

(3)

$$= \left(\frac{5(4)}{3}\right)^2 - \frac{\log x}{3x^3} + O\left(\frac{1}{x^3}\right) \checkmark$$

6.

$$\text{iii) } \sum_{n \leq x} \frac{\phi(n)}{n^{1/4}} = \sum_{n \leq x} \frac{1}{n^{1/4}} \sum_{d|n} \mu(d) \frac{n}{d} \checkmark$$

$$= \sum_{q \cdot d \leq x} \frac{1}{(qd)^{1/4}} \mu(d) \frac{qd}{d} \checkmark$$

$$= \sum_{d \leq x} \frac{\mu(d)}{d^{1/4}} \sum_{q \leq x/d} q^{3/4} \checkmark$$

$$= \sum_{d \leq x} \frac{\mu(d)}{d^{1/4}} \left(\frac{4}{7} \left(\frac{x}{d}\right)^{7/4} + O\left(\left(\frac{x}{d}\right)^{3/4}\right) \right) \checkmark$$

$$= \frac{4x^{7/4}}{7} \sum_{d \leq x} \frac{\mu(d)}{d^2} + O\left(x^{3/4} \sum_{d \leq x} \frac{|\mu(d)|}{d}\right)$$

need to take absolute value
here as $\mu(d)$ can go -ve.

$$= \frac{4x^{7/4}}{7} \left(\sum_{d=1}^{\infty} \frac{\mu(d)}{d^2} - \sum_{d > x} \frac{\mu(d)}{d^2} \right) + O\left(x^{3/4} \sum_{d \leq x} \frac{1}{d}\right) \checkmark$$

$$= \frac{4x^{7/4}}{7 \cdot \frac{6}{\pi^2}} - \frac{4x^{7/4}}{7} \sum_{d > x} \frac{\mu(d)}{d^2} + O\left(x^{3/4} (\log x + C + O\left(\frac{1}{x}\right))\right) \checkmark$$

$$= \frac{4x^{7/4}}{7 \cdot \frac{6}{\pi^2}} - x^{7/4} O\left(\sum_{d > x} \frac{1}{d^2}\right) + O(x^{3/4} \log x) \checkmark$$

$$= \frac{4x^{7/4}}{7 \cdot \frac{6}{\pi^2}} - x^{7/4} O\left(\frac{1}{x}\right) + O(x^{3/4} \log x) \checkmark$$

$$= \frac{4x^{7/4}}{7 \cdot \frac{6}{\pi^2}} + O(x^{3/4} \log x) \checkmark$$

which is of the required form
with $C = \frac{4}{7} \frac{1}{\frac{6}{\pi^2}}$ \checkmark

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