Please make sure that the assignment number is correctly entered on your PT3 form as

$$\boxed{\text{M823 03}}.$$

Question 1 (on Chapter 6)

(i) (a) Construct a table of values for all the nonprincipal Dirichlet characters mod 16.

(b) Verify from your table that

$$\sum_{\chi \bmod 16} \chi(3) = 0 \quad \text{and} \quad \sum_{\chi \bmod 16} \chi(11) = 0.$$

(c) For each real-valued nonprincipal character χ mod 16, verify that

$$L(1,\chi) \neq 0.$$

(d) For each real-valued nonprincipal character χ mod 16, verify that

$$A(225) \geq 1.$$

$$\left(\text{Recall that } A(n) = \sum_{d|n} \chi(d).\right)$$ [12]

(ii) For each real-valued nonprincipal character χ mod k, let

$$A(n) = \sum_{d|n} \chi(d) \quad \text{and} \quad F(x) = \sum_{n \leq x} \frac{A(n)}{n}.$$

Prove that

$$F(x) = L(1,\chi)\log x + O(1).$$ [8]

Question 2 (on Chapter 7)

(i) By adapting the proof of Theorem 4.12, prove that

$$\sum_{\substack{p \leq x \\ p \equiv 4 \pmod 9}} \frac{1}{p} = \tfrac{1}{6}\log\log x + A + O(1/\log x),$$

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for some constant A. [8]

(ii) Let $f(x) = \sum_{\substack{p \leq x \\ p \equiv 4 \pmod 9}} 1$ and $g(x) = \sum_{\substack{p \leq x \\ p \equiv 4 \pmod 9}} \log p$.

(a) Find $f(50)$ and $g(50)$.

(b) Prove that $g(x) = f(x)\log x - \int_2^x t^{-1} f(t)\,dt$.

(c) Assuming that $f(x) \sim \frac{1}{6}\pi(x)$, prove that $g(x) \sim \frac{1}{6}x$.

(You may assume the Prime Number Theorem: $\pi(x) \sim x/\log x$.) [12]