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If we put $T_1=\begin{pmatrix}1\\0\\0\end{pmatrix}, T_2=\begin{pmatrix}0\\1\\0\end{pmatrix}, T_3=\begin{pmatrix}0\\0\\1\end{pmatrix}$

Then every transitive generator is a linear combination of these T_i, and a generator for ρ_t is the set

$$\left\{\begin{bmatrix}0&-1&0\\+1&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&0&0\\-1&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&-1\\0&1&0\end{bmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

and every linear combination of these is the infinitesimal generator of some ρ_t. 3

I don't see a formula expressing the $\frac{d}{dt}(\rho_t)\Big|_{t=0}$ in terms of X_a, T_a (half marks OK)

iv) The generator of ... is

$\gamma\cos\theta X_1 + \gamma\sin\theta\sin\phi X_2 + \gamma\sin\theta\cos\phi X_3 + v^1T_1 + v^2T_2 + v^3T_3 = \xi^1X_1 + \xi^2X_2 + \xi^3X_3 + \eta^1T^1 + \eta^2T^2 + \eta^3T^3$

Identify η^i with v_i, $i=1,2,3$.

$\gamma\cos\theta = \xi^1$, $\gamma\sin\theta\sin\phi = \xi^2$, $\gamma\sin\theta\cos\phi = \xi^3$

$\gamma = \sqrt{(\xi^1)^2+(\xi^2)^2+(\xi^3)^2}$

$\Rightarrow \cos\theta = \dfrac{\xi^1}{\sqrt{(\xi^1)^2+(\xi^2)^2+(\xi^3)^2}}$ ✓

$\Rightarrow$

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$\dfrac{\gamma\sin\theta\sin\phi}{\gamma\sin\theta\cos\phi} = \dfrac{\xi^2}{\xi^3} = \tan\phi$

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Note we take γ as +ve. If γ is −ve about an axis, then $\cos\theta = -\dfrac{\xi^1}{+\sqrt{(\xi^1)^2+(\xi^2)^2+(\xi^3)^2}} = \cos(\pi+\theta)$

then γ will be +ve about the axis rotated by π.