

12

$$\begin{aligned}
 \text{ii) } e^x e^{x^2/2} e^{x^3/3} \dots &= e^{x + x^2/2 + x^3/3 + \dots} \\
 &= e^{\sum_{k=1}^{\infty} x^k/k} \\
 &= \frac{1}{e^{-\sum_{k=1}^{\infty} x^k/k}}
 \end{aligned}$$

$$\begin{aligned}
 \log(x) &= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots \\
 \Rightarrow \log(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \\
 \Rightarrow \log(1-x) &= -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} - \frac{(-x)^4}{4} + \dots \\
 &= -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right) \\
 &= -\sum_{k=1}^{\infty} \frac{x^k}{k}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \exp(\log(1-x)) &= \exp\left(-\sum_{k=1}^{\infty} \frac{x^k}{k}\right) \\
 1-x &= \exp\left(-\sum_{k=1}^{\infty} \frac{x^k}{k}\right)
 \end{aligned}$$

$$\frac{1}{1-x} = e^{\sum_{k=1}^{\infty} x^k/k}$$

$$\begin{aligned}
 \text{ii) } e^{-\sum_{k=1}^{\infty} x^k/k} &= \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \\
 &= \sum_{k=0}^{\infty} x^k
 \end{aligned}$$

$$\begin{aligned}
 e^x e^{x^2/2} e^{x^3/3} \dots &= \prod_{k=1}^{\infty} e^{x^k/k} \\
 &= \prod_{k=1}^{\infty} \left(\sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{x^k}{k}\right)^n \right)
 \end{aligned}$$

3.

Careful with writing fractions as $\binom{a}{b}$ is a mathematical symbol, but is not %.

Coefficient of x^{17} in sum of products