If λ is a characteristic root of B, $\det(\lambda I - B) = 0$, so there exists a possibly complex vector $x \neq 0$ such that $(\lambda I - B)x = 0$, thus $Bx = \lambda x$.

Multiplying by $\bar{x}^T$ we have

$$\bar{x}^T B x = \lambda \bar{x}^T x \quad (1)$$

but $Bx = \lambda x \Rightarrow \bar{x}^T (\bar{B})^T = \bar{\lambda} \bar{x}^T$ and since B is Hermitian $B = (\bar{B})^T$ thus

$$\bar{x}^T B x = \bar{\lambda} \bar{x}^T x$$

From these two equations, (1) & (2) we see $(\lambda - \bar{\lambda}) \bar{x}^T x = 0$, and since $\bar{x}^T x \neq 0$, $\lambda = \bar{\lambda}$, so λ is real

Now let A be skew symmetric then iA is Hermitian, so if λ is any characteristic root of A then $\det(i\lambda I - iA) = 0$ so $i\lambda$ is purely real; ie λ is purely imaginary

$\therefore \det(I - A) \neq 0$

Also $I - A = I + A^T = (I + A)^T$, since $A + A^T = 0 \Rightarrow A = -A^T$

$$\det(I - A) = \det(I + A)^T = \det(I + A), \neq 0$$