

Also this is the only solution, for suppose there were another solution. The composition of S and T are uniquely decided by the distribution of the generators. There are $3, 3^7, 3^{13}, 3^{17}, 3^{19}, 3^{23}, 3^{29}$. Since each power of a generator is also a generator if that power is relatively prime to 31, and these are the only generators. ✓

For the reason ^{on prev. page} every odd power of these generators is a member of T , and every even power is a member of S . ✓
Multiply any two generators: the result is an even power, so is a member of S .
(or since both are members of T , their product is a member of S). But there are exactly the sets S and T constructed overleaf, opposite. ✓

It is easy to see that S must contain all quadratic residues, since if $x \in S$ then $x^2 \in S$ and if $x \notin S$, $x \in T$ $\therefore x^2 \in S$. $\therefore |S| \geq 15$.
Since S is a subgroup of A , $|S| \mid |A|$. But $|A|$ is 30 so
either $|S| = 15$ and S is only the quadratic residues
or $|S| = 30$ and T is empty - a contradiction.

Therefore the solution given (S = quadratic residues, T = non-residues) is unique.