

(3)

$$\begin{aligned}
 &= \langle c_1 \partial_1 + c_2 \partial_2, -a_1 \partial_1 - \partial_2 b_1 \omega_1 \wedge \omega_2 \rangle \\
 &= \langle c_1 \partial_1 + c_2 \partial_2, -a_1 (\partial_2 b_1) \omega_2 \rangle \\
 &= -a_1 c_2 (\partial_2 b_1).
 \end{aligned}$$

$$\Rightarrow (d(V \lrcorner \omega) + V(d\omega))(X)$$

$$\begin{aligned}
 &= c_1 a_1 (\partial_1 b_1) + c_1 (\partial_1 a_1) b_1 + c_2 (\partial_2 a_1) b_1 \\
 &= (L_V \omega)(X) \text{ as required.}
 \end{aligned}$$

5
20

$$\begin{aligned}
 2) i) \alpha \wedge \beta &= (3\theta^1 - 2\theta^2 + \theta^3 - \theta^4) \wedge (\theta^1 \wedge \theta^2 + \theta^1 \wedge \theta^3 - \theta^2 \wedge \theta^4) \\
 &= 3\theta^1 \wedge \theta^1 \wedge \theta^2 + 3\theta^1 \wedge \theta^1 \wedge \theta^3 - 3\theta^1 \wedge \theta^2 \wedge \theta^4 \\
 &\quad - 2\theta^2 \wedge \theta^1 \wedge \theta^2 - 2\theta^2 \wedge \theta^1 \wedge \theta^3 + 2\theta^2 \wedge \theta^2 \wedge \theta^4 \\
 &\quad + \theta^3 \wedge \theta^1 \wedge \theta^2 + \theta^3 \wedge \theta^1 \wedge \theta^3 - \theta^3 \wedge \theta^2 \wedge \theta^4 \\
 &\quad - \theta^4 \wedge \theta^1 \wedge \theta^2 - \theta^4 \wedge \theta^1 \wedge \theta^3 + \theta^4 \wedge \theta^2 \wedge \theta^4
 \end{aligned}$$

$$\begin{aligned}
 &= 0 + 0 - 3\theta^1 \wedge \theta^2 \wedge \theta^4 + 0 + 2\theta^1 \wedge \theta^2 \wedge \theta^3 \\
 &\quad + 0 + \theta^1 \wedge \theta^2 \wedge \theta^3 + 0 + \theta^2 \wedge \theta^3 \wedge \theta^4 \\
 &\quad - \theta^1 \wedge \theta^2 \wedge \theta^4 - \theta^1 \wedge \theta^3 \wedge \theta^4 + 0 \\
 &= -4\theta^1 \wedge \theta^2 \wedge \theta^4 + 2\theta^1 \wedge \theta^2 \wedge \theta^3 \\
 &\quad + \theta^2 \wedge \theta^3 \wedge \theta^4 - \theta^1 \wedge \theta^3 \wedge \theta^4.
 \end{aligned}$$

3

$$\begin{aligned}
 ii) v_1 \lrcorner \beta &= v_1 \lrcorner (\theta^1 \wedge \theta^2 + \theta^1 \wedge \theta^3 - \theta^2 \wedge \theta^4) \\
 &= \langle v_1, \theta^1 \rangle \theta^2 - \langle v_1, \theta^3 \rangle \theta^1 + \langle v_1, \theta^1 \rangle \theta^3 \\
 &\quad - \langle v_1, \theta^3 \rangle \theta^1 - \langle v_1, \theta^2 \rangle \theta^4 + \langle v_1, \theta^4 \rangle \theta^2 \\
 &= \theta^2 - 2\theta^1 + \theta^3 + 3\theta^1 - 2\theta^4 + 0 \\
 &= \theta^1 + \theta^2 + \theta^3 - 2\theta^4
 \end{aligned}$$

4

$$\begin{aligned}
 iii) \beta(v_2, v_3) &= (\theta^1 \wedge \theta^2 + \theta^1 \wedge \theta^3 - \theta^2 \wedge \theta^4)(v_2, v_3) \\
 &= \langle v_2, \theta^1 \rangle \langle v_3, \theta^2 \rangle - \langle v_2, \theta^2 \rangle \langle v_3, \theta^1 \rangle \\
 &\quad + \langle v_2, \theta^1 \rangle \langle v_3, \theta^3 \rangle - \langle v_2, \theta^3 \rangle \langle v_3, \theta^1 \rangle \\
 &\quad - \langle v_2, \theta^2 \rangle \langle v_3, \theta^4 \rangle + \langle v_2, \theta^4 \rangle \langle v_3, \theta^2 \rangle \\
 &= 1 \cdot 1 - 0 \cdot 0 + 1 \cdot 0 - 1 \cdot 0 \\
 &\quad - 1 \cdot 0 + 0 \cdot 0
 \end{aligned}$$