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$= (D_1 D_2(\alpha) + D_2 D_1(\alpha)) \wedge \beta + \alpha \wedge (D_1 D_2(\beta) + D_2 D_1(\beta))$

(Since k_1 & k_2 odd $\Rightarrow (-1)^{k_1} = (-1)^{k_2} = -1$)

$= D(\alpha) \wedge \beta + \alpha \wedge D(\beta)$.

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iv) $D_V \alpha = V \lrcorner (d\alpha) + d(V \lrcorner \alpha)$.

$d\alpha$ and $V \lrcorner \alpha$ are anti derivations of degree 1 and -1 resp. so $D_V\alpha$ is a derivation of degree $(1-1) = 0$ by iii).

$$D_V f = V \lrcorner (df) + d(V \lrcorner \alpha)$$
$$= \langle V^a \partial_a, \partial_b f \, dx^b \rangle + 0$$
$$= V^a \partial_b f \, \delta^b_a$$ (summation convention in force)
$$= Vf. = L_V f.$$

$$D_V(\alpha) = V \lrcorner (d\alpha) + d(V \lrcorner \alpha)$$

Put $\alpha = f_a dx^a$, $V = V^c \partial_c$

$$D_V(\alpha) = V \lrcorner (\partial_b f_a \, dx^b \wedge dx^a) + d(\langle V^c \partial_c, f_a dx^a \rangle)$$
$$= \partial_b f_a V^c \langle \partial_c, dx^b \rangle dx^a - \partial_b f_a V^c \langle \partial_c, dx^a \rangle dx^b + d(V^c f_c)$$
$$= (\partial_c f_a) V^c dx^a - (\partial_a f_b) V^b dx^a + (\partial_a V^c) f_c + V^c \partial_a f_c) dx^a$$
$$= ((\partial_c f_a) V^c - (\partial_a f_b) V^b + (\partial_a V^c) f_c + V^c (\partial_a f_c)) dx^a$$
$$= ((\partial_c f_a) V^c + (\partial_a V^c) f_c) dx^a + (-(\partial_a f_b) V^b + (\partial_a f_c) V^c) dx^a$$

(Second term is zero)

$$= V^c \partial_c f_a \, dx^a + (\partial_a V^c) f_c \, dx^a$$
$$= (V f_a) dx^a + f^c dV^c = L_V \alpha.$$

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D_V defined entirely by effect on functions and one forms

$\therefore D_V \alpha = L_V \alpha$. for all forms α.