$\mathbf{n} = r_u \times r_v / \|r_u \times r_v\| = \left(2u/d, 2v/d, (u^2+v^2-1)/d\right)$

$n_u = \left(1+v^2-u^2, -2uv, 2u\right)2/d^2$

$n_v = \left(2uv, 1-u^2+v^2, -2v\right)\left(-2/d^2\right)$

But $n_u = k.\mathbf{r}_u$ and $n_v = -k.\mathbf{r}_v$, where $k = \left(-2/d^2\right)$,

so the shape operator is $\begin{pmatrix} k & 0 \\ 0 & -k \end{pmatrix}$, which has trace = 0, so the mean curvature of the surface is 0.

Question 5

(i) For a general metric $Edu^2 + 2Fdudv + Gdv^2$, $\frac{\partial}{\partial u}.\frac{\partial}{\partial u} = E$, and $\frac{\partial}{\partial v}.\frac{\partial}{\partial v} = G$, and

$EG - F^2 > 0$. So in the present case, $F = 0$, and $E = 1$, so the given G must be positive, so it can be written as a square.

(ii) $\Gamma^a_{bc} = \frac{1}{2}\left\{g^{a1}\left(\partial_b g_{c1} + \partial_c g_{b1} - \partial_1 g_{bc}\right) + g^{a2}\left(\partial_b g_{c2} + \partial_c g_{b2} - \partial_2 g_{bc}\right)\right\}$,

where $\left(g_{ij}\right) = \begin{pmatrix} 1 & 0 \\ 0 & G^2 \end{pmatrix}$ and $\left(g^{ij}\right)$ is the inverse of this, to wit, $\begin{pmatrix} 1 & 0 \\ 0 & G^{-2} \end{pmatrix}$

The formula for Since g_{11}, g_{12}, g_{21} are constant, as are g^{11}, g^{12}, g^{21}, differentiating them gives 0. In calculating each Γ^1_{bc} various terms the first bracket vanishes unless $b = c = 2$ and the second bracket vanishes because $g^{12} = 0$. The result is $\Gamma^1_{22} = \frac{-1}{2}\partial_1 G^2 = -G\partial_1 G$, and all other Γ^1_{bc} vanish.

The Γ^2_{bc}'s are more varied. They work out to be $\Gamma^2_{11} = 0$, $\Gamma^2_{12} = \Gamma^2_{21} = G^{-1}\partial_1 G$, $\Gamma^2_{22} = G^{-1}\partial_2 G$.

(iii) The geodesic equations are $\frac{d^2\xi^a}{dt^2} + \Gamma^a_{bc}\frac{d\xi^a}{dt}\frac{d\xi^c}{dt} = 0$. We have to check they are satisfied by $\xi^1 = t$, $\xi^2 = c$. The only terms that remain after the fact that the only non-zero derivative of a ξ is $\frac{d\xi^a}{dt} = 1$ involve Γ^a_{11}, but these are both zero. So the equation is satisfied.