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iv) Since the transformation is a linear mapping, (see PPE) it suffices to check that the basis matrices are orthogonal under the mapping.

we define a dot product

$$\gamma_\mu \cdot \gamma_\nu = \frac{1}{2} Tr(\gamma_\mu \gamma_\nu)$$

Since in this representation x_1, x_2, x_3, x_4 are just multiples of $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ respectively if $\gamma_\mu \cdot \gamma_\nu = 0$, $x_\mu \cdot x_\nu = 0$.

By inspection $\gamma_\mu \cdot \gamma_\mu = 1$

$$\gamma_0 \cdot \gamma_1 = \frac{1}{2} Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right) = 0.$$

$$\gamma_0 \cdot \gamma_2 = \frac{1}{2} Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right) = 0$$

$$\gamma_0 \cdot \gamma_3 = \frac{1}{2} Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = 0$$

$$\gamma_1 \cdot \gamma_2 = \frac{1}{2} Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right) = \frac{1}{2} Tr\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = 0$$

$$\gamma_1 \cdot \gamma_3 = \frac{1}{2} Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = \frac{1}{2} Tr\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = 0$$

$$\gamma_2 \cdot \gamma_3 = \frac{1}{2} Tr\left(\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = \frac{1}{2} Tr\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = 0$$

$$Tr(\gamma_\mu \gamma_\nu) = Tr(\gamma_\nu \gamma_\mu)$$

∴ The γ_i are all orthogonal.

$$(U\gamma_\mu U^+)(U\gamma_\nu U^+) = \frac{1}{2} Tr\left((U\gamma_\mu U^+)(U\gamma_\nu U^+)\right)$$

$$= \frac{1}{2} Tr(U\gamma_\mu \gamma_\nu U^+).$$

An refers to "3-space"

You need simply to show that x_μ's is invariant