The other possibilities are that we can consider $a^{-1} \circ i \circ a$ to define an isomorphism thus:

$$(a^{-1} \circ i \circ a)(g) = a^{-1} t \; a g t^{-1}$$
$$= a^{-1} t \; a g t^{-1} a^{-1} t a \, (a^{-1} t^{-1} a)$$
$$= a^{-1} t \; a (g t^{-1} a^{-1} t a)(a^{-1} t a)^{-1} \qquad *$$

Can we consider $a^{-1} \circ i \circ a$ to act on the element $g t^{-1} a^{-1} t a$ (I thought it possible, now I don't).

I am anyway puzzled why I can't write $(a^{-1} \circ i \circ a)(g) = (a^{-1} \circ i)\, ag = a^{-1} \circ (i(ag)) = a^{-1}(t^{-1} a g t)$
$= a^{-1} t^{-1} a g t$

As I want $(a^{-1} \circ a)(g) = g$ which works if a^{-1}, a act on the left (or both on the right). If I use this however, I don't think I can write the soln in the form opposite, but must use $*$.

Standard way $\quad \phi, \psi \in A(G)$

$$\phi \circ \psi\,(g_1 g_2) = \phi \circ \psi(g_1)\; \phi \circ \psi(g_2)$$

since $\phi \circ \psi$ is another element of $A(G)$

If $\quad \phi_\tau \in I(G) \qquad \phi_\tau(x) = \tau x \tau^{-1} \quad \binom{x \in G}{\tau \in G}$

$$\phi \circ \phi_\tau \circ \phi^{-1}(g) = \phi \circ \phi_\tau(\phi^{-1}(g))$$
$$= \phi\circ(\tau \phi^{-1}(g) \tau^{-1}) \quad \text{everything's ok}$$
$$= \phi(\tau)\; \phi \circ \phi^{-1}(g)\; \phi(\tau^{-1})$$
$$= \phi(\tau)\, g\, \phi(\tau)^{-1} \quad \text{prop. of Aut}$$
$$= \tau' g \tau'^{-1}$$