$= (Vf)\langle X, \omega_1\rangle + fV\langle X, \omega_1\rangle - f\langle L_V X, \omega_1\rangle$
$= Vf\langle X, \omega_1\rangle + f\langle L_V X, \omega_1\rangle + f\langle X, L_V\omega_1\rangle - f\langle L_V X, \omega_1\rangle$
$= Vf\langle X, \omega_1\rangle + f\langle X, L_V\omega_1\rangle$
$= Vf\,\omega_1(X) + f(L_V\omega_1)(X)$
$= Vf\,\omega_1 + fL_V\omega_1$
since X is arbitrary.

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iv) Evaluate on RHS and LHS separately.
RHS: $\langle X, L_V\omega\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, L_{a_1\partial_1}(b_1\omega_1)\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, a_1(\partial_1(b_1))\omega_1 + b_1 L_{a_1\partial_1}(\omega_1)\rangle.$
We need to find $L_{a_1\partial_1}(\omega_1)$.
Curve through (x^1, x^2) with tangent vector $a_1\partial_1$ is given by $(a_1 t + x^1, x^2) = \phi$.

$\phi^*(\omega^1) = d\phi^1 = d(a_1 t + x^1)$
$= (\partial_1 a_1)t\,\omega_1 + (\partial_2 a_1)^t\omega_2$
$\Rightarrow L_V\omega^1 = (\partial_1 a_1)\omega_1 + (\partial_2 a_1)\omega_2$

$\Rightarrow \langle c_1\partial_1 + c_2\partial_2, a_1(\partial_1 b_1)\omega_1 + b_1 L_{a_1\partial_1}(\omega_1)\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, (a_1(\partial_1 b_1) + b_1(\partial_1 a_1))\omega_1 + b_1(\partial_2 a_1)\omega_2\rangle$
$= c_1 a_1(\partial_1 b_1) + c_1 b_1(\partial_1 a_1) + c_2 b_1(\partial_2 a_1) = (L_V\omega)(X)$

RHS $(d(V\lrcorner\omega))(X) = \langle X, d(V\lrcorner\omega)\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, d(\langle a_1\partial_1, b_1\omega_1\rangle)\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, d(a_1 b_1)\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, (a_1\partial_1 b_1 + (\partial_1 a_1)b_1)\omega_1 + (a_1\partial_2 b_1 + (\partial_2 a_1)b_1)\omega_2\rangle$
$= c_1 a_1(\partial_1 b_1) + c_1(\partial_1 a_1)b_1 + c_2 a_1\partial_2 b_1 + c_2(\partial_2 a_1)b_1$
and
$(V\lrcorner d\omega)(X) = \langle c_1\partial_1 + c_2\partial_2, a_1\partial_1 \lrcorner d(b_1\omega^1)\rangle$
$= \langle c_1\partial_1 + c_2\partial_2, a_1\partial_1 \lrcorner (\partial_1 b_1\,\omega^1\wedge\omega^1 - \partial_2 b_1\,\omega^1\wedge\omega^2)\rangle.$