

$$4) i) \{x, y+z\} = xM(y+z)^T \\ = xMy^T + xMz^T \\ = \{x, y\} + \{x, z\}$$

$$ii) \{x+y, z\} = (x+y)Mz^T \\ = xMz^T + yMz^T \\ = \{x, z\} + \{y, z\}$$

$$iii) \{a x, y\} = a xMy^T = (ax)My^T \\ = \{ax, y\}$$

$$\{x, ay\} = xM(ay)^T = xMy^T a^T \\ = xMy^T \bar{a} \text{ since } \bar{a} \text{ is } a^T$$

a complex number

$$\{x, ay\} = \{x, y\} \bar{a}$$

None of properties i) to iii) demand any conditions on M , since operation is linear, automatically

$$iv) \{x, y\} = xMy^T = \bar{x} \bar{M} y^T$$

$$My^T = (\bar{m}_{11}y_1 + \bar{m}_{12}y_2 + \dots + \bar{m}_{1n}y_n, \dots, \bar{m}_{m1}y_1 + \dots + \bar{m}_{mn}y_n) \\ = \left(\sum_{r=1}^n \bar{m}_{ir} y_r \right)_i^T$$

which is a vector in R^m , which i is the index.

$$\bar{x} My^T = \bar{x} \left(\sum_{r=1}^n \bar{m}_{ir} y_r \right)_i^T = \sum_{i=1}^m \sum_{r=1}^n \bar{x}_i \bar{m}_{ir} y_r$$

$$\{y, x\} = yMx^T$$

$$Mx^T = \left(\sum_{r=1}^n m_{ir} \bar{x}_r \right)_i^T, \text{ a vector in } R^n$$

$$yMx^T = y \left(\sum_{r=1}^n m_{ir} \bar{x}_r \right)_i^T = \sum_{i=1}^m \sum_{r=1}^n y_i m_{ir} \bar{x}_r$$

$$\bar{x} My^T = yMx^T \text{ demands}$$

$$\sum_{i=1}^m \sum_{r=1}^n \bar{x}_i \bar{m}_{ir} y_r = \sum_{i=1}^m \sum_{r=1}^n y_i m_{ir} \bar{x}_r$$