$f(17) = 7\times17^2+29\times17+18 = 2534 \equiv 18 \pmod{37}$
$f(18) = 7\times18^2+29\times18+18 = 2808 \equiv 33 \pmod{37}$
$f(19) = 7\times19^2+29\times19+18 = 3096 \equiv 25 \pmod{37}$
$f(20) = 7\times20^2+29\times20+18 = 3398 \equiv 31 \pmod{37}$
$f(21) = 7\times21^2+29\times21+18 = 3714 \equiv 14 \pmod{37}$
$f(22) = 7\times22^2+29\times22+18 = 4044 \equiv 11 \pmod{37}$
$f(23) = 7\times23^2+29\times23+18 = 4388 \equiv 22 \pmod{37}$
$f(24) = 7\times24^2+29\times24+18 = 4746 \equiv 10 \pmod{37}$
$f(25) = 7\times25^2+29\times25+18 = 5118 \equiv 12 \pmod{37}$
$f(26) = 7\times26^2+29\times26+18 = 5504 \equiv 28 \pmod{37}$
$f(27) = 7\times27^2+29\times27+18 = 5904 \equiv 21 \pmod{37}$
$f(28) = 7\times28^2+29\times28+18 = 6318 \equiv 28 \pmod{37}$
$f(29) = 7\times29^2+29\times29+18 = 6746 \equiv 12 \pmod{37}$
$f(30) = 7\times30^2+29\times30+18 = 7188 \equiv 10 \pmod{37}$
$f(31) = 7\times31^2+29\times31+18 = 7644 \equiv 22 \pmod{37}$
$f(32) = 7\times32^2+29\times32+18 = 8144 \equiv 11 \pmod{37}$
$f(33) = 7\times33^2+29\times33+18 = 8592 \equiv 25 \pmod{37}$
$f(34) = 7\times34^2+29\times34+18 = 9096 \equiv 31 \pmod{37}$
$f(35) = 7\times35^2+29\times35+18 = 9608 \equiv 25 \pmod{37}$
$f(36) = 7\times36^2+29\times36+18 = 10134 \equiv 33 \pmod{37}$ ✓

$\therefore x \equiv 6, 11$ are solns $\pmod{37}$ ✓
Compare with $x \equiv 0, 1 \pmod{3}$
We see $x \equiv 6 \pmod{111}$ is a soln ✓
We need $6+37k$, $11+37k$ to be
0 or 1 $\pmod{3}$, and less than or equal to 111.
43, 80, given by $6+37k$
48, 85 given by $11+37k$.
From which $x \equiv 43, 48, 85 \pmod{111}$
are also solns $\therefore x \equiv 6, 43, 48, 85 \pmod{111}$ ✓
and $x = 6 + 111k$
$43 + 111k$
$48 + 111k$, $85 + 111k$ ✓
are also solns.

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