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c) Remember that for all curves γ on $GL(n, \mathbb{R})$, $\gamma(0) = I$. Let $A = \gamma(t)$

$d\det \gamma'(0) = (\det A)'(0)$

We find $(\det A)'(0)$ by differentiating rows one at a time at $t=0$, and taking the determinants of the matrices, each with one row differentiated, then adding them all up.

$$(\det A)'(0) = \det \begin{bmatrix} a_{11}' & a_{12}' & \cdots & a_{1n}' \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{t=0}$$

$$+ \det \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21}' & a_{22}' & \cdots & a_{2n}' \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{t=0}$$

$$+ + \cdots + \det \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \\ a_{n1}' & a_{n2}' & \cdots & a_{nn}' \end{bmatrix}_{t=0}$$

At $t=0$, all $a_{ij}(t) = 0$ except a_{ii} $(=1)$ so each matrix reduces to a diagonal and a row of functions differentiated