ii) We must show $\omega \otimes \alpha$ is alternating & multilinear

$\alpha(v_{a_1}, v_{a_2}, v_{a_3}) = \varepsilon_{a_1 a_2 a_3} \alpha(v_1, v_2, v_3)$

Since α is a 3 form so is alternating

$\Rightarrow (\omega \otimes \alpha)(v_{a_1}, v_{a_2}, v_{a_3})$

$= \alpha(v_{a_1}, v_{a_2}, v_{a_3})\, \omega = \varepsilon_{a_1 a_2 a_3} \alpha(v_1, v_2, v_3)\, \omega$

$= \varepsilon_{a_1 a_2 a_3} (\omega \otimes \alpha)(v_1, v_2, v_3)$. ✓

$\Rightarrow \omega \otimes \alpha$ is alternating.

$(\omega \otimes \alpha)(k v_1, v_2, v_3) = \alpha(k v_1, v_2, v_3)\, \omega$

$= k \alpha(v_1, v_2, v_3)\, \omega = k (\omega \otimes \alpha)(v_1, v_2, v_3)$

Proof of linearity for v_2 and v_3 is identical. ✓

$\Rightarrow (\omega \otimes \alpha)$ is multilinear and alternating, and takes its values in V $\therefore$ is a vector valued 3-form. 3

iii) Must show Θ_L is alternating & multilinear, and $\Theta_L \in \mathbb{R}$

$\Theta_L(v_{a_1}, v_{a_2}, v_{a_3}) = \langle \varepsilon_{a_1 a_2 a_3} L(v_1, v_2, v_3), \Theta \rangle$ ✓

$= \varepsilon_{a_1 a_2 a_3} \langle L(v_1, v_2, v_3), \Theta \rangle$

$= \varepsilon_{a_1 a_2 a_3} \Theta_L(v_1, v_2, v_3) \Rightarrow \Theta_L$ alternating.

$\Theta_L(k v_1, v_2, v_3) = \langle L(k v_1, v_2, v_3), \Theta \rangle$

$= \langle k L(v_1, v_2, v_3), \Theta \rangle = k \langle L(v_1, v_2, v_3), \Theta \rangle$

$= k \Theta_L(v_1, v_2, v_3)$ ✓

$\Rightarrow \Theta_L$ multilinear. 3

$L(v_1, v_2, v_3) \in V \Rightarrow \langle L(v_1, v_2, v_3), \Theta \rangle \in \mathbb{R}$

$\Rightarrow \Theta_L$ is an ordinary 3-form.

iv) $\sum_a e_a \otimes \Theta_L^a = \sum_a e_a \otimes \langle L(v_1, v_2, v_3), \Theta^a \rangle$

Let $L(v_1, v_2, v_3) = L^j e_j$ (summation convention wrt j in force).