17

$d(L_X\theta^1) + L_X(\omega^1_a)\wedge\theta^a + \omega^1_a \wedge L_X(\theta^1) + \omega^1_2\wedge L_X(\theta^2) = 0$

$d(\lambda\theta^2) + L_X(\omega^1_1)\wedge\theta^1 + L_X(\omega^1_2)\wedge\theta^2 + \omega^1_1\wedge\lambda\theta^2 - \omega^1_2\wedge\lambda\theta^1 = 0$

$d\lambda\wedge\theta^2 + \lambda d\theta^2 + L_X(\omega^1_1)\wedge\theta^1 + L_X(\omega^1_2)\wedge\theta^2 + \omega^1_1\wedge\lambda\theta^2$
$- \omega^1_2\wedge\lambda\theta^1 = 0$

$d\lambda\wedge\theta^2 + \lambda(-\omega^2_1\wedge\theta^1) + L_X(\omega^1_1)\wedge\theta^1 + L_X(\omega^1_2)\wedge\theta^2$
$+ \omega^1_1\wedge\lambda\theta^2 - \omega^1_2\wedge\lambda\theta^1 = 0$

$(d\lambda + L_X(\omega^1_2) + \lambda\omega^1_1)\wedge\theta^2 + (-\lambda\omega^2_1 + L_X(\omega^1_1) - \lambda\omega^1_2)\wedge\theta^1 = 0$

By Exercise 52 chapter seven

$\omega^1_1 = 0, \ \omega^1_2 = -\omega^2_1$ for orthonormal basis

$d\lambda + L_X(\omega^1_2) = 0$, since second term vanishes

$\Rightarrow -L_X(\omega^1_2) = L_X(\omega^2_1) = d\lambda$. OK - but do read the solution

Put $\omega = \omega^2_1$ then $L_X(\omega) = d\lambda$. which is clearer 5

v) $dL_X(\omega) = d^2\lambda$

d commutes with Lie derivative

$dL_X(\omega) = L_X(d\omega) = d^2\lambda = 0$, automatically

But $d\omega = -K\theta^1\wedge\theta^2$

$L_X(-K\theta^1\wedge\theta^2) = 0$

$= L_X(-K)\theta^1\wedge\theta^2 - K(L_X\theta^1)\wedge\theta^2$
$- K\theta^1\wedge L_X\theta^2 = 0$

$L_X(-K)\theta^1\wedge\theta^2 - K\lambda\theta^2\wedge\theta^2 - K\theta^1\wedge(-\lambda\theta^1) = 0$

The second two terms are both zero $\Rightarrow L_X(-K)\theta^1\wedge\theta^2 = 0$ ✓

$\Rightarrow L_X(-K) = 0$ since $\theta^1\wedge\theta^2$ is not zero everywhere ($\theta^1\wedge\theta^2(U_1, U_2) = 1$ everywhere on the domain of $U_1, U_2, \theta^1, \theta^2$.

3

20