$$= \frac{1}{6}(-2\theta^1 \wedge \theta^2) \wedge (2\theta^1 + 2\theta^2 + 3\theta^3)$$

$$= \frac{1}{6}(-2\theta^1 \wedge \theta^2 \wedge 3\theta^3) = -\theta^1 \wedge \theta^2 \wedge \theta^3$$

The transition matrix from $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ to $\{e_1, e_2, e_3\}$ is

$$\begin{pmatrix} -\sqrt{3}/2 & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 \\ \sqrt{3}/2 & -\sqrt{2} & 1/\sqrt{3} \end{pmatrix}$$ with determinant

-1, so we have $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ is $-$vely oriented and the volume form has a $-$ve sign to take account of this change the sign of $\hat{e}_3$

3 ✓ so we have a new basis $\{\hat{e}_1, \hat{e}_2, -\hat{e}_3\}$ e_3 and the volume form is $\Omega = \theta^1 \wedge \theta^2 \wedge \theta^3$ positively oriented wrt $\{e_1, e_2, e_3\}$

v) $*(e^3) = g^{-1}(e^3) \lrcorner (\theta^1 \wedge \theta^2 \wedge \theta^3)$

$$= e_3 \lrcorner (\theta^1 \wedge \theta^2 \wedge \theta^3)$$

$$= \langle e_3, \theta^1 \rangle \theta^2 \wedge \theta^3 - \langle e_3, \theta^2 \rangle \theta^1 \wedge \theta^3 + \langle e_3, \theta^3 \rangle \theta^1 \wedge \theta^2$$

$$= 0 - 0 + 1 \cdot \theta^1 \wedge \theta^2$$

$$= \theta^1 \wedge \theta^2$$

θ^1 is a one form and $\{e^1, e^2, e^3\}$ is a basis for one forms so we can express θ^1 in terms of $\{e^1, e^2, e^3\}$ and similarly for θ^2 and θ^3

$$\theta^1 = ae^1 + be^2 + ce^3$$

$$= ag(e_1) + bg(e_2) + cg(e_3)$$

$$= a\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}^T + b\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}^T + c\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}^T$$