

$$= (df) \theta^1 \wedge \dots \wedge \theta^p + f \sum_{i=1}^p \theta^1 \wedge \dots \wedge d(\theta^i) \wedge \dots \wedge \theta^p$$

$$= D_2 f \theta^1 \wedge \dots \wedge \theta^p + f \sum_{i=1}^p \theta^1 \wedge \dots \wedge D_2(\theta^i) \wedge \dots \wedge \theta^p$$

Since $D_1(f) = D_2(f)$ and $D_1(\theta) = D_2(\theta)$

$$= D_2(f \theta^1 \wedge \theta^2 \wedge \dots \wedge \theta^p)$$

Since $D_1(f) = D_2(f)$ and $D_1(\theta) = D_2(\theta)$

$\therefore D_1(\alpha) = D_2(\alpha)$ for all forms α .

$$\text{iii) } D(\alpha \wedge \beta) = D_1(D_2(\alpha \wedge \beta)) + D_2(D_1(\alpha \wedge \beta))$$

$$= D_1(D_2(\alpha) \wedge \beta + (-1)^p \alpha \wedge D_2(\beta))$$

$$+ D_2(D_1(\alpha) \wedge \beta + (-1)^p \alpha \wedge D_1(\beta))$$

If α is a p -form, β is a q -form

$D_1(\alpha)$ is a $p+k_1$ form

$D_2(\alpha)$ is a $p+k_2$ form

$D_1(\beta)$ is a $q+k_1$ form

$D_2(\beta)$ is a $q+k_2$ form.

$$\Rightarrow D(\alpha \wedge \beta) = D_1 D_2(\alpha) \wedge \beta + (-1)^{p+k_2} D_2(\alpha) \wedge D_1(\beta)$$

$$+ (-1)^p D_1(\alpha) \wedge D_2(\beta) + (-1)^{2p} \alpha \wedge D_1 D_2(\beta)$$

$$+ D_2 D_1(\alpha) \wedge \beta + (-1)^{p+k_1} D_1(\alpha) \wedge D_2(\beta)$$

$$+ (-1)^p D_2(\alpha) \wedge D_1(\beta) + (-1)^{2p} \alpha \wedge D_2 D_1(\beta)$$

$$= (D_1 D_2(\alpha) \wedge \beta + D_2 D_1(\alpha) \wedge \beta) + (\alpha \wedge D_1 D_2(\beta) + \alpha \wedge D_2 D_1(\beta))$$

$$+ ((-1)^{p+k_2} D_2(\alpha) \wedge D_1(\beta) + (-1)^p D_1(\alpha) \wedge D_2(\beta))$$

$$+ (-1)^{p+k_1} D_1(\alpha) \wedge D_2(\beta) + (-1)^p D_2(\alpha) \wedge D_1(\beta))$$

$$= (D_1 D_2(\alpha) + D_2 D_1(\alpha)) \wedge \beta + \alpha \wedge (D_1 D_2(\beta) + D_2 D_1(\beta))$$

$$+ (-1)^p ((-1)^{k_2} D_2(\alpha) \wedge D_1(\beta) + D_2(\alpha) \wedge D_1(\beta))$$

$$+ (-1)^p ((-1)^{k_1} D_1(\alpha) \wedge D_2(\beta) + D_1(\alpha) \wedge D_2(\beta))$$