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ is an orthonormal basis so will

$g(\hat{e}_a)=\hat{e}^a$, $\Omega=\hat{e}^1\wedge\hat{e}^2\wedge\hat{e}^3$

$g(\hat{e}_1)=(-\sqrt{3/2}, 0, \sqrt{2/3})\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$

$=+\sqrt{2/3}(-1.5, 0, 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}=\sqrt{2/3}(-1, 0.5, 0)$

$g(\hat{e}_2)=(1/\sqrt{2}, \sqrt{2}, -\sqrt{2})\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$

$=(1 \;\; 2 \;\; -2)\frac{}{\sqrt{2}}\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}=\frac{1}{\sqrt{2}}(0, 1, 0)$

$=(0, 1/\sqrt{2}, 0)$

$g(\hat{e}_3)=(0, 0, 1/\sqrt{3})\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}=\frac{1}{\sqrt{3}}(2, 2, 3)$

$\Omega=g(\hat{e}_1)\wedge g(\hat{e}_2)\wedge g(\hat{e}_3)$

$=\sqrt{2/3}(-1, 0.5, 0)\wedge\frac{1}{\sqrt{2}}(0, 1, 0)\wedge\frac{1}{\sqrt{3}}(2, 2, 3)$

$=\frac{1}{\sqrt{6}}(-2, 1, 0)\wedge(0, \frac{1}{\sqrt{2}}, 0)\wedge(\frac{2, 2, 3}{\sqrt{3}})$

$=\frac{1}{6}(-2\theta^1+\theta^2)\wedge(\theta^2)\wedge(2\theta^1+2\theta^2+3\theta^3)$