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ii) $\theta_1(6.5) = \int^{6.5} \theta(t)\,dt$

$$= \int_1^{6.5} \sum_{p \le t} \log p \, dt \checkmark$$

Now $\sum_{p \le t} \log p = \sum_{p \le \text{Int}(t)} \log p$

So, if we split the integral up into several integrals, the upper limit of each being an integer one greater than the lower, and treat the last one separately then summation and integral will commute so ✓

$$\theta_1(6.5) = \int_1^2 \sum_{p \le 1} \log p \, dt + \int_2^3 \sum_{p \le 2} \log p \, dt$$

$$+ \int_3^4 \sum_{p \le 3} \log p \, dt + \int_4^5 \sum_{p \le 4} \log p \, dt + \int_5^6 \sum_{p \le 5} \log p \, dt$$

$$+ \int_6^{6.5} \sum_{p \le 6} \log p \, dt$$

$$= \sum_{p \le 1} \log p \int_1^2 dt + \sum_{p \le 2} \log p \int_2^3 dt$$

$$+ \sum_{p \le 3} \log p \int_3^4 dt + \sum_{p \le 4} \log p \int_4^5 dt + \sum_{p \le 5} \log p \int_5^6 dt$$

$$+ \sum_{p \le 6} \log p \int_6^{6.5} dt \checkmark$$