(2)

$\|f_1\|^2 = g(f_1, f_1)$

$= (-1.5, 0, 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} -1.5 \\ 0 \\ 1 \end{pmatrix} = (-1, 0.5, 0)\begin{pmatrix} -1.5 \\ 0 \\ 1 \end{pmatrix}$

$= 1.5 \Rightarrow \|f_1\| = \sqrt{3/2}$

$\|f_2\|^2 = g(f_2, f_2) = (0, -3/2, 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ -3/2 \\ 1 \end{pmatrix}$

$= (0.5, -1, 0)\begin{pmatrix} 0 \\ -1.5 \\ 1 \end{pmatrix} = 1.5 \Rightarrow \|f_2\| = \sqrt{3/2}$

$\cos\theta = \dfrac{g(f_1, f_2)}{\|f_1\|\|f_2\|} = \dfrac{(-1.5, 0, 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ -1.5 \\ 1 \end{pmatrix}}{(\sqrt{3/2})^2}$

$= \frac{2}{3}(-1, 0.5, 0)\begin{pmatrix} 0 \\ -1.5 \\ 1 \end{pmatrix} = 2/3 \cdot -0.75 = -\frac{1}{2}$

ie $\cos\theta = -1/2$, θ = angle between f_1, f_2 ✓ 6

iii) $g(e_3, e_3) = g_{33} = 3$.

Put $\hat{e}_3 = k e_3$ then $g(\hat{e}_3, \hat{e}_3) = k^2 g(e_3, e_3) = 3k^2 = 1$ ✓
then $k = 1/\sqrt{3} \Rightarrow \hat{e}_3 = (0, 0, 1/\sqrt{3})$.
f_1, f_2 are both orthogonal to e_3, hence also to $\hat{e}_3$. Take $\hat{e}_1$ to be a scalar multiple of f_1 such that $g(\hat{e}_1, \hat{e}_1) = 1$
Then complete the basis by finding $\hat{e}_2$ with $g(\hat{e}_2, \hat{e}_2) = 1$
$g(\hat{e}_2, \hat{e}_1) = 0$
$g(\hat{e}_2, \hat{e}_3) = 0$

Put $\hat{e}_1 = k f_1$