2

Im ϕ is a group, since $\phi(I)=\begin{bmatrix}1&0\\0&1\end{bmatrix}\in \mathrm{Im}\,\phi$

$\phi(A^{-1})=\phi(\bar{A}^T)=\overline{\phi(A^T)}=\begin{bmatrix}\overline{\det A}&0\\0&\det A\end{bmatrix}=\phi(A)^{-1}$

And from the morphism property we have closure. $\therefore$ Im ϕ is a group and it is abelian since

$$\begin{bmatrix}e^{i\theta}&0\\0&e^{-i\theta}\end{bmatrix}\begin{bmatrix}e^{i\phi}&0\\0&e^{-i\phi}\end{bmatrix}=\begin{bmatrix}e^{i(\theta+\phi)}&0\\0&e^{-i(\theta+\phi)}\end{bmatrix}=\begin{bmatrix}e^{i\phi}&0\\0&e^{-i\phi}\end{bmatrix}\begin{bmatrix}e^{i\theta}&0\\0&e^{-i\theta}\end{bmatrix}$$

$\therefore C(\mathrm{Im}\,\phi)=\mathrm{Im}\,\phi$

To find $C(SU(2))$

$$\left.\begin{matrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}=\begin{pmatrix}a&a+b\\c&c+d\end{pmatrix}\\ \begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a+c&b+d\\c&d\end{pmatrix}\end{matrix}\right\}\text{equal}$$

equating entries, $a=a+c\Rightarrow c=0$
$a+b=b+d\Rightarrow a=d$.

$$\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}=\begin{pmatrix}a+b&b\\c+d&d\end{pmatrix}$$

$$\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a&b\\a+c&b+d\end{pmatrix}$$

equating entries $a+b=a\Rightarrow b=0$

$\therefore$ any $x\in C(SU(2))$ has the form $\begin{pmatrix}a&0\\0&a\end{pmatrix}$, but $\det x=1\Rightarrow a^2=1\Rightarrow a=\pm1$

$\therefore C(SU(2))=\{I,-I\}$.

and $C(SU(2))\subseteq$ Im ϕ.

standard matrix group Im $\phi\cong U(1)$ 9½

Interlude

2i) If B is Hermitian ($B=(\bar{B})^T$) and