Exhaustion is painful as p gets large, and soon becomes impossible. In this case we can complete the square:

$$7x^2 + 29x + 18 \equiv 0 \mod 37$$

Since $7 \times 16 \equiv 1 \mod 37$, multiplying by 16 gives:

$$x^2 + 20x + 29 \equiv 0 \mod 37$$

$$\therefore (x+10)^2 \equiv 100 - 29 \mod 37$$
$$\equiv 34 \mod 37$$

$$\therefore x + 10 \equiv 16 \text{ or } 21 \mod 37$$
$$\therefore x \equiv 6 \text{ or } 11 \mod 37$$

~~(eg, by using~~ $\cancel{x^{\phi(37)/2}}$

Finding the square root of 34 mod 37 is a bit tricky but there are clever (and fast) ways to do it using GCDs.