

To Prove that  $\{e_1, e_2, e_3, e_4\}$  is a basis.

$$ae_1 + be_2 + ce_3 + de_4 = 0$$

$$\Rightarrow \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 0 & b & c \\ 0 & -b & a & d \\ 0 & -c & -d & a \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow a=b=c=d=0$$

$\{e_1, e_2, e_3, e_4\}$  is a linearly independent set.

To prove that the set spans  $H$ , let  $g \in H$

$$g = a_1 + b_1 + c_1 + d_1$$

$$\text{then } \phi(g) = ae_1 + be_2 + ce_3 + de_4.$$

$\therefore$  The set is a basis for  $H$ .