From matrix theory, $Tr(BA) = Tr(AB)$
$Tr(BAC) = Tr(CBA)$
etc

$\therefore \frac{1}{2} Tr(u \gamma_\mu \gamma_\nu u^+) = \frac{1}{2} Tr(u^+ u \gamma_\mu \gamma_\nu)$

$= \frac{1}{2} Tr(\gamma_\mu \gamma_\nu)$ since $u^+ u = I$.

$\therefore (u \gamma_\mu u^+)(u \gamma_\nu u^+) = \gamma_\mu \cdot \gamma_\nu = 1 \quad \mu = \nu$
$= 0 \quad \mu \neq \nu$.

$\therefore$ the map is orthogonal.

v) At a given $\sigma = (-u) X u^+$ is a reflection of X in the plane perpendicular ① to that vector defined by the matrix u. Let σ act on the basis vectors of X (σ is a linear mapping, hence it is sufficient to consider the effect of σ on the basis vectors).

Pick an ② orthonormal basis $\{\gamma_0, u_1, u_2, u_3\}$ of X with each $u_i = v \gamma_i v^+$ for some $v \in SU(2)$ such that ③ $u_1 = u$. Then

$\sigma(u_i) = -u\, u_i\, u^+$.

If $u_i = u \quad \sigma(u_i) = -u_i \; (= u_1)$.

If $u_i \neq u \quad \sigma(u_i) = -u\, u_i\, u^+$

$= -v^+ \gamma_i v^+ v \gamma_j v^+ (v \gamma_i v^+)^+$

$= -v \gamma_i \gamma_j v^+ v \gamma_i^+ v^+$

$= -v \gamma_i \gamma_j \gamma_i^+ v^+$

$= v(-\gamma_i \gamma_j \gamma_i^+) v^+$

So we only have to check that

$-\gamma_i \gamma_j \gamma_i^+ = \gamma_j \quad$ if $j, i, j \neq 1$

$-\gamma_1 \gamma_2 \gamma_1^+ = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

① what do you mean "vector defined by matrix u"? We know that a hermitian matrix (e.g.) defines a vector, but u is not hermitian.

② what do you mean an orthonormal basis $\{\ \}$ of X"? A basis is for a linear space.

③ If you are looking for a basis of hermitian matrices, u will not do as it is not hermitian. $u \in SU(2)$ GROUP, not $u \in su(2)$ ALGEBRA.