find solutions (mod $p_i^{\alpha_i}$). For each congruence (mod $p_i^{\alpha_i}$) to have solutions it must have solutions mod p_i^a $a=1,\dots,\alpha_i$ ✓
If it has a soln (mod p), then it will have soln mod p^2 if $f'(r)\not\equiv 0 \bmod p$ or $f'(r)\equiv 0 \pmod p$ and $f(r)\equiv 0 \bmod p^2$ ✓
In the 1st case each solution (mod p) gives rise to a unique solution (mod p^2), ✓ and the second case has each solution (mod p) giving rise to p soln (mod p^2). ✓

The substance of the last paragraph generalises so if $f(r)\equiv 0 \pmod{p^k}$ has a soln r, and $f'(r)\not\equiv 0 \pmod p$ ✓ then we can find soln (mod p^{k+1}) If $f'(r)\equiv 0 \pmod p$, $f(r)\equiv 0 \pmod{p^{k+1}}$ each soln (mod p^k) ✓ gives rise to p solutions (mod p^{k+1}). If $f'(r)\equiv 0 \pmod p$, $0 \not\equiv f(r) \pmod{p^{k+1}}$, then r cannot give rise to a soln (mod p^{k+1}). ✓

For example, find solutions to $f(x)=2x^2+3\equiv 0 \pmod{25}$
Find solns to $f(x)=2x^2+3\equiv 0 \pmod 5$
$f(0)=3 \pmod 5$
$f(1)=5\equiv 0 \pmod 5$
$f(2)=2\cdot 2^2+3=1 \pmod 5$
$f(3)=2\cdot 3^2+3\equiv 1 \pmod 5$
$f(4)=2\cdot 4^2+3\equiv 0 \pmod 5$
1, 4 are solns
Now $f(1)\not\equiv 0 \pmod{25}$
$f(4)\equiv 35\not\equiv 0 \pmod{25}$
$f'(x)=4x \Rightarrow f'(1)=4\not\equiv 0 \pmod 5$
$f'(4)=16\not\equiv \pmod 5$