P65787708 ①

TMA M823 04

1) i(a) $\sum_{r=1}^{72} r(r|73) = \sum_{r=1}^{72}(73-r)(73-r|73)$ since $73 \equiv 1 \pmod 4$ (see overleaf).

$\Rightarrow \sum_{r=1}^{72} r(r|73) = \frac{1}{2}\left(\sum_{r=1}^{72} r(r|73) + (73-r)(73-r|73)\right)$

$= \frac{1}{2}\left(\sum_{r=1}^{72} 73(73-r|73) + \sum_{r=1}^{72} r(r|73) - r(73-r|73)\right)$

$= \frac{1}{2}\left(73\sum_{r=1}^{72}(73-r|73) + \sum_{r=1}^{72} r((r|73)-(73-r|73))\right)$ ✱

½ of the numbers $1 \le r \le 72$ are quadratic residues: $(r|73) = 1$

½ are quadratic non residues: $(r|73) = -1$

$\Rightarrow \sum_{r=1}^{72}(73-r|73) = 0$

✱ $= \frac{1}{2}\sum_{r=1}^{72} r((r|73)-(73-r|73))$ ①

$73 \equiv 1 \pmod 4 \Rightarrow (r|73) = (73-r|73)$

$\Rightarrow$ The sum ① is zero.

$\therefore \sum_{r=1}^{72} r(r|73) = 0.$

2½

(b) $\sum_{r=1}^{70} r^2(r|71) = \sum_{r=1}^{70}(71-r)^2(71-r|71)$

$= \sum_{r=1}^{70}(5041 - 142r + r^2)(71-r|71)$

$\Rightarrow \sum_{r=1}^{70} r^2(r|71) = \frac{1}{2}\left(\sum_{r=1}^{70} r^2(r|71) + (5041-142r+r^2)(71-r|71)\right)$

$= \frac{1}{2}\left(\sum_{r=1}^{70} 5041(71-r|71) - 142\sum_{r=1}^{70} r(71-r|71) + \right.$

$\left. + \sum_{r=1}^{70} r^2((r|71)+(71-r|71))\right)$

Again, $(r|71) = -(71-r|71)$ since $71 \equiv -1 \pmod 4$