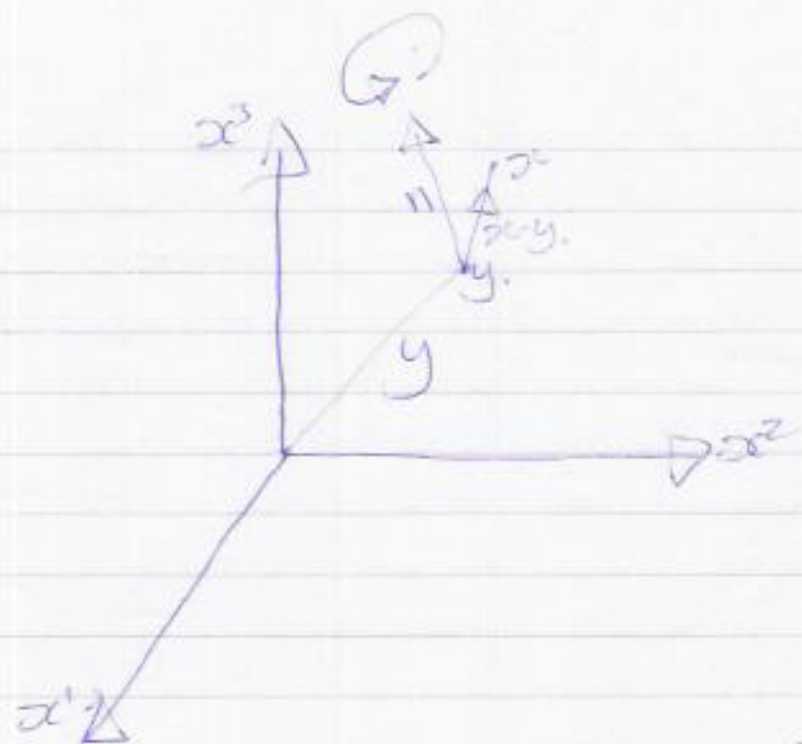


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Rotations preserve lengths so that  $|R(x-y)| = |x-y|$ , and  $p_+(y) = y + R(y-y) = y$ , so  $R$  fixes the point  $y$ . So for all  $t$ ,  $p_+(x)$  is a vector  $R_t(x-y)$  of length  $|x-y|$  attached at the point  $y$ . More over, we can resolve  $x-y$  into two parts: that <sup>part</sup> parallel to  $n$ , and that part perpendicular to  $n$ . Call the parallel part  $(x-y)_||$ , and the perpendicular part  $(x-y)_\perp$ .

$R_t$  leaves  $n$  fixed, and any multiple of  $n$ , including  $(x-y)_||$ , so  $R_t(x-y)_||$  is a vector length  $|(x-y)_|||$  attached at the point  $y$  in direction  $n$ . The length of  $R_t(x-y)_\perp$  is also preserved, then Pythagoras theorem implies  $R_t(x-y)_\perp$  is perpendicular to  $R_t(x-y)_||$ , so  $R_t(x-y)$  rotates the vector  $(x-y)_\perp$  around the point of contact  $y + (x-y)_||$ , keeping it perpendicular to  $(x-y)_||$ .

\*Since length  $R_t(x-y)$  preserved