$$\int_1^x (t-[t])\left(\frac{\lg t}{t^3}\right)' dt = \int_1^\infty (t-[t])\left(\frac{\lg t}{t^3}\right)' dt$$
$$- \int_x^\infty (t-[t])\left(\frac{\lg t}{t^3}\right)' dt$$

Now $\int_1^\infty (t-[t])\left(\frac{\lg t}{t^3}\right)' dt =$ constant

and for $x \geqslant e^{1/3}$, $\left(\frac{\lg t}{t^3}\right)' \leq 0$

$$\therefore\ 0 \leq -\int_x^\infty (t-[t])\left(\frac{\lg t}{t^3}\right)' dt \leq \left[-\frac{\lg t}{t^3}\right]_x^\infty = \frac{\lg x}{x^3}$$

$$\therefore\ \int_1^x (t-[t])\left(\frac{\lg t}{t^3}\right)' dt = K + O\left(\lg x/x^3\right)$$