$\therefore$ Morphism property is satisfied.

$\phi_3(g_1) = \phi_3(g_2) \Rightarrow {}^t(\bar{g}_1)^{-1} = {}^t(\bar{g}_2)^{-1}$. The transpose is unique since corresponding entries match $\therefore (\bar{g}_1)^{-1} = (\bar{g}_2)^{-1}$

$$((\bar{g}_1)^{-1})^{-1} = ((\bar{g}_2)^{-1})^{-1}$$

$$\therefore \bar{g}_1 = \bar{g}_2$$

Since inverses are unique, so $g_1 = g_2$ and ϕ_3 is one-one. If $(g^+)^{-1} \in \phi(G)$ there exists $g \in G$ such that $\phi_3(g) = (g^+)^{-1}$ $\therefore \phi_3$ is onto $\mathrm{Im}\,\phi_3$. $\therefore \phi_3$ is an isomorphism.

ii) $\phi(g_1)\phi(g_2) = Tg_1T^{-1}Tg_2T^{-1}$
$= Tg_1g_2T^{-1} = \phi(g_1g_2)$

(are you an ex nuc student!) $\therefore$ Morphism property is satisfied.

$\phi(g_1) = \phi(g_2) \Rightarrow Tg_1T^{-1} = Tg_2T^{-1}$

$\Rightarrow T^{-1}Tg_1T^{-1}T = T^{-1}Tg_2T^{-1}T$

$\Rightarrow g_1 = g_2$ since T^{-1} exists

$\therefore \phi$ is one one.

If $TgT^{-1} \in \phi(G)$ there exists $g \in G$ such that $\phi(g) = TgT^{-1}$ $\therefore \phi$ is onto $\mathrm{Im}\,\phi$ by construction.

iii) If $T \in G$, then since $g \in G$, $T^{-1}gT$ and $TgT^{-1} \in G$ by closure property of groups. But ϕ is an isomorphism from ii) (one-one and onto), for suppose $g \in G$ then $\phi(T^{-1}gT) = TT^{-1}gTT^{-1} = g$
$\therefore \phi(G) = G$ and ϕ is an automorphism

iv) $\phi_3(g) = (g^+)^{-1}$. If $g \in U(n)$ then g^{-1} exists and $g^{-1} = g^+$ $\therefore \phi_3(g) = g = IgI^{-1}$ $\therefore \phi_3$ is an inner isomorphism with $T = I$.