(12)

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i m_{ij} \bar{x}_j = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i \bar{m}_{ji} \bar{x}_j$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i m_{ij} \bar{x}_j - x_i \bar{m}_{ji} \bar{x}_j = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i (m_{ij} - \bar{m}_{ji}) \bar{x}_j = 0 \quad (1)$$

As before $m_{ij} - \bar{m}_{ji} = 0$ for all i, j ✓
for take $x = e_i$ then

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i (m_{ij} - \bar{m}_{ji}) \bar{x}_j = (m_{ii} - \bar{m}_{ii}) = 0 \Rightarrow m_{ii} \text{ real} \quad (2)$$

take $x = (0\ 0\ 0 \ldots \underset{r}{1}\ 0\ 0 \ldots \underset{k}{1}\ 0\ 0 \ldots 0)$ ✓

$$\sum_{i=1}\sum_{j=1} x_i m_{ij} \bar{x}_j = m_{rr} + m_{rk} + m_{kr} + m_{kk} = a \text{ real no.}$$

m_{rr} and m_{kk} are real $\therefore$ we need
$m_{rk} + m_{kr} = a$, real
$\therefore$ if $m_{rk} = c + bi$ then $m_{kr} = (a-c) - bi$

Now put $x = (0\ 0 \ldots \underset{r}{1}\ 0 \ldots 0\ \underset{k}{i}\ 0 \ldots 0)$
then $\sum_{i=1}\sum_{j=1} x_i m_{ij} \bar{x}_j$ must be real. ✓

$= m_{rr} + m_{kk}(-i) + m_{kr}(i) + m_{kk}(i \times -i)$
$= m_{rr} + m_{kk} + i(m_{kr} - m_{rk})$
$m_{rr} + m_{kk}$ is real $\therefore i(m_{kr} - m_{rk})$ is real
Put $m_{kr} = a + bi$
$m_{rk} = c + di$

$i(a + bi - c - di) = d - b + i(a - c)$
so $a = c$ and $m_{rk} = a + bi \Rightarrow m_{kr} = a - bi$
$\therefore M$ is Hermitian (since m_{ii} real from (2)) ✓

(For $x_r = (0 \ldots 0, \underset{r}{a}, 0 \ldots 0)$ we require
$\sum\sum x_i m_{ij} \bar{x}_j = m_{rr} > 0 \Rightarrow m_{rr}$ is +ve) By
definition we want $xMx^{+} > 0$, for $x \neq 0$
M must be positive definite PTO ✓