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$= (x^3\partial_2 + x^2\partial_3)(x^1+x^3)(dx^1\wedge dx^3)$
$+ (x^1+x^3)(0 + dx^1\wedge dx^2)$
$= x^2\, dx^1\wedge dx^3 + (x^1+x^3)dx^1\wedge dx^2$

$L_V((x^2+x^3)dx^2\wedge dx^3)$
$= (L_V(x^2+x^3))(dx^2\wedge dx^3) + (x^2+x^3)(dV^2\wedge dx^3 + dx^2\wedge dV^3)$
$= (x^3\partial_2 + x^2\partial_3)(x^2+x^3)(dx^2\wedge dx^3) + (x^2+x^3)(0+0)$
$= (x^2+x^3)dx^2\wedge dx^3$

$\therefore L_V\omega = \cancel{x^3 dx^1\wedge dx^2} + x^1 dx^1\wedge dx^3 + \cancel{x^2 dx^1\wedge dx^3}$
$- \cancel{x^2 dx^1\wedge dx^3} - x^1 dx^1\wedge dx^2 - \cancel{x^3 dx^1\wedge dx^2}$
$+ x^2 dx^2\wedge dx^3 + x^3 dx^2\wedge dx^3$
$= -x^1 dx^1\wedge dx^2 + x^1 dx^1\wedge dx^3 + (x^2+x^3)dx^2\wedge dx^3$

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iii) $d\omega = d(x^1+x^2)\wedge dx^1\wedge dx^2$
$- d(x^1+x^3)\wedge dx^1\wedge dx^3 + d(x^2+x^3)\wedge dx^2\wedge dx^3$
$= 0$ since every term is of the form $dx^a\wedge dx^a\wedge dx^b$
or $dx^a\wedge dx^b\wedge dx^a$.

$(x^1+x^2)dx^1\wedge dx^2 = d(\tfrac{1}{2}(x^1)^2 dx^2 - \tfrac{1}{2}(x^2)^2 dx^1)$
$(x^1+x^3)dx^1\wedge dx^3 = d(\tfrac{1}{2}(x^1)^2 dx^3 - \tfrac{1}{2}(x^3)^2 dx^1)$
$(x^2+x^3)dx^2\wedge dx^3 = d(\tfrac{1}{2}(x^2)^2 dx^3 - \tfrac{1}{2}(x^3)^2 dx^2)$.
So the one form we require
is $\theta = -\frac{1}{2}((x^2)^2 - (x^3)^2)dx^1 + \frac{1}{2}((x^1)^2 - (x^3)^2)dx^2$
$+ \frac{1}{2}(-(x^1)^2 + (x^2)^2)dx^3$

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iv) ~~iii)~~ $W \lrcorner \Omega = (W_1\partial_1 + W_2\partial_2 + W_3\partial_3)(dx^1\wedge dx^2\wedge dx^3)$
$= W_1 dx^2\wedge dx^3 - W_2 dx^1\wedge dx^3 + W^3 dx^1\wedge dx^2$
$= (x^1+x^2)dx^1\wedge dx^2 - (x^1+x^3)dx^1\wedge dx^3$
$+ (x^2+x^3)dx^2\wedge dx^3$