(6)

then $\sum_a e_a \otimes \langle L, \Theta^a \rangle$

$= \sum_a e_a \otimes \langle L^j e_j \Theta^a \rangle$

$= \sum_a \langle L^j e_j, \Theta^a \rangle e_a$

$= \sum_a L^j \langle e_j, \Theta^a \rangle e_a$ ✓ 6

$= \sum_a L^j \delta^j_a e_a. = L^j e_j$. as required.

v) Define $\alpha(\Theta_L) = \sum e_a \otimes \Theta^a_L$ then α is an isomorphism of 3-forms on to the set of vector values 3 forms since

1) $\alpha(\Theta_L) = 0 \Rightarrow \sum e_a \otimes \Theta^a_L = 0$

$\Rightarrow L^j e_j = 0$ for each $j \Rightarrow L = 0$

$\Rightarrow \ker(\alpha) = 0$ $\therefore$ α is one to one.

α onto since for each L there exists a 3-form Θ_L such that $\Theta_L = \langle L, \Theta \rangle$ from iii)

then $\alpha(\Theta_L) = L$. 1

$\therefore \dim(L) =$ dimension of 3 forms

ie $\dim(L) = \frac{4!}{3!1!} = 4$. no - see solutions

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Also $\alpha(\Theta_{1_L} + \Theta_{2_L})$

$= \sum_a e_a \otimes (\Theta_{1_L} + \Theta_{2_L})^a$

$= \sum_a \langle \Theta_{1_L} + \Theta_{2_L}, \Theta^a \rangle e_a$

$= \sum_a \langle \Theta_{1_L}, \Theta^a \rangle e_a + \sum_a \langle \Theta_{2_L}, \Theta^a \rangle e_a$

$= \alpha(\Theta_{1_L}) + \alpha(\Theta_{2_L})$ & so

$\Rightarrow$ morphism prop satisfied