By theorem 5.23, each S_k is divisible by 823. Cancel 822! to give
$S_1 823^{821} + S_2 823^{820} - \ldots + S_{820}(823)^2$
$- S_{821} 823 = 0$
The expression equals 0 so equals $0 \pmod{823^3}$, but every term except the last is divisible by 823^3, and so therefore is the last. ie 823^2 divides S_{821} ✓

$\therefore 823^2$ divides $822! \times \frac{(r-s)}{823\ s}$ ✓

$\Rightarrow 823^3$ divides $822! \frac{(r-s)}{s}$ ✓

$\Rightarrow 823^3$ divides $r-s$, since 823 prime ✓

$\Rightarrow r-s \equiv 0 \pmod{823^3}$
$\Rightarrow r \equiv s \pmod{823^3}$ ✓ 4 pts.

iv) $f(x) = 3x^2 + 2x + 100 \equiv 0 \pmod{343}$
First find solns (mod 7)
$f(0) = 100 \equiv 2 \pmod 7$
$f(1) = 3+2+100 = 105 \equiv 0 \pmod 7$ ✓
$f(2) = 12+4+100 = 116 \equiv 4 \pmod 7$
$f(3) = 27+6+100 = 133 \equiv 0 \pmod 7$ ✓
$f(4) = 48+8+100 = 156 \equiv 2 \pmod 7$
$f(5) = 75+10+100 = 185 \equiv 3 \pmod 7$
$f(6) = 108+12+100 = 220 \equiv 3 \pmod 7$

There are 2 solns, $x = 1, 3$ ✓ because f is quadratic and all coefficients are coprime to 7.
$f'(x) = 6x+2$
$f'(1) = 6+2 = 8 \not\equiv 0 \pmod 7$ ✓
$f'(3) = 18+2 = 20 \not\equiv 0 \pmod 7$ ✓