$A_{mm}$ skew Hermitian $\Rightarrow \mathrm{Dim}\, A_{mm} = m^2$ ✓

$A_{nn}$ skew Hermitian $\Rightarrow \mathrm{Dim}\, A_{nn} = n^2$ ✓

$\mathrm{Dim}\, A_{mn} = \mathrm{Dim}\, A_{nm} = 2nm$

$\mathrm{Dim}\, su(m,n) = m^2 + n^2 + 2nm - 1$ (since $Tr = 0$)

$= (n+m)^2 - 1$ ✓

iv) $A = \begin{bmatrix} A_{mm} & A_{m,n} \\ A_{n,m} & A_{nn} \end{bmatrix}$, $B = \begin{bmatrix} B_{mm} & B_{m,n} \\ B_{n,m} & B_{nn} \end{bmatrix}$,

We must see if $[A,B]K + K[A,B]^{+} = 0$ *

$(AB - BA)K + K(AB - BA)^{+}$

$= ABK - BAK + K(B^{+}A^{+} - A^{+}B^{+})$

$= (ABK + KB^{+}A^{+}) - (BAK + KA^{+}B^{+})$

$= ABK + (-BK)A^{+} - (BAK - AKB^{+})$

$K = K^{-1} \Rightarrow A^{+} = K^{-1}(A)K = -KAK$, $B^{+} = -KBK$, Note $K^2 = I$

$ABK + BKKAK - BAK + AKKBK = 0$

$\therefore$ * is satisfied and $[A,B]$ is an $(m+n)\times(m+n)$ matrix since $A, B$ are $su(m,n)$ is a Lie Algebra.

v) $su(m+n)$ is the set of matrices satisfying:

$AI + IA^{+} = 0$ ie $A + A^{+} = 0$, the set of skew Hermitian matrices

$A \in su(m,n) \cap su(m+n)$

A is of form $\underbrace{\begin{bmatrix} A_{mm} & A_{m,n} \\ A_{n,m} & A_{nn} \end{bmatrix}}_{\substack{A_{mm}, A_{nn} \text{ skew hermitian} \\ A_{nm} = A_{mn}^{+}}}$ and $\underbrace{\begin{bmatrix} A_{mm} & A_{mn} \\ A_{nm} & A_{nn} \end{bmatrix}}_{\substack{A_{m,m}, A_{nn} \text{ skew hermitian}, \\ A_{nm} = -A_{mn}^{+}}}$