Question 5 (20 marks)

It can be shown that coordinates may be introduced on any (abstract) surface such that the metric takes the form

$$ds^2 = (d\xi^1)^2 + G(\xi^1, \xi^2)^2 (d\xi^2)^2.$$

(i) The coefficient of $(d\xi^2)^2$ is expressed as the square of the function G for ease of calculation. Explain why this form of the expression is justified. [5]

(ii) Compute the connection coefficients of the Levi-Civita connection for this surface, expressing your answers in terms of G and its partial derivatives. [5]

(iii) Show that the ξ^1 coordinate curves $t \mapsto (t, \text{constant})$ are geodesics. [5]

(iv) Calculate R^1_{212} and deduce that the Gaussian curvature K is given by

$$K = -\frac{1}{G}\frac{\partial^2 G}{\partial \xi^{1 2}}.$$

[5]

$$dt = \frac{\partial f}{\partial \xi^1} d\xi^1 + \frac{\partial f}{\partial \xi^2} d\xi^2 = 0$$

$$(ds)^2 \quad (V_1 \;\; V_2)\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} V_1 \end{pmatrix}$$