

(5)

$$\begin{aligned}
 \operatorname{Re}(x, x) &= \frac{1}{2} \operatorname{Re} \left(\sum_i \sum_j x_i m_{ij} \bar{x}_j + \sum_i \sum_j \bar{x}_i m_{ij} x_j \right) \quad [\text{Put } i \leftrightarrow j] \\
 &= \frac{1}{2} \operatorname{Re} \left(\sum_i \sum_j x_i m_{ij} \bar{x}_j + \sum_i \sum_j x_i m_{ji} \bar{x}_j \right) \\
 &= \frac{1}{2} \operatorname{Re} \left(\sum_i \sum_j x_i (m_{ij} + m_{ji}) \bar{x}_j \right) \\
 &= \frac{1}{2} \sum_i \sum_j x_i (m_{ij} + m_{ji}) \bar{x}_j \quad \text{by symmetry.}
 \end{aligned}$$

and we require ^{this} to be greater than or equal to zero. If x is not the zero vector then we require it to be greater than zero, so M must be +ve definite.

what do you mean by +ve definite?

(It's a tautology to say the standard

def: $x^T M x \geq 0$ only $= 0$ for $x = 0$ $\forall x$.)

By using prop. that hermit. n may

be diagonalized $U^H M U = \operatorname{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$

we can easily show that this property

is equivalent to all $\lambda_i > 0$.