P65787708 (8)

and then must solve the system

$$x \equiv a_1 \pmod{p_1}$$
$$x \equiv a_2 \pmod{p_2}$$
$$\vdots$$
$$x \equiv a_n \pmod{p_n}$$ ✓

The system is solved using the Chinese remainder theorem and we have, uniquely mod $\prod_{j=1}^{n} p_j$ ✓

$$x = \sum_{i=1}^{n} a_i \left(\prod_{\substack{j=1 \\ j \neq i}} p_j\right) m_i$$

where m_i is the inverse of $\prod_{\substack{j=1 \\ j \neq i}}^{n} p_j$ ✓ $\pmod{p_i}$. If one of the congruences $f(x) \equiv 0 \pmod{p_i}$ has no solution, there are no solutions to $f(x) \equiv 0 \pmod{p}$. There are in general more than one solution to each congruence. If there are $s(1)$ solutions to the 1st, $s(2)$ solutions to (2), ... $s(n)$ solutions to (n) then we will be able to form $s(1)s(2)\ldots s(n)$ n-tuples of solutions, and there will be the number of solutions $\pmod{m}\ (= \prod p_i)$ ✓

How to extend the method to an arbitrary modulus?

We write $m = \prod p_i^{a_i}$, and take our system to solve as

$$f(x) \equiv 0 \pmod{p_1^{a_1}}$$
$$f(x) \equiv 0 \pmod{p_2^{a_2}}$$
$$\vdots$$
$$f(x) \equiv 0 \pmod{p_n^{a_n}}.$$ ✓

If we can find solns to each of these congruences, then by the Chinese remainder theorem, we can ✓