

(9)

$$\Rightarrow W_1 = x^2 + x^3$$

$$W_2 = x^1 + x^3$$

$$W_3 = x^1 + x^2$$

$$\Rightarrow W = (x^2 + x^3)\partial_1 + (x^1 + x^3)\partial_2 + (x^1 + x^2)\partial_3$$

$$dW = 0 \Rightarrow W \text{ is closed}$$

$$\text{and } d\langle W, \gamma \rangle = \langle W, d\gamma \rangle + d\langle W, \gamma \rangle$$

$$= d\langle W, \gamma \rangle = dW = 0$$

$W$  is a vector field whose flow preserves  $\Omega$  is flow of vector field  $W$  is volume preserving (20)

5) i) For each of i) ii) iii) take  $\alpha = f\theta^1 \wedge \dots \wedge \theta^P$

$$\beta = g\theta^{P+1} \wedge \dots \wedge \theta^{P+K} \quad \text{see opp}$$

$$\begin{aligned} a) D(\alpha \wedge \beta) &= d(fg\theta^1 \wedge \dots \wedge \theta^P \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K}) \\ &= (df)g + fg(d\theta^1 \wedge \dots \wedge \theta^P \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K}) \\ &= df \wedge \theta^1 \wedge \dots \wedge \theta^P \wedge g\theta^{P+1} \wedge \dots \wedge \theta^{P+K} \\ &\quad + (-1)^P f\theta^1 \wedge \dots \wedge \theta^P \wedge dg \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K} \end{aligned}$$

$$= (d\alpha) \wedge \beta + (-1)^P \alpha \wedge d\beta$$

$D = d$  is an anti-derivation of degree 1.

$$\begin{aligned} b) D(\alpha \wedge \beta) &= L_V(fg\theta^1 \wedge \dots \wedge \theta^P \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K}) \\ &= V(fg)\theta^1 \wedge \dots \wedge \theta^P \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K} \\ &\quad + fg(L_V\theta^1 \wedge \dots \wedge dV^{\Gamma} \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K}) \\ &\quad + fg\theta^1 \wedge \dots \wedge \theta^P \wedge \left( \sum_{\Gamma} \theta^{P+1} \wedge \dots \wedge \theta^{P+K} \right) dV^{\Gamma} \end{aligned}$$

$$\begin{aligned} &= ((Vf)g + f(Vg))\theta^1 \wedge \dots \wedge \theta^{P+K} \\ &\quad + f \sum \theta^1 \wedge \dots \wedge dV^{\Gamma} \wedge \theta^{P+1} \wedge \dots \wedge \theta^{P+K} \\ &\quad + f\theta^1 \wedge \dots \wedge \theta^P \wedge g \left( \sum \theta^{P+1} \wedge \dots \wedge dV^{\Gamma} \wedge \theta^{P+K} \right) \end{aligned}$$

$$\begin{aligned} &= (Vf)\theta^1 \wedge \dots \wedge \theta^P \wedge g\theta^{P+1} \wedge \dots \wedge \theta^{P+K} \\ &\quad + f\theta^1 \wedge \dots \wedge \theta^P \wedge (Vg)\theta^{P+1} \wedge \dots \wedge \theta^{P+K} \\ &\quad + fL_V(\theta^1 \wedge \dots \wedge \theta^P) \wedge \beta \\ &\quad + \alpha \wedge gL_V(\theta^{P+1} \wedge \dots \wedge \theta^{P+K}) \end{aligned}$$