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$[A,[A,B]] = 0 = [B,[A,B]] \Rightarrow [A+B,[A,B]] = 0$

So $A+B$ commutes with $[A,B]$ see opp.

$\therefore e^{A+B+[A,B]/2} = e^{A+B} e^{[A,B]/2}$

$e^{-A-B+[A,B]/2} = e^{-A-B} e^{[A,B]/2}$

$\therefore e^{A+B+[A,B]/2} e^{-A-B+[A,B]/2} = e^{A+B} e^{[A,B]/2} e^{-A-B} e^{[A,B]/2}$

Again since $A+B$, $[A,B]$ commute $e^{[A,B]/2} e^{-A-B} = e^{-A-B} e^{[A,B]/2}$. We have $e^{A+B} e^{-A-B} e^{[A,B]/2} e^{[A,B]/2}$

Now $(A+B)$ commutes with $-(A+B)$, $[A,B]/2$ commutes with $[A,B]/2$

$\therefore e^{A+B} e^{-A-B} e^{[A,B]/2} e^{[A,B]/2} = e^{[A,B]}$.

$\therefore e^A e^B e^{-A} e^{-B} = e^{[A,B]}$

$e^A e^B = e^{[A,B]} e^B e^A$

(Multiplying by e^B, then e^A on right)

Since A, B commute with $[A,B]$

$e^A e^B = e^B e^A e^{[A,B]}$.

(Not an easy question is it? Or is there an easier way?)

If a solution requires more than 1 line, perhaps it's too long.

My solution: (i),(ii) $\Rightarrow$ $e^A B e^{-A} = B + [A,B]$, hence $e^A e^B = e^B e^A e^{[A,B]}$ (using $[\,,\,]$ as given.)

(→ by exponentiation)

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