$$= (a_2, b_2, c_2, d_2)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2d_1 & -2c_1 \\ 0 & -2d_1 & 0 & 2b_1 \\ 0 & +2c_1 & -2b_1 & 0 \end{pmatrix}$$

The matrix above represents $ad\, g_1$

Also, if g_1, g_2 are as before and $x = x_0 + ix_1 + jx_2 + kx_3$
Then g_1, g_2 are represented as before
and $x = (x_0, x_1, x_2, x_3)^T$. Then:

$$= \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & b_1 & c_1 \\ 0 & -b_1 & 0 & d_1 \\ 0 & -c_1 & -d_1 & 0 \end{pmatrix}\begin{pmatrix} a_2 & 0 & 0 & 0 \\ 0 & 0 & b_2 & c_2 \\ 0 & -b_2 & 0 & d_2 \\ 0 & -c_2 & -d_2 & 0 \end{pmatrix}$$

$$- \begin{pmatrix} a_2 & 0 & 0 & 0 \\ 0 & 0 & b_2 & c_2 \\ 0 & -b_2 & 0 & d_2 \\ 0 & -c_2 & -d_2 & 0 \end{pmatrix}\begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & b_1 & c_1 \\ 0 & -b_1 & 0 & d_1 \\ 0 & -c_1 & -d_1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a_1a_2 & 0 & 0 & 0 \\ 0 & -(b_1b_2+c_1c_2) & -c_1d_2 & b_1d_2 \\ 0 & -c_2d_1 & -(b_1b_2+d_1d_2) & -b_1c_2 \\ 0 & b_2d_1 & -b_2c_1 & -(c_1c_2+d_1d_2) \end{pmatrix}$$

$$- \begin{pmatrix} a_1a_2 & 0 & 0 & 0 \\ 0 & -(b_1b_2+c_1c_2) & -c_2d_1 & b_2d_1 \\ 0 & -c_1d_2 & -(b_1b_2+d_1d_2) & -b_2c_1 \\ 0 & b_1d_2 & -b_1c_2 & -(c_1c_2+d_1d_2) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & c_2d_1-c_1d_2 & b_1d_2-b_2d_1 \\ 0 & -(c_2d_1-c_1d_2) & 0 & b_2c_1-b_1c_2 \\ 0 & -(b_1d_2-b_2d_1) & -(b_2c_1-b_1c_2) & 0 \end{pmatrix}$$