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$\therefore$ Both can be lifted to solns of
$f(x) = 0 \pmod{49}$
$f(1) = 15 \times 7 \Rightarrow k = 15$
we solve $8q + 15 \equiv 0 \pmod 7$
$q \equiv -1 \equiv 6 \pmod 7$
$a = 1 + 6 \times 7 = 43$
and $f(43) = 3 \times 43^2 + 2 \times 43 + 100$
$= 5733 \equiv 0 \bmod 49$
$f'(43) = 6 \times 43 + 2 = 260 \not\equiv 0 \pmod{49}$
$\therefore$ 43 can be lifted.
$f(43) = 117 \times 49 \Rightarrow k = 117$
we solve $260q + 117 \equiv 0 \pmod{\cancel{49}\, 7}$
$15q + 19 \equiv 0 \pmod{\cancel{49}\, 7}$
$15q \equiv -19 \equiv 30 \pmod{\cancel{49}\, 7}$
$q \equiv 2 \pmod{\cancel{49}\, 7}$
$a = 43 + 2 \times 49 = 141$
and $f(141) = 3 \times 141^2 + 2 \times 141 + 100$
$= 60025 \equiv 0 \pmod{343}$

q is defined mod 7, you only need to solve for it mod 7 at each lift.

The second case is $x = 3$
$f(3) = 133 = 19 \times 7 \Rightarrow k = 19$
We solve $20q + 19 \equiv 0 \pmod 7$
$-q \equiv 2 \pmod 7$
$q \equiv 5 \pmod 7$
$a = 3 + 5 \times 7 = 38$
$f(38) = 3 \times 38^2 + 2 \times 38 + 100$
$= 4508 \equiv 92 \times 49 \equiv 0 \pmod{49}$

$f'(38) = 6 \times 38 + 2 = 230 \equiv 34 \pmod{49}$
$\therefore$ This soln can be lifted uniquely
by Th 5.30 We solve $230q + 92 \equiv 0 \pmod{\cancel{49}\, 7}$
$34q + 43 \equiv 0 \pmod{\cancel{49}\, 7}$
$-15q \equiv 6 \pmod{\cancel{49}\, 7}$