

Your analysis assumes that one limit is approached always from below and the other always from above; however, this need not be true.  
Better proof:

$$\varepsilon \frac{\Theta(x)}{x} > \frac{1}{2} \frac{\Theta(x)}{\frac{1}{2}x^2} - \frac{(1-\varepsilon)^2}{2} \frac{\Theta(x-\varepsilon x)}{\frac{1}{2}(x-\varepsilon x)^2}$$

Taking limits as  $x \rightarrow \infty$ ,

$$\varepsilon > \frac{1}{2} - \frac{1}{2}(1-\varepsilon)^2$$

$$\Rightarrow (1 - \frac{1}{2}\varepsilon) \leq 1$$