$= b_1(a_1)b_2\omega_2(X)$, so the LHS and the RHS agree.

(iii) LHS $= L_{a_1\partial_1}(f\omega_1)(X) = a_1\partial_1(f\omega_1 X) - f\omega_1([a_1\partial_1, X])$

$= a_1\partial_1(f)\omega_1(X) + fa_1\partial_1(\omega_1 X) - f\omega_1([a_1\partial_1, X])$

RHS $= a_1\partial_1(f)\omega_1(X) + fa_1\partial_1(\omega_1 X) - f\omega_1([a_1\partial_1, X])$

So LHS = RHS as claimed.

(iv) Use the result that $(L_V\omega)(X) = V(\omega X) - \omega([V, X])$ to argue that

LHS $= a_1\partial_1(b_1\omega_1(X)) - b_1\omega_1([a_1\partial_1, c_1\partial_1 + c_2\partial_2])$

$= a_1\partial_1(b_1c_1) - b_1(a_1\partial_1 c_1) + b_1 c_1(\partial_1 a_1) + b_1 c_2(\partial_2 a_1)$

$= a_1\partial_1(b_1)(c_1) + a_1b_1\partial_1(c_1) - b_1(a_1\partial_1 c_1) + b_1 c_1(\partial_1 a_1) + b_1 c_2(\partial_2 a_1)$

$= a_1\partial_1(b_1)(c_1) + b_1 c_1(\partial_1 a_1) + b_1 c_2(\partial_2 a_1)$

RHS $= d(a_1b_1)X + a_1\partial_1 \rfloor d(b_1\omega_1)(X)$

$= (\partial_1(a_1b_1)\omega_1 + \partial_2(a_1b_1)\omega_2)X + a_1\partial_1 \rfloor (-\partial_2 b_1)(\omega_1 \wedge \omega_2)(X)$

$= (\partial_1(a_1b_1)\omega_1 + \partial_2(a_1b_1)\omega_2)X - \partial_2 b_1(\omega_1 \wedge \omega_2)(a_1\partial_1, X)$

$= c_1\,\partial_1(a_1b_1) + c_2\partial_2(a_1b_1) - (\partial_2 b_1)a_1c_2$

$= c_1\,(\partial_1(a_1)b_1 + a_1\partial_1 b_1) + c_2(\partial_2 a_1(b_1) + a_1\partial_2 b_1) - (\partial_2 b_1)a_1c_2$

$= c_1\,(\partial_1(a_1)b_1 + a_1\partial_1 b_1) + c_2\partial_2 a_1(b_1)$.

so the LHS and the RHS agree.

Question 2

These are largely routine calculations, using $\theta^i e_j = \delta_{ij}$.

The answers are:

(i) $\dot{\alpha} \wedge \beta = 3\theta^1\theta^2\theta^3 - 4\theta^1\theta^2\theta^4 - \theta^1\theta^3\theta^4 + \theta^2\theta^3\theta^4$

(ii) $v_1 \rfloor \beta = \theta + \theta^2 + \theta^3 - 2\theta^4$

(iii) $\beta(v_2, v_3) = 0$

(iv) It may be worth pointing out that $\beta \wedge \beta = 2\theta^1 \wedge \theta^2 \wedge \theta^3 \wedge \theta^4$ is not zero. This can happen because β is of even degree. You now have to compute det$\langle \theta^i, v_j \rangle$, which works out to be 9, so the answer is 2.9 = 18.

Question 3