σ is as defined.
Let a be a Lie element which is homogeneous of degree m. We use induction on m:

$[\ldots[x_1, x_2]\ldots x_m]\sigma = [\ldots[x_1, x_2]\ldots x_{m-1}]\sigma, x_m]$
$+ [[\ldots[x_1 x_2]\ldots x_{m-1}] x_m \sigma]$
$= (m-1)[\ldots[x_1 x_2]\ldots x_m] + [\ldots[x_1 x_2]\ldots x_m]$
$= m[x_1 \ldots [x_1, x_2]\ldots x_m]$

And the hypothesis is true for $m=1$ from defn of σ.
We apply σ to $\log(1+z)$ to get

$$\log(1+z) = \sum_m \sum_{p_i, q_i} \frac{(-1)^{m-1}}{m(\sum p_i + q_i)} [\ldots \overbrace{[AA]\ldots A}^{p_1} \overbrace{]B]\ B]}^{q_1} \ldots \overbrace{B]\ B]}^{q_m}$$

When we expand this series we find

$$\log(1+z) = 1 + A + B + \frac{[A, B]}{2} + \frac{[A[A, B]]}{12} + \ldots$$

All the terms of degree three or higher are zero. $1+z = e^A e^B$

$$\log(e^A e^B) = 1 + A + B + \frac{[A, B]}{2}$$

since log and exp are inverses and one-one near zero we have,
$e^A e^B = e^{A+B+[A,B]/2}$ ①
Let γ be the operator $\gamma(A) = -A$
$\gamma(B) = -B$
Applying this to the formula ①
we have
$e^{-A} e^{-B} = e^{-A-B+[A,B]/2}$ ②
Multiplying ① and ② together gives
$e^A e^B e^{-A} e^{-B} = e^{A+B+[A,B]/2} e^{-A-B+[A,B]/2}$