

$= (A_1 \circ A_2) \circ A_3(g)$ (Composition of functions is associative). $A(G)$ is a group.

b) $I(G)$ is that set of elements of $A(G)$ such that for $i \in I(G)$, there exists $T \in G$ such that $i(g) = T g T^{-1}$.

Closure

$$i_1 \circ i_2(g) = i_2(T_1 g T_1^{-1}) = T_2 T_1 g T_1^{-1} T_2^{-1} = T_2 T_1 g (T_2 T_1)^{-1}$$

$\therefore i_1 \circ i_2 \in I(G)$

Identity

$$i_e(g) = g = I g I^{-1} \therefore i_e \in I(G)$$

Inverses

$i \in I(G) \Rightarrow$ there exists $T \in G$ such that $i(g) = T g T^{-1}$, but then $T^{-1} \in G$ also since G is a group. If we define i' then

$$i'(g) = T^{-1} g T \text{ then } i \circ i'(g) = i'(T g T^{-1}) = T^{-1} T g T T^{-1} = g$$

and $i'^{-1} \in I(G)$

Normality: Let $a \in A(G)$, $i \in I(G)$

$$a \circ i \circ a(g)$$

Every automorphism $a \in A(G)$ can be represented by an invertible matrix, a say.

$$(a^{-1} \circ i \circ a)(g) = a^{-1} T g T^{-1} a = (a^{-1} T) g (a^{-1} T)^{-1} \in I(G)$$

the point is that not every automorphism a in $A(G)$ can be represented by an invertible matrix (cf. Part (iv)(c)).

$I(G)$ is a normal subgroup of $A(G)$

d) If $C \in C(G)$ then C commutes with every element of G . $C(g)T = TC(g)$

(NO)