which we can write in matrix form as (13)

$$(x_0', x_1', x_2', x_3') = (x_0, x_1, x_2, x_3)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a^2+b^2-c^2-d^2 & 2(bc+ad) & 2(-ac+bd) \\ 0 & 2(bc-ad) & a^2-b^2+c^2-d^2 & 2(ab+cd) \\ 0 & 2(bd+ac) & 2(-ab+cd) & a^2-b^2-c^2+d^2 \end{pmatrix}$$

The transformation matrix is obviously isomorphic to a member of O(3) for the reasons explained on p60/61, and it is a member of SO(3) since it is a product of two reflections (the proof of this is virtually the same as in 2) ✓

You were only asked to show 'smooth'

If q is a curve in S^3, $q = a + bi + cj + dk$ and q is differentiable then each component is separately differentiable, and $ab, a^2, c^2, da, b^2, a^2+b^2$ is the sums and products of the component functions are also differentiable ∴ the transformation matrix is differentiable, since all of it's entries are, so the transformation matrix is a differentiable curve in SO(3) and ρ is smooth. ✓

only this part is answer to (i) & you did not need explicit form

ii) $d\rho(\gamma'(0))(\underline{x}) = (\rho \circ \gamma)'(0)(\underline{x})$

$$= (x_1, x_2, x_3)\begin{pmatrix} a^2+b^2-c^2-d^2 & 2(bc+ad) & 2(-ac+bd) \\ 2(bc-ad) & a^2-b^2+c^2-d^2 & 2(ab+cd) \\ 2(bd+ac) & 2(-ab+cd) & a^2-b^2-c^2+d^2 \end{pmatrix}'$$

$\gamma(0) = 1$

$\gamma'(0) = b'(0)\underline{i} + c'(0)\underline{j} + d'(0)\underline{k}$