$2\frac{1}{4}$

Let $z = e^A e^B - 1 = \sum_{p,q} \frac{A^p}{p!}\frac{B^q}{q!}$ $\quad p+q \geq 1$

then $z^m = \sum_{p_i q_i} \frac{A^{p_1} B^{q_1} A^{p_2} B^{q_2}}{p_1!\, q_1!\, p_2!\, q_2!} \cdots \frac{A^{p_m} B^{q_m}}{p_m!\, q_m!}, \; p_i+q_i \geq 1$

and $\log(1+Z) = \sum_m \sum_{p_i q_i} \frac{(-1)^{m-1}}{m} \frac{A^{p_1} B^{q_1}}{p_1!\, q_1!} \cdots \frac{A^{p_m} B^{q_m}}{p_m!\, q_m!}$

We need to prove that $\log(1+Z)$ is a Lie element, that is it can be expressed as a sum of elements in the Lie algebra L. To do this we need the theorem:

Let $S = \{A, B\}$ that is, the set generated by all sums and products of A, B. Let δ be the diagonal mapping of S into $S \times S$ such that $x \in S$, $x\delta \to x \times 1 + 1 \times x$. Then $a \in S$ is a Lie element iff $a\delta = a \times 1 + 1 \times a$.

We have $[a \times 1 + 1 \times a, b \times 1 + 1 \times b] = [ab] \times 1 + 1 \times [ab]$ which implies that the set of elements of S satisfying this condition is a subalgebra of L. this includes A, B. Let $\{y_1, y_2 \ldots\}$ be a basis for L. Then since S is the universal enveloping algebra, (ie it contains L), the elements $y_1^{k_1} y_2^{k_2} \ldots$ etc, with $k_i \geq 0$, $y_i^0 = 1$ form a basis for S. Hence the products $(y_1^{k_1} y_2^{k_2} \ldots y_m^{k_m}) \times (y_1^{l_1} y_2^{l_2} \ldots y_m^{l_m})$ form a basis for $S \times S$. We have

$$(y_1^{k_1} y_2^{k_2} \ldots y_m^{k_m})\delta = (y_1 \times 1 + 1 \times y_1)^{k_1} (y_2 \times 1 + 1 \times y_2)^{k_2} \ldots (y_m \times 1 + 1 \times y_m)^{k_m}$$

$$= y_1^{k_1} y_2^{k_2} \ldots y_m^{k_m} \times 1 + k_1 y_1^{k_1 - 1} y_2^{k_2} \ldots y_m^{k_m} \times y_1 + k_2 y_1^{k_1} y_2^{k_2 - 1} \ldots y_m^{k_m} \times y_2 + \ldots + k_m y_1^{k_1} \ldots y_m^{k_m - 1} \times y_m + \ldots *$$