(28)

5) i) The coefficient of $(d\xi^2)^2$ must be positive since if it is negative then on a curve along the ξ^2-axis, $(ds)^2 < 0$ ie all distances are negative, which is obviously impossible, so we write G^2 instead of G to emphasise its positiveness. ($G \neq 0$ anywhere since then we would have a degenerate metric.)

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ii) $g_{11} = 1 \quad g_{22} = G^2, \; g_{12} = g_{21} = 0$

The connection coefficients we must find are $\Gamma^1_{11}, \Gamma^1_{21}, \Gamma^1_{22}, \Gamma^2_{11}, \Gamma^2_{12}, \Gamma^2_{22}$ (note $\Gamma^1_{12} = \Gamma^1_{21}$, $\Gamma^2_{12} = \Gamma^2_{21}$).

$$\Gamma^1_{11} = \frac{g^{1d}}{2}\left(\frac{\partial g_{1d}}{\partial x^1} + \frac{\partial g_{1d}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^d}\right) = 0$$

Since must have $d = 1$ and $g_{11} = 1$.

$$\Gamma^1_{21} = \frac{g^{1d}}{2}\left(\frac{\partial g_{1d}}{\partial x^1} + \frac{\partial g_{2d}}{\partial x^1} - \frac{\partial g_{21}}{\partial x^d}\right),$$ again must have $d = 1$

$$\Gamma^1_{21} = \frac{1}{2}\left(0 + 0 - 0\right) = 0$$

$$\Gamma^1_{22} = \frac{g^{1d}}{2}\left(\frac{\partial g_{2d}}{\partial x^2} + \frac{\partial g_{2d}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^d}\right)$$

Must have $d = 1$

$$\Gamma^1_{22} = \frac{g^{11}}{2}\left(\frac{\partial g_{21}}{\partial x^2} + \frac{\partial g_{21}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^1}\right) = -G\frac{\partial G}{\partial \xi^1}$$