Suppose $i_1, i_2 \in I(G)$ such that
$i_1(g) = T_1 g T_1^{-1}$
$i_2(g) = T_2 g T_2^{-1}$ for all $g \in G$
then if $T_1 C(G) = T_2 C(G)$
we must have $T_1^{-1} T_2 \in C(G)$
∴ $T_1^{-1} T_2$ commutes with every element of G ∴ $T_1^{-1} T_2 g (T_1^{-1} T_2)^{-1} = g$ so $i_1^{-1} i_2$ is the identity on G ∴ $T_1^{-1} = T_2^{-1}$
∴ $T_1 = T_2$
∴ the map $T \to T C(G)$ is one to one
If $T \in C(G)$ then $T g T^{-1} = g$ so conjugation by any element of $C(G)$ is the identity on G
If $T \in G - C(G)$ then since $I(G)$ is just the set of all conjugations of elements of G by elements of G, there exists $g \in G$ such that $T g T^{-1} \neq g$
∴ $T \notin C(G)$
∴ We can define an isomorphism thus. Take $T C(G) \in G/C$ with $T \notin C$ then $i : g \to T g T^{-1}$ is not the identity on G and $i \in I(G)$, and from the top of the page the correspondence
$T \to T C(G)$ is one-one.
We now know it is onto, and
$$(\sigma(i_1)\sigma(i_2))(g) = CT_1 CT_2\, g\, (CT_1)^{-1} (CT_2)^{-1}$$
$$= CT_1T_2\, g\, (CT_2T_1)^{-1}$$
$$= \sigma(i_1)\sigma(i_2)(g)$$
∴ the map σ from $I(G)$ to G/C satisfies the morphism property so $I(G)$ and $G/C(G)$ are isomorphic

8½