8

$$= \sum_{n \le [x]} (x-n)\Lambda_1(n) = \sum_{n \le x} (x-n)\Lambda_1(n)$$ 1.

$$\Theta_1(6.5) = \sum_{n \le 6.5} (6.5-n)\Lambda_1(n)$$

$$= (6.5-1)\Lambda_1(1) + (6.5-2)\Lambda_1(2) + (6.5-3)\Lambda_1(3) + (6.5-4)\Lambda_1(4) + (6.5-5)\Lambda_1(5) + (6.5-6)\Lambda_1(6)$$

$$= 0 + 4.5\log 2 + 3.5\log 3 + 0 + 1.5\log 5$$

$$= 4.5\log 2 + 3.5\log 3 + 1.5\log 5.$$ ✓ 1.

This is the answer from a).

c) $\Theta_1(x) - \Theta_1(x-\varepsilon x) = \int_1^x \Theta(t)\,dt - \int_1^{x-\varepsilon x} \Theta(t)\,dt.$ ✓

$$= \int_{x-\varepsilon x}^{x} \Theta(t)\,dt.$$ ✓

$\Theta(t)$ is an increasing function ✓

$\therefore \Theta(x) \ge \Theta(y)$ if $x \ge y$ ✓✓

$$\therefore \int_{x-\varepsilon x}^{x} \Theta(t)\,dt \le (x-(x-\varepsilon x))\Theta(x)$$ ✓

$$= \varepsilon x\,\Theta(x).$$ ✓

ie $\Theta_1(x) - \Theta_1(x-\varepsilon x) \le \varepsilon x\,\Theta(x)$ ✓

c) $L = \lim_{x\to\infty} \dfrac{\Theta_1(x)}{\frac{1}{2}x^2} = \lim_{x\to\infty} \dfrac{\Theta_1(x-\varepsilon x)}{\frac{1}{2}(x-\varepsilon x)^2}$ ✓ for ε small >0 & fixed

$\Theta_1(x) - \Theta_1(x-\varepsilon x) \le \varepsilon x\,\Theta(x)$ ✓

which implies

$$\frac{\Theta_1(x)}{x^2} - \frac{\Theta_1(x-\varepsilon x)}{x^2} \le \frac{\varepsilon\,\Theta(x)}{x}$$ ✓