4) $\phi^*((x^1+x^2)dx^1\wedge dx^2)$
$=(\phi^1+\phi^2)\,\phi^*(dx^1)\wedge\phi^*(dx^2)$
$=(\phi^1+\phi^2)\,d\phi^1\wedge d\phi^2$
$=(y^1y^2+(y^1)^2-(y^2)^2)(y^1dy^2+y^2dy^1)\wedge(2y^1dy^1-2y^2dy^2)$
$=(y^1y^2+(y^1)^2-(y^2)^2)(-2(y^1)^2-2(y^2)^2)dy^1\wedge dy^2$
$=(-2y^1y^2((y^1)^2+(y^2)^2)-2((y^1)^4-(y^2)^4))dy^1\wedge dy^2$

$-\phi^*((x^1+x^3)dx^1\wedge dx^3)$
$=-(\phi^1+\phi^3)\phi^*(dx^1)\wedge\phi^*(dx^3)$
$=-(\phi^1+\phi^3)\,d\phi^1\wedge d\phi^3$
$=-(y^1y^2+(y^1)^2+(y^2)^2)(y^1dy^2+y^2dy^1)\wedge(2y^1dy^1+2y^2dy^2)$
$=-(y^1y^2+(y^1)^2+(y^2)^2)(-2(y^1)^2+2(y^2)^2)dy^1\wedge dy^2$
$=2y^1y^2((y^1)^2-(y^2)^2)+2((y^1)^4-(y^2)^4)dy^1\wedge dy^2$

$\phi^*((x^2+x^3)dx^2\wedge dx^3)$
$=(\phi^2+\phi^3)\,\phi^*(dx^2)\wedge\phi^*(dx^3)$
$=(\phi^2+\phi^3)\,d\phi^2\wedge d\phi^3$
$=((y^1)^2-(y^2)^2+(y^1)^2+(y^2)^2)(2y^1dy^1-2y^2dy^2)(2y^1dy^1+2y^2dy^2)$
$=(2(y^1)^2)(8y^1y^2)\,dy^1\wedge dy^2$
$=2y^1y^2(8(y^1)^2)\,dy^1\wedge dy^2$

Adding above three terms gives

$\phi^*(\omega)=(2y^1y^2(-2(y^2)^2+8(y^1)^2)\,dy^1\wedge dy^2$
$=4y^1y^2(4(y^1)^2-(y^2)^2)dy^1\wedge dy^2.$ ✓ 5

ii) $L_V((x^1+x^2)dx^1\wedge dx^2)$
$=(L_V(x^1+x^2))\,(dx^1\wedge dx^2)+(x^1+x^2)(dV^1\wedge dx^2+dx^1\wedge dV^2)$
$=(x^3\partial_2+x^2\partial_3)(x^1+x^2))(dx^1\wedge dx^2)+(x^1+x^2)(0+dx^1\wedge dx^3)$
$=x^3\,dx^1\wedge dx^2+(x^1+x^3)\,dx^1\wedge dx^3.$

$L_V((x^1+x^3)dx^1\wedge dx^3)$
$=L_V(x^1+x^3)\,dx^1\wedge dx^3+(x^1+x^3)(dV^1\wedge dx^3+dx^1\wedge dV^3)$