Please make sure that the assignment number is correctly entered on your PT3 form as

$\boxed{\text{M823 04}}$.

Question 1 (on Chapter 9)

(i) Without evaluating the Legendre symbols, prove the following.

(a) $$(1|73) + 2(2|73) + 3(3|73) + \cdots + 72(72|73) = 0.$$

(*Hint*: As r runs through the numbers $1, 2, \ldots, 72$, so does $73 - r$.)

(b) $$(1|71) + 2^2(2|71) + 3^2(3|71) + \cdots + 70^2(70|71) \\ \equiv 71\{(1|71) + 2(2|71) + \cdots + 70(70|71)\}.$$ [5]

(ii) By considering the number $N = 16p_1^2p_2^2 \ldots p_n^2 - 2$, prove that there are infinitely many primes of the form $8k - 1$. [4]

(iii) The set $\{1, 2, \ldots, 30\}$ is to be split into two disjoint non-empty sets S and T in such a way that:

(a) the product (mod 31) of any two elements of S lies in S;

(b) the product (mod 31) of any two elements of T lies in S;

(c) the product (mod 31) of any element of S and any element of T lies in T.

Prove that the *only* solution is

$$S = \{1, 2, 4, 5, 7, 8, 9, 10, 14, 16, 18, 19, 20, 25, 28\},$$
$$T = \{3, 6, 11, 12, 13, 15, 17, 21, 22, 23, 24, 26, 27, 29, 30\}.$$ [6]

(iv) Find *all* the solutions of the congruence

$$7x^2 + 29x + 18 \equiv 0 \pmod{111}.$$ [5]

Question 2 (revision)

Write an account of **one** of the following topics:

(i) the solution of polynomial congruences of the form $f(x) \equiv 0 \pmod{m}$;

(ii) some methods for proving that there are infinitely many primes of the form $an + b$ $(n = 0, 1, 2, \ldots)$, when $(a, b) = 1$.

Your account should be in the form of an essay of 3–6 pages. The aim of this question is to test your overall understanding of the material, rather than your ability to copy out pages from the textbook. Thus you should aim, whenever possible, for clear non-technical descriptions in words, quoting formulas only when you need to. [20]

Now that you have completed your assignments for M823, please complete the course evaluation sheet which was sent to you with this *Assignment Booklet*. Please return it to the Courses Office, Faculty of Mathematics and Computing, as directed. Thank you.