$g(\hat{e}_1,\hat{e}_1) = g(kf_1, kf_1) = k^2 g(f_1, f_1) = \frac{3k^2}{2}$ from ii)

We require $g(\hat{e}_1,\hat{e}_1) = \frac{3k^2}{2} = 1 \Rightarrow k = \sqrt{\frac{2}{3}}$

$\Rightarrow \hat{e}_1 = (-\sqrt{3/2}, 0, \sqrt{2/3})$

Put $\hat{e}_2 = (a, b, c)$

$$g(\hat{e}_2,\hat{e}_2) = (a, b, c)\begin{pmatrix}2 & 1 & 2\\ 1 & 2 & 2\\ 2 & 2 & 3\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}$$

$$= (2a+b+2c, a+2b+2c, 2a+2b+3c)\begin{pmatrix}a\\ b\\ c\end{pmatrix}$$

$$= 2a^2 + ab + 2ac + ab + 2b^2 + 2bc + 2ac + 2bc + 3c^2$$
$$= 2a^2 + 2b^2 + 3c^2 + 2ab + 4ac + 4bc = 1$$

for a more direct method, see solutions

$g(\hat{e}_2,\hat{e}_1) = g(\hat{e}_2, e_1) = 0$

$$(a, b, c)\begin{pmatrix}2 & 1 & 2\\ 1 & 2 & 2\\ 2 & 2 & 3\end{pmatrix}\begin{pmatrix}-1.5\\ 0\\ 1\end{pmatrix} = (a, b, c)\begin{pmatrix}-1\\ 0.5\\ 0\end{pmatrix}$$

$$= -a + 0.5b = 0 \Rightarrow -2a + b = 0$$

$g(\hat{e}_2, \hat{e}_3) = g(\hat{e}_2, e_3) = 0$

$$(a, b, c)\begin{pmatrix}2 & 1 & 2\\ 1 & 2 & 2\\ 2 & 2 & 3\end{pmatrix}\begin{pmatrix}0\\ 0\\ 1\end{pmatrix} = (a, b, c)\begin{pmatrix}2\\ 2\\ 3\end{pmatrix}$$

$$= 2a + 2b + 3c = 0$$

We have 3 eqs.

$-2a + b = 0 \Rightarrow b = 2a$

$2a + 2b + 3c = 0 \Rightarrow 6a + 3c = 0 \Rightarrow 2a + c = 0$

$\Rightarrow c = -2a$

Then $\hat{e}_2$ takes the form $(a, 2a, -2a)$.