3) i) $ad\,g : L \to L$

$$\alpha x \to ad\,g(\alpha x) = [g, \alpha x]$$
$$= g\alpha x - \alpha x g = \alpha(gx - xg)$$
$$= \alpha[g, x]$$
$$= \alpha\, ad\,g(x)$$

$$x + y \to ad\,g(x+y) = [g, x+y]$$
$$= g(x+y) - (x+y)g$$
$$= gx + gy - xg - yg$$
$$= gx - xg + gy - yg$$
$$= [g, x] + [g, y]$$
$$= ad\,g(x) + ad\,g(y)$$

$\therefore$ $ad\,g$ is a linear transformation

Moreover, a Lie algebra is closed under $[\ ,\ ]$ $\therefore$ $ad\,g$ is an endomorphism of L

ii) $ad : L \to End\,L$

$$[X, Y] \to ad[X, Y]$$

$$ad[X,Y](g) = [[X,Y], g]$$
$$= (XY - YX)g - g(XY - YX)$$

and $[adX, adY](g)$
$$= (adX\,adY - adY\,adX)g$$
$$= adX\,adY(g) - adY\,adX(g)$$
$$= adX(Yg - gY) - adY(Xg - gX)$$
$$= X(Yg - gY) - (Yg - gY)X - Y(Xg - gX) + (Xg - gX)Y$$
$$= XYg - \cancel{XgY} - \cancel{YgX} + gYX - YXg + \cancel{YgX} + \cancel{XgY} - gXY.$$
$$= (XYg - YXg - gXY + gYX$$
$$= (XY - YX)g - g(XY - YX)$$
$$= ad[X,Y](g)$$