

$$-0 \cdot 1 - 1 \cdot 1 = -1 = 0$$

$$\begin{aligned} \text{iv) } \beta \wedge \beta &= (\theta^1 \wedge \theta^2 + \theta^1 \wedge \theta^3 - \theta^2 \wedge \theta^4) \\ &= \wedge (\theta^1 \wedge \theta^2 + \theta^1 \wedge \theta^3 - \theta^2 \wedge \theta^4) \\ &= \theta^1 \wedge \theta^2 \wedge \theta^1 \wedge \theta^2 + \theta^1 \wedge \theta^2 \wedge \theta^1 \wedge \theta^3 \\ &\quad - \theta^1 \wedge \theta^2 \wedge \theta^2 \wedge \theta^4 + \theta^1 \wedge \theta^3 \wedge \theta^1 \wedge \theta^3 + \theta^1 \wedge \theta^3 \wedge \theta^2 \wedge \theta^4 \\ &\quad - \theta^2 \wedge \theta^4 \wedge \theta^1 \wedge \theta^2 - \theta^2 \wedge \theta^4 \wedge \theta^1 \wedge \theta^3 \\ &\quad + \theta^2 \wedge \theta^4 \wedge \theta^2 \wedge \theta^4 \end{aligned}$$

$$\begin{aligned} &= -\theta^1 \wedge \theta^3 \wedge \theta^2 \wedge \theta^4 - \theta^2 \wedge \theta^4 \wedge \theta^1 \wedge \theta^3 \\ &= \theta^1 \wedge \theta^2 \wedge \theta^3 \wedge \theta^4 - \theta^1 \wedge \theta^2 \wedge \theta^4 \wedge \theta^3 \\ &= 2\theta^1 \wedge \theta^2 \wedge \theta^3 \wedge \theta^4 \end{aligned}$$

$$\begin{aligned} \Rightarrow \beta \wedge \beta(v_1, v_2, v_3, v_4) &= 2\theta^1 \wedge \theta^2 \wedge \theta^3 \wedge \theta^4(v_1, v_2, v_3, v_4) \\ &= 2 \det(\langle v_i, \theta^j \rangle) \text{ see opp.} \end{aligned}$$

3) i) If λ, μ are vector valued 3 forms on a vector space V , define $\lambda(v_1, v_2, v_3) + \mu(v_1, v_2, v_3)$
 $= (\lambda + \mu)(v_1, v_2, v_3)$
 $\Rightarrow \lambda + \mu$ is a vector valued 3 form.

Define also, for $c \in \mathbb{R}$
 $(c \cdot \lambda)(v_1, v_2, v_3) = c(\lambda(v_1, v_2, v_3))$
 then $c \cdot \lambda$ is a vector valued 3-form.

Also the zero vector valued three form is a member of this space, since $0 \in V \Rightarrow 0(v_1, v_2, v_3) = 0$ and $0(v_1, v_2, v_3) \in L$.
 \Rightarrow set of vector valued 3-forms is a vector space.