P6578708 (3)

$1 \in S$, for put $x=y=1$, then $1 \cdot 1 = 1$ which makes $1 \in T$ impossible (we would have $T \ni 1 \cdot 1 = 1 \in S$, clearly impossible for S, T disjoint).
Also from 1) the closure axiom to make S a group is satisfied.
Now if $x \in \mathbb{Z}_{31}$ (where $\mathbb{Z}_{31}$ is the set $\{1, \ldots, 30\}$ then x has a unique multiplicative inverse x^{-1} (mod 31), since 31 prime.
$\Rightarrow x x^{-1} = 1 \pmod{31}$ is defined
Suppose $x \in S$, $x^{-1} \in T$, then from 3. we have $x x^{-1} = 1 \in T$ but $1 \in S$, so we have a contradiction and $x^{-1} \in S$, so the inverse axiom is satisfied.
Modular multiplication is associative $\therefore$ ~~S~~ is a subgroup of $\{\mathbb{Z}_{31}, \times\} = A$, say)
A is cyclic, with generator 3, since $3^{\phi(31)/2} = 14348907 \equiv -1 \pmod{31}$

Not quite enough need to check $3^{\phi(31)/p}$ for all $p \mid \phi(31)$ i.e. p = 2, 3, 5

$\therefore A$ has a unique subgroup, also cyclic, for every divisor of $|A|$ in fact every subgroup of A is cyclic. The composition of each subgroup depends on the powers of the generator in it. Is $3 \in S$? No, for if it were then $3^2, 3^2 \cdot 3$, in fact every power of 3 would be in S and T would be empty
$\therefore 3 \in T \therefore$ from 2, $3^2 \in S$, from 3, $3^2 \cdot 3 \in T$ etc, with every odd power of 3 a member of T, every even power a member of S.
$\Rightarrow S = \{3^2, 3^4, 3^6, 3^8, 3^{10}, 3^{12}, 3^{14}, 3^{16}, 3^{18}, 3^{20}, 3^{22}, 3^{24}, 3^{26}, 3^{28}, 3^{30}\}$
reduced mod 31