(14)

Since M is Hermitian so operates promiscuously on x and y.
Put $x = p^i$ in turn $(i = 1, \ldots, n)$
then $\{p^i, y\} = p^{i\dagger} M y = (M p^i)^\dagger y = \lambda_i p_i^\dagger y = 0$ for all p^i $\Rightarrow$ y is orthogonal to every basis vector in a space of dimension n so is identically zero (since y has n components)

The only problem is if any eigenvalue is zero. We know that $|\det P| = 1$
$\therefore |\det D| = |\prod \lambda_i| = |\det M|$ so if no eigenvalue is zero $|\det M| \neq 0$ and everything I've said holds.
ie we require $|\det M| \neq 0$, or that no eigenvalue is zero

You were asked in the question to "treat each property in isolation". This part has nothing to do with Hermiticity — nondegeneracy $\Leftrightarrow$ $\det M \neq 0$ so your conclusion is right.

7½