

a matrix and so we can live with the more concrete definition; and there is in fact a theorem which says we can always embed a Lie Algebra in an associative algebra where ordinary multiplication is defined anyway - so there is no loss of generality in writing the commutator as $XY - YX$ explicitly. The proof that ad gives a representation, however, relies only on the Jacobi identity and antisymmetry.

Question 4: Note the equivalence of the transformation in Question 3 (iii) and part (ii) of this question. This is not an accident, but a special case of a general result seen later on; the adjoint map in the algebra is the differential of the Adjoint map (a map of the form $u \mapsto u^\dagger$) in the group.

Question 5: The group $SU(m,n)$ is not a unitary group - but almost (sometimes called pseudo-unitary). It is the subgroup with determinant 1 of $U(m,n)$, defined without the restriction to determinant 1. The elements are the transformations which preserve forms such as $|x_0|^2 - |x_1|^2 - |x_2|^2 - |x_3|^2$, or in the real case $x_0^2 - x_1^2 - x_2^2 - x_3^2$ (so the Lorentz Group is a real form of a pseudo-unitary group, a pseudo-orthogonal group) and so you can believe that they are very important. They also occur in Physics in problems of superfluidity, which you may have heard about, as well as nuclear physics.

(iii) "Physicist's" evaluation of dimensionality, i.e. count the number of real free parameters, is rigorous and acceptable here. The most elegant proof that $\dim u(m,n) = \dim u(m+n) = (m+n)^2$ is simply the following:

The linear map $f: A \rightarrow KA$ from $u(m,n)$ to $u(m+n)$ gives a vector space isomorphism. QED
 $\dim su(m,n) = \dim u(m+n) - 1$, due to Trace condition.

Have a nice Summer!



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M824 TMA02 2000 Scores

Question	1	2	3	4	5
Mean	8.2	7.0	7.1	8.8	7.2
St Dev	2.3	2.5	2.7	2.0	2.7
No. Scripts	11	Av. Mark	76.6	St. Dev.	22.8