b) $\phi_2(g) = {}^t g^{-1} = {}^t(g^\dagger) = {}^t({}^t\bar{g}) = \bar{g} = \phi_1(g)$
Since g is any member of $U(n)$, $\phi_1 = \phi_2$

c) Suppose ϕ_1 is an inner isomorphism then there exists $T \in U(n)$ such that $TgT^{-1} = \bar{g}$ for all g. Then we have $Tg = \bar{g}T$

$\sum_r t_{ir} g_{rj} = \sum_r \bar{g}_{ir} t_{rj}$

$\therefore \sum_r t_{ir} g_{rj} - \bar{g}_{ir} t_{rj} = 0$ for all g

Take g to be the matrix with a 1 in the k,m position and all other entries zero, then Tg is the kth column of T in column m similarly $\bar{g}T$ is the mth row of T, and if we let g run through all the elementary matrices described, the only matrix T satisfying $\bar{g}T = Tg$ for all such g is the identity matrix, but put $g = iI$ then $-iI \neq Ii$. ϕ_1 is not an inner automorphism. Bit complicated — just consider $U(n) \ni g = iI$ & that will.

i) $A_1, A_2 \in A(G) \Rightarrow A_1(G) = A_2(G) = G$.
$A_1 \circ A_2(G) = A_1(G) = G \therefore A_1 \circ A_2 \in A(G)$
Call that A such that $A(g) = g$ for all g the identity automorphism, A_e then $A_e(G) = G \Rightarrow A_e \in A(G)$.
$A \in A(G) \Rightarrow A$ is one-one and onto therefore if $A(g) = g'$ there exists A^{-1} such that $A^{-1}(g') = g$ and since A is onto G, A^{-1} is onto G $\therefore A^{-1} \in A(G)$
$A_1(A_2 \circ A_3)(g) = A_1(A_2 \circ A_3(g)) = A_1 \circ A_2 \circ A_3(g)$ ✓
$=$