of all coefficients whose powers of $x$ multiply to give $x^{17}$,

$$e^{x}e^{x^2/2}e^{x^3/3}\cdots e^{x^{16}/16}e^{x^{17}/17}e^{x^{18}/18}\cdots$$

$$=e^{x}e^{x^{17}/17}\prod_{\substack{k=2\\k\neq 17}}^{\infty}e^{x^k/k}$$

$$=\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)\left(\sum_{m=0}^{\infty}\left(\frac{x^{17}}{17}\right)^m\frac{1}{m!}\right)\prod_{\substack{k=2\\k\neq 17}}^{\infty}\left(\sum_{r=0}^{\infty}\left(\frac{x^k}{k}\right)^r\frac{1}{r!}\right)$$

We want to find coefficient of $x^{17}$ in this expansion. We write it as

$$\left(1+x+\frac{x^2}{2!}+\cdots\frac{x^{17}}{17!}+\cdots\right)\left(1+\frac{x^{17}}{17}+\cdots\right)\left(1+\frac{x^2}{2}+\frac{1}{2!}\left(\frac{x^2}{2}\right)^2\cdots\right)$$
$$\left(1+\frac{x^3}{3}+\frac{1}{2!}\left(\frac{x^3}{3}\right)^2+\cdots\right)\cdots$$

coefficient of $x^{17}$ is

$$\left(\frac{1}{17!}+\frac{1}{17}+\underbrace{\left(\frac{(1/2)}{15!1!}+\frac{(1/3)}{14!1!}+\frac{(1/2)^2}{13!2!}+\frac{(1/4)}{13!1!}+\cdots\right)}\right)$$

The denominator of no term in this bracket is divisible by 17, $\therefore$ by property of addition of fractions so when we add all the terms up, the denominator will still not be divisible by 17. Therefore coefficient of $x^{17}$ is $\frac{1}{17!}+\frac{1}{17}+\frac{r}{s}$ where $17\nmid s$.

2.

From earlier in ii) the coefficient of every power of $x$ is 1.

$$\frac{1}{17!}+\frac{1}{17}+\frac{r}{s}=1.$$