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which shows $(I-A)(I+A)^{-1}$ orthogonal.

iii) Since Q orthogonal $I - Q^TQ = 0$

$\therefore 2I - 2Q^TQ = 0$

$\therefore 2I + ((I-Q)-(I+Q))^T Q = 0$

$(I+Q)^T + (I-Q)^T + ((I-Q)-(I+Q))^T Q = 0$

$(I+Q)^T(I-Q) + (I-Q)^T(I+Q) = 0$

Since $\det(I+Q) \neq 0$ $(I+Q)^{-1}$ exists

$((I+Q)^T)^{-1}$ exists

$(I-Q) + ((I+Q)^T)^{-1}(I-Q)^T(I+Q) = 0$ multiplying on left ~~right~~ by $(I+Q)^T$

$(I-Q)(I+Q)^{-1} + ((I+Q)^T)^{-1}(I-Q)^T = 0$

$(I-Q)(I+Q)^{-1} + ((I-Q)(I+Q)^{-1})^T = 0$

which is of the form $B + B^T = 0$

$\therefore (I-Q)(I+Q)^{-1}$ is skew-symmetric. (10)

3) i) $\phi_1(g_1)\phi_1(g_2) = \bar{g}_1\bar{g}_2 = \phi_1(g_1g_2)$ $\therefore$ morphism property is satisfied.

$\phi_1(g_1) = \phi_1(g_2) \Rightarrow \bar{g}_1 = \bar{g}_2 \Rightarrow g_1 = g_2$ $\therefore \phi_1$ is one-one

all the 'onto' If $\bar{g}_1 \in \phi_1(G)$ there exists $g_1 \in G$ such that $\phi_1(g_1) = \bar{g}_1 \Rightarrow \phi_1$ is onto Im ϕ.

$\therefore \phi_1$ is an isomorphism

properties are given in Qn. ('image' property)

$\phi_2(g_1)\phi_2(g_2) = {}^tg_1^{-1}\,{}^tg_2^{-1} = {}^t(g_2^{-1}g_1^{-1}) = {}^t(g_1g_2)^{-1}$
$= \phi_2(g_1g_2)$

$\therefore$ morphism property is satisfied.

$\phi_2(g_1) = \phi_2(g_2) \Rightarrow {}^tg_1^{-1} = {}^tg_2^{-1}$

corresponding entries in each matrix match, so, $g_1^{-1} = g_2^{-1}$, but then $g_1g_2^{-1} = 1$

$\Rightarrow g_1 = (g_2^{-1})^{-1} = g_2$ $\therefore \phi_2$ is one-one.

If ${}^tg^{-1} \in \phi_2(G)$ there exists $g \in G$ such that $\phi_2(g) = {}^tg^{-1}$ $\therefore \phi$ is onto.

$\therefore \phi_2$ is an isomorphism

$\phi_3(g_1)\phi_3(g_2) = (g_1^+)^{-1}(g_2^+)^{-1} = (g_2^+g_1^+)^{-1}$
$= ({}^t\bar{g}_2\,{}^t\bar{g}_1)^{-1} = {}^t(\bar{g}_1\bar{g}_2)^{-1} = \phi_3(g_1)\phi_3(g_2)$