22

$$\Gamma^2_{11} = \frac{g^{2d}}{2}\left(\frac{\partial g_{1d}}{\partial x^1} + \frac{\partial g_{1d}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^d}\right) = 0.$$

$$\Gamma^2_{12} = \frac{g^{2d}}{2}\left(\frac{\partial g_{2d}}{\partial x^1} + \frac{\partial g_{1d}}{\partial x^2} - \frac{\partial g_{12}}{\partial x^d}\right)$$

$d = 2$, from that $g^{2d} = 0$ unless $d = 2$

$$\Gamma^2_{12} = \frac{g^{2d}}{2}\left(\frac{\partial g_{2d}}{\partial x^1} + \frac{\partial g_{1d}}{\partial x^d} - \frac{\partial g_{12}}{\partial x^d}\right)$$

$d = 2$ again

$$\Gamma^2_{12} = \frac{1}{2G^2} 2G \frac{\partial G}{\partial \xi^1} = \frac{1}{G}\frac{\partial G}{\partial \xi^1}$$

$$\Gamma^2_{22} = \frac{g^{2d}}{2}\left(\frac{\partial g_{2d}}{\partial x^2} + \frac{\partial g_{2d}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^d}\right)$$

$d = 2$ again

$$\Gamma^2_{22} = \frac{1}{2G^2}\left(2G\frac{\partial G}{\partial \xi^2} + 2G\frac{\partial G}{\partial \xi^2} - 2G\frac{\partial G}{\partial \xi^2}\right)$$

$$= \frac{1}{G}\frac{\partial G}{\partial \xi^2}$$

5

Then $\Gamma^1_{11} = \Gamma^1_{21} = \Gamma^2_{11} = 0$

$\Gamma^1_{22} = -G\frac{\partial G}{\partial \xi^1}$, $\Gamma^2_{12} = \frac{1}{G}\frac{\partial G}{\partial \xi^1}$ $\Gamma^2_{22} = \frac{1}{G}\frac{\partial G}{\partial \xi^2}$

iii) The geodesic equations are

$$\frac{d^2 y^1}{dt^2} + \Gamma^1_{11}\left(\frac{dy^1}{dt}\right)^2 + 2\Gamma^1_{21}\frac{dy^1}{dt}\frac{dy^2}{dt} + \Gamma^1_{22}\left(\frac{dy^2}{dt}\right)^2 = 0$$

$\Gamma^1_{11} = \Gamma^1_{21} = 0 \Rightarrow$

$$\frac{d^2 y^1}{dt^2} + \Gamma^1_{22}\left(\frac{dy^2}{dt}\right)^2 = 0$$

$$\frac{d^2 y^2}{dt^2} + \Gamma^2_{11}\left(\frac{dy^1}{dt}\right)^2 + 2\Gamma^2_{21}\left(\frac{dy^1}{dt}\frac{dy^2}{dt}\right) + \Gamma^2_{22}\left(\frac{dy^2}{dt}\right)^2 = 0.$$

$\Gamma^2_{11} = 0$

$$\frac{d^2 y^2}{dt^2} + 2\Gamma^2_{21}\left(\frac{dy^1}{dt}\frac{dy^2}{dt}\right) + \Gamma^2_{22}\left(\frac{dy^2}{dt}\right)^2 = 0$$

5 see back of sheet

PTO.