

(11)

Interchange the roles of  $i$  and  $r$  in the second summation

$$\sum_{i=1}^n \sum_{r=1}^n \bar{x}_i \bar{m}_{ir} y_r = \sum_{i=1}^n \sum_{r=1}^n y_r m_{ri} \bar{x}_i = \sum_{i=1}^n \sum_{r=1}^n \bar{x}_i m_{ri} y_r$$

$$\text{then } \sum_{i=1}^n \sum_{r=1}^n (\bar{x}_i \bar{m}_{ir} y_r - \bar{x}_i m_{ri} y_r) = 0$$

$$\therefore \sum_{i=1}^n \sum_{r=1}^n \bar{x}_i (\bar{m}_{ir} - m_{ri}) y_r = 0 \quad (1)$$

For this summation to be true for each  $i, r$  we must have

$$\bar{m}_{ir} = m_{ri} \quad (\text{take } x_i = (0, 0, 0, \dots, \underset{\substack{\uparrow \\ \text{rth place}}}{1}, 0, 0, \dots, 0)^T \\ y_r = (0, 0, \dots, \underset{\substack{\uparrow \\ \text{rth place}}}{1}, 0, 0, \dots, 0)^T)$$

then if  $\bar{m}_{ir} \neq m_{ri}$ , (1)  $\neq 0$ , but then  $M = M^+$  is  $M$  is Hermitian. ✓

$$1) \{x, x\} = x M x^+$$

$$M x^+ = \left( \sum_{j=1}^n m_{ij} \bar{x}_j \right)^T$$

$$x M x^+ = \sum_{i=1}^n \sum_{j=1}^n x_i m_{ij} \bar{x}_j \quad (1)$$

$$\text{If } x M x^+ \text{ is real, so is } (x M x^+)^+ = x M^+ x^+$$

$$M^+ x^+ = \left( \sum_{j=1}^n \bar{m}_{ji} \bar{x}_j \right)$$

$$x M^+ x^+ = \sum_{i=1}^n \sum_{j=1}^n x_i \bar{m}_{ji} \bar{x}_j \quad (2)$$

Equating (1) and (2) ✓