If $XY = YX$

$$e^X e^Y = \sum_{n=0}^{\infty} \frac{X^n}{n!} \sum_{m=0}^{\infty} \frac{Y^m}{m!}$$

$$= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{X^n}{n!} \frac{Y^m}{m!} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{Y^m}{m!} \frac{X^n}{n!}$$ since $XY = YX$ $\Rightarrow X^2Y = YX^2$ etc

$$= \sum_{m=0} \frac{Y^m}{m!} \sum \frac{X^n}{n!} = e^Y e^X$$

$$e^{X+Y} = \sum_{n=0}^{\infty} \frac{(X+Y)^n}{n!} = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{m=0}^{n} \binom{n}{m} X^m Y^{n-m}$$

since $XY = YX \Rightarrow X^2 Y = YX^2$ etc

$$\therefore e^{X+Y} = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} X^m Y^{n-m}$$

$$= \sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{X^m}{m!} \frac{Y^{n-m}}{(n-m)!} = \sum_{m=0}^{n}\sum_{n=0}^{\infty} \frac{X^m}{m!} \frac{Y^{n-m}}{(n-m)!}$$

In this expansion every term in $e^X e^Y$ occurs $\therefore e^{X+Y} = e^X e^Y$