∴ ad is a map to the set of endomorphisms of $L$ ∴ ad is a representation of $L$.

Needs linearity too −1

~~X~~ No

So $sl(1)$ see later

iii) Let $q = a + b\underline{i} + c\underline{j} + d\underline{k}$

$$\phi(q) = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 0 & c & c \\ 0 & -b & 0 & d \\ 0 & -c & -d & 0 \end{bmatrix}$$

We have taken as a basis $e_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, e_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Let $q_1 = a_1 + b_1\underline{i} + c_1\underline{j} + d_1\underline{k}$

$q_2 = a_2 + b_2\underline{i} + c_2\underline{j} + d_2\underline{k}$

Then $\phi[q_1, q_2]$

$= \phi((a_1 + b_1\underline{i} + c_1\underline{j} + d_1\underline{k})(a_2 + b_2\underline{i} + c_2\underline{j} + d_2\underline{k}) - (a_2 + b_2\underline{i} + c_2\underline{j} + d_2\underline{k})(a_1 + b_1\underline{i} + c_1\underline{j} + d_1\underline{k}))$

$= \phi((a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 - a_2a_1 + b_2b_1 + c_2c_1 + d_2d_1)$
$+ \underline{i}(a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2 - a_2b_1 - b_2a_1 - c_2d_1 + d_2c_1)$
$+ \underline{j}(a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2 - a_2c_1 + b_2d_1 - c_2a_1 - d_2b_1)$
$+ \underline{k}(a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2 - a_2d_1 - b_2c_1 + c_2b_1 - d_2a_1))$

$= \phi(0 + \underline{i}(2c_1d_2 - 2d_1c_2) + \underline{j}(2d_1b_2 - 2b_1d_2) + \underline{k}(2b_1c_2 - 2c_1b_2))$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2(c_1d_2 - d_1c_2) & 2(d_1b_2 - b_1d_2) \\ 0 & -2(c_1d_2 - d_1c_2) & 0 & 2(b_1c_2 - c_1b_2) \\ 0 & -2(d_1b_2 - b_1d_2) & -2(b_1c_2 - c_1b_2) & 0 \end{bmatrix}$$