

- (i) d is an anti-derivation of degree 1;
 L_V is a derivation of degree 0;
 $V\rfloor$ is an anti-derivation of degree -1;

(ii) Either argue by induction or computation.

Induction: The step from p to $p+1$ can be done by noticing that it is enough to consider the effect on a single $(p+1)$ -form, and this is a product of a 1-form and a p -form. For both of these it is true that $D_1 = D_2$.

Computation: You can write the effect of D on a p -form as the sum of p terms where in each one it acts on a 1-form. On these, it is true that $D_1 = D_2$.

- (iii) Compute $D(\alpha \wedge \beta)$, where α is a p -form. When you meet terms like $D_1(D_2\alpha \wedge \beta)$ remember that $D_2\alpha$ is a $(k_2 + p)$ -form. Because the k 's are odd you will find that the terms you don't want occur in cancelling pairs.

- (iv) You can show that D_V is a derivative by calculation. But the better answer is to observe that (by

(i)) $V\rfloor$ and d can be taken as D_1 and D_2 as in (iii).

To show that $D_V = L_V$ it is now enough to show that they agree on functions (trivial) and on 1-forms.

This is not entirely trivial, but any 1-form is a sum of expressions like $f dx$, so it is enough to check it on them, and that is a simple calculation of functions. On 1-forms, it is enough, by linearity, to work with $f dx$, where $x = x^i$.

$$D_V(f dx) = V\rfloor d(f dx) + d(V\rfloor f dx) = V\rfloor df \wedge dx + d(f \langle V, dx \rangle) = V\rfloor df \wedge dx + V_1 df + f dV_1$$

where $V = \sum_i V_i dx^i$. So

$$\begin{aligned} D_V f dx(W) &= df \wedge dx(V, W) + V_1 \langle df, W \rangle + f \langle dV_1, W \rangle = \\ &\langle df, V \rangle \langle dx, W \rangle - \langle df, W \rangle \langle dx, V \rangle + V_1 \langle df, W \rangle + f \langle dV_1, W \rangle = \\ &\langle df, V \rangle \langle dx, W \rangle + f \langle dV_1, W \rangle \end{aligned}$$

$$\text{On the other side, } L_V(f dx)W = L_V f \langle dx, W \rangle + f(L_V dx)W =$$

$$Vf \langle dx, W \rangle + f(V \langle dx, W \rangle - dx([V, W])).$$

So it is enough to show that $\langle dV_1, W \rangle = (V \langle dx, W \rangle - dx([V, W]))$ which it does (write V and W in components).