First, let me admit that the maximum marks available for TMA 02 seem to be 95! I have raised the marks to 100 by increasing every section of Question 1 to 5, for a total of 20. Thanks to those of you who pointed this out.

In this question I gave full marks to any correct proof of the required identities, even if you did not use the indicated vector fields and 1-forms. But it might be as well to remember that questions should be taken literally (and may even be easier that way). The important point is to be sure you understand and remember the formulae.

The only known typo in TMA 04 is in Q4 (ii), which should say that 'It follows from (i) that

$$2g\big(R(U,V)W,X\big)=g\big(R(U,V)W,X\big)-g\big(R(U,V)X,W\big)'.$$

The '= 0' is wrong and should be deleted. Thanks to L de L for spotting this.

Dare I hope that there are no mistakes on TMA 03!?

If you are severely pressed for time, on TMA 03 Q 1 is largely routine (do it!) Q2 is hard but very instructive, Q3 is important but not too easy, Q4 is important, and Q5 is not as hard as it looks.

Good luck

Jeremy

M827 TMA 02 2001 Solutions

Question 1

LHS: $(L_V(\omega_1\wedge\omega_2))(X,Y) = (L_V\,\omega_1\wedge\omega_2 + \omega_1\wedge L_V\,\omega_2)(X,Y)$

$= L_V\,\omega_1 X\,\omega_2 Y - L_V\,\omega_1 Y\,\omega_2 X + \omega_1 X L_V\,\omega_2 Y - \omega_1 Y L_V\,\omega_2 X$

$= (V\omega_1 X)\omega_2 Y - \omega_1([V,X])\omega_2 Y - (V\omega_1 Y)\omega_2 X + \omega_1([V,Y])\omega_2 X + \omega_1 X(V\omega_2 Y) -$

$\omega_1 X\omega_2([V,Y]) - \omega_1 Y(V\omega_2 X) + \omega_1 Y\,\omega_2([V,X])$

$= V(\omega_1 X)\omega_2 Y - V(\omega_1 Y)\omega_2 X + V(\omega_2 Y)\omega_1 X - V(\omega_2 X)\omega_1 Y) - (\omega_1([V,X])\omega_2 Y -$

$\omega_1([V,Y])\omega_2 X + \omega_1 X\omega_2([V,Y]) - \omega_1 Y\,\omega_2([V,X]))$

RHS: $L_V((\omega_1\wedge\omega_2)(X,Y)) - (\omega_1\wedge\omega_2)(L_V X,Y) - (\omega_1\wedge\omega_2)(X,L_V Y)$

$= V(\omega_1 X\omega_2 Y - \omega_1 Y\omega_2 X) - \omega_1([V,X])\omega_2 Y + \omega_1 Y\,\omega_2([V,X]) - \omega_1 X\omega_2([V,Y]) +$

$\omega_1([V,Y])\omega_2 X.$

LHS = RHS , confirming the result.

(ii) LHS $= V\rfloor(\chi_1\wedge\chi_2)(X) = \chi_1\wedge\chi_2(V,X) = b_1\omega_1\wedge b_2\omega_2(a_1\partial_1,X) =$

$b_1b_2((\omega_1\,a_1\partial_1)\omega_2(X) - \omega_2(a_1\partial_1)\omega_1(X)) = b_1b_2\,a_1\,\omega_2(X)$

RHS $= (V\rfloor\chi_1)\wedge\chi_2(X) - \chi_1\wedge(V\rfloor\chi_2)(X) = b_1\omega_1(a_1\partial_1)b_2\omega_2(X) -$

$b_1\omega_1(X)b_2\omega_2(a_1\partial_1)$