vi) $e_r = (0 \ldots 0, \underset{r}{1}, \ldots \ldots 0)$

$e_s = (0 \ldots, \underset{s}{1}, 0 \ldots \ldots \ldots 0)$

$\sum_{i=1}\sum_{j=1} e_{ri} m_{ij} e_{sj} = m_{rr}$ if $r = s$, $\{x_r, x_r\} = 1 \Rightarrow m_{rr} = 1$

$= m_{rs}$ if $r \neq s$, $\{x_r, x_s\} = 0 \Rightarrow m_{rs} = 0$

By running through all the elements of the standard basis and setting $\langle e_i, e_i \rangle$ and $\langle e_j, e_i \rangle = 0$ (for $i \neq j$) we see that the matrix M is the identity

vii) Every Hermitian matrix may be diagonalized by a unitary matrix P such that $D = P^{-1}MP$, then the columns of P are linearly independent orthogonal eigenvectors of M, for if p^i are such, then

$Mp^{iT} = PP^{-1}Mp^{iT} = Pd^{iT} = \lambda_i p_i^T$ (1)

where λ_i is the eigenvalue corresponding to the eigenvector p^i. Thus the eigenvectors p^i generate a space of dimension n, and

$\{x, p^i\} = xMp^{iT} = xPP^{-1}Mp^{iT}$

$= x\lambda_i p_i^{+} = \lambda_i x p_i^{+} = 0$

by hypothesis. Thus in a space of dimension n, x is orthogonal to every basis vector, so to every linear combination of basis vectors so is identically zero. (since x has n components).

$\{x, y\} = xMy^{+} = M^{+}xy^{+} = Mxy^{+}$