(11)

$\frac{l}{2}(2\varepsilon+\varepsilon^2) + \frac{\varepsilon_1}{2}((1+\varepsilon)^2+1) > \varepsilon(1-\varepsilon_1)$

$l\varepsilon(1+\varepsilon/2) + \frac{\varepsilon_1}{2}((1+\varepsilon)^2+1+2\varepsilon) > \varepsilon$ ✓

We can take ε_1 to be arbitrarily small (since ε is given at this stage) by definition of 'tending to a limit' therefore $l\varepsilon(1+\varepsilon/2) \geq \varepsilon$

Better to use the same argument I gave for $l(1-\varepsilon/2) \leq 1$

$l(1+\varepsilon/2) \geq 1$ as required ✓

We have $(1-\varepsilon/2)l \leq 1 \Rightarrow l \leq \frac{1}{(1-\varepsilon/2)}$ ✓

$(1+\varepsilon/2)l \geq 1 \Rightarrow l \geq \frac{1}{(1+\varepsilon/2)}$ ✓

$\therefore \frac{1}{1+\varepsilon/2} \leq l \leq \frac{1}{1-\varepsilon/2}$ ✓

We can now let ε be arbitrarily small and by doing this we see $l=1$. ✓

$\therefore 1 = \lim_{x\to\infty} \frac{\Theta_1(x)}{\frac{1}{2}x^2}$

$\Theta_1(x) \sim \frac{x^2}{2}$ ✓

4.

(16)

3) i) We want $2000^{2000} \pmod{13}$

$2000^{2000} \equiv 11^{2000} \pmod{13}$ since $2000 \equiv 11 \pmod{13}$ ✓

$\equiv (11^{12})^{166} \times 11^8 \pmod{13}$ by rearrangement ✓

$\equiv 1^{166} \times 11^8 \pmod{13}$ by Fermats Little Th. ✓✓

$\equiv (-2)^8 \pmod{13}$ since $11 \equiv -2 \pmod{13}$ ✓

$\equiv 256 \pmod{13}$ ✓

$\equiv 9 \pmod{13}$ ✓

ie remainder is 9. ✓

4.