TMA 01 M824.

1) Take $A_1 = (a_{ij})$ $A_2 = (a'_{ij})$ using Curtis's notation, where $a_{ij}, a'_{ij} \in \mathbb{C}$ then $A_1, A_2 \in GL(n, \mathbb{C})$, which is a group.

$$\phi_0(A_1 A_2) = \begin{bmatrix} \det(A_1 A_2) & 0 \\ 0 & \overline{\det(A_1 A_2)} \end{bmatrix}$$

$$= \begin{bmatrix} \det A_1 \det A_2 & 0 \\ 0 & \overline{\det A_1 \det A_2} \end{bmatrix}, \text{ since } \det(A_1 A_2) = \det A_1 \det A_2$$

$$= \begin{bmatrix} \det A_1 & 0 \\ 0 & \overline{\det A_1} \end{bmatrix} \begin{bmatrix} \det A_2 & 0 \\ 0 & \overline{\det A_2} \end{bmatrix}$$

$$= \phi_0(A_1)\phi_0(A_2).$$

Moreover $\det(\phi_0(A)) = \det \begin{bmatrix} \det A & 0 \\ 0 & \overline{\det A} \end{bmatrix}$

* $= \det A \, \overline{\det A} = |\det A|^2 \neq 0$

since if $A \in GL(n, \mathbb{C})$, $|\det A| \neq 0$ $\therefore (\phi_0(A))^{-1}$ exists

Also $\operatorname{Im}\phi_0 = \begin{bmatrix} a & 0 \\ 0 & \bar{a} \end{bmatrix}$, $a\bar{a} = 1$ and ϕ is

PTO

onto $\operatorname{Im}\phi_0$ by construction $\therefore \phi_0$ induces the homomorphism described. $(\phi_0(I_n) = I_2)$.

If $A \in \operatorname{Ker}\phi_0$ then $\begin{bmatrix} \det A & 0 \\ 0 & \overline{\det A} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

the identity element of $GL(2, \mathbb{C})$ so $\det A = 1$ $\therefore$ $A \in SU(n)$ and $\operatorname{Ker}\phi = SU(n)$.

$\phi(A) = \begin{bmatrix} \det A & 0 \\ 0 & \overline{\det A} \end{bmatrix} \in \operatorname{Im}\phi$ and

$\det(\phi(A)) = \det A \, \overline{\det A} = 1$, since $A \in U(n) \Rightarrow |\det A| = 1$

$\operatorname{Im}(\phi) \cong T$, the standard maximal torus in $SU(2)$, and every element of $\operatorname{Im}(\phi)$ has the form $\begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix}$

$\cong U(1)$

Torus, but name! – S