

$$B(t) = e^{tA} B e^{-tA} = \sum_{m=0}^{\infty} \frac{t^m A^m}{m!} B \sum_{n=0}^{\infty} \frac{t^n A^n}{n!} \quad \text{MT}$$

$$\frac{d}{dt} B(t) = \frac{d}{dt} \left( \sum_{m=0}^{\infty} \frac{t^m A^m}{m!} \right) B \sum_{n=0}^{\infty} \frac{t^n A^n}{n!}$$

$$= \sum_{m=1}^{\infty} \frac{t^{m-1} A^m}{(m-1)!} B \sum_{n=0}^{\infty} \frac{t^n A^n}{n!} + \sum_{m=0}^{\infty} \frac{t^m A^m}{m!} B \sum_{n=1}^{\infty} \frac{t^{n-1} A^n}{(n-1)!}$$

$$= A \sum_{m=0}^{\infty} \frac{t^m A^m}{m!} B \sum_{n=0}^{\infty} \frac{t^n A^n}{n!} + \sum_{m=0}^{\infty} \frac{t^m A^m}{m!} B \sum_{n=0}^{\infty} \frac{t^n A^n}{n!} A$$

$$= A B(t) - B(t) A = [A, B(t)]$$