

⑧

$$= - \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ i & 0 \end{pmatrix} = Y_2$$

$$\begin{aligned} -Y_1 Y_3 Y_1^+ &= - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= - \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= - \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Y_3 \end{aligned}$$

$$\begin{aligned} -Y_2 Y_3 Y_2^+ &= - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ &= - \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = - \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Y_3 \end{aligned}$$

These are the verifications for $j > i > 0$
Those for $i > j > 0$ are exactly similar.

$\therefore \sigma(u_i) = -u_i \quad u_i = u \quad (= u_i)$
 $\sigma(u_i) = u_i \quad u_i \neq u$
and $\sigma(u)$ is a reflection
 $\therefore -\sigma(u_i) = u_i \quad u_i = u = u_i \quad (-\sigma(u) = uu_i u)$
 $= -u_i \quad u_i \neq u$
is a rotation around the axis u_i
 $u_i (-\sigma(u_i))$ is a composition of two reflections, hence is a rotation.

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