(1)

M827 TMA 03

1) $\langle v+w, v-w\rangle = \langle v, v-w\rangle + \langle w, v-w\rangle$
$= \langle v, v\rangle - \langle v, w\rangle + \langle w, v\rangle - \langle w, w\rangle$
$= \langle v, v\rangle - \langle v, w\rangle - \langle v, w\rangle - \langle w, w\rangle$
$= 0.$
$\Rightarrow v+w, v-w$ are orthogonal. ✓ 2

ii) $g(f_1, e_3) = f_1^T G e_3$
$= (k_1, 0, 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (k_1, 0, 1)\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$
$= 2k_1 + 3 \Rightarrow k_1 = -3/2$ for f_1, e_3 to be orthogonal. ✓
$g(f_2, e_3) = f_2^T G e_3 =$
$= (0, k_2, 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (0, k_2, 1)\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$
$= 2k_2 + 3 \Rightarrow k_2 = -3/2$ for f_2, e_3 to be orthogonal. ✓
With $f_1 = (-1.5, 0, 1)$ $f_2 = (0, -1.5, 1)$
$\{f_1, f_2\}$ is a linearly independent set of dimension 2. If $v = v_1 f_1 + v_2 f_2$ and $w = w_1 e_3$
$g(v, w) = g(v_1 f_1 + v_2 f_2, w_1 e_3)$
$= g(v_1 f_1, w_1 e_3) + g(v_2 f_2, w_1 e_3)$
$= v_1 w_1 g(f_1, e_3) + v_2 w_1 g(f_2, e_3)$
$= 0$
Obviously vectors of the form of v span the orthogonal complement to the ~~sub~~space spanned by e_3
$\therefore \{f_1, f_2\}$ is the required basis ✓