

where * represents a linear combination of base elements of the form $y_1^{j_1} y_2^{j_2} \dots y_s^{j_s} x y_1^{k_1} y_2^{k_2} \dots y_t^{k_t}$ with $\sum l_i > 1$. The second through the $(m+1)$ st term do not occur in expressions of this type for any other base element $y_1^{j_1} y_2^{j_2} \dots y_s^{j_s}$. It follows that in order that a shall be a linear combination of the base elements of the form $y_1^{k_1} \dots y_m^{k_m} x$ and $x y_1^{k_1} \dots y_s^{k_s}$, it is necessary that in the expression for a in terms of the chosen basis only base elements $y_1^{k_1} \dots y_m^{k_m}$ with $k_i = 1$ and all the other $k_j = 0$ occur with non-zero coefficient. Hence a is a linear combination of the y_i , hence $a \in L$, hence $a \delta = ax + x a$ if $a \in L$.

Now we can prove that $\log(1+z)$ is a Lie element

$$\begin{aligned} e^{A\delta} e^{B\delta} &= e^{A\delta} e^{B\delta} \\ &= e^{(Ax + xA)} e^{(Bx + xB)} \\ &= e^{Ax} e^{x A} e^{Bx} e^{x B} \\ &= (e^{Ax})(1 \times e^A)(e^{Bx})(1 \times e^B) \\ &= (e^A \times 1)(e^B \times 1)(1 \times e^A)(1 \times e^B) \\ &= (e^A e^B \times 1)(1 \times e^A e^B) \end{aligned}$$

$$\begin{aligned} \text{Hence } (\log(1+z))\delta &= \log((1+z)\delta) \\ &= \log(1+z \times 1) + \log(1 \times (1+z)) \\ &= \log(1+z) \times 1 + 1 \times \log(1+z) \end{aligned}$$

$$\therefore \log(1+z) \in L$$

Define the mapping σ by $A\sigma = A$
 $B\sigma = B$

if $x_i \in \mathcal{A}^{\mathbb{Z}}$, $(x_1, x_2, \dots, x_m)\sigma = [x_1, x_2, \dots, x_m]$. This mapping is linear. We prove:

A homogeneous element a of degree $m > 0$ is a Lie element iff $a\sigma = ma$ where