$$= \frac{-1}{2x^2}\log x + 0 + \int_1^x \frac{dt}{2t^3}$$

$$= -\frac{1}{2x^2}\log x + \left[\frac{-1}{4t^2}\right]_1^x$$

$$= -\frac{\log x}{2x^2} - \frac{1}{4x^2} + \frac{1}{4}$$

Using (1) on previous page, we have

$$\sum_{n\le x}\frac{\log n}{n^3} = \cancel{\frac{1}{4}}\,k - \frac{\log x}{2x^2} - \frac{1}{4x^2} + O\left(\frac{\log x}{x^3}\right),$$ 5.

ii) $$\sum_{n\le x}\frac{d(n)}{n^4} = \sum_{n\le x}\frac{1}{n^4}\sum_{q\mid n} 1 = \sum_{qd\le x}\frac{1}{q^4 d^4}$$

Keep d fixed, then q ranges over all values $\le x/d$. Then

$$\sum_{n\le x}\frac{d(n)}{n^4} = \sum_{d\le x}\frac{1}{d^4}\sum_{q\le x/d}\frac{1}{q^4}$$

$$= \sum_{d\le x}\frac{1}{d^4}\left(\frac{-1}{3(x/d)^3} + O\left(\frac{1}{x/d}\right)^4 + \zeta(4)\right)$$

$$= \zeta(4)\sum_{d\le x}\frac{1}{d^4} - \frac{1}{3x^3}\sum_{d\le x}\frac{1}{d} + O\left(\frac{1}{x^4}\sum_{d\le x} 1\right)$$

$$= \zeta(4)\left(\frac{-1}{3x^3} + \zeta(4) + O\left(\frac{1}{x^4}\right)\right)$$

$$- \frac{1}{3x^3}\left(\log x + C + O\left(\frac{1}{x}\right)\right) + O\left(\frac{1}{x^3}\right)$$

$$= (\zeta(4))^2 - \frac{\log x}{3x^3} + O\left(\frac{1}{x^4}\right) - \frac{\zeta(4)}{3x^3} - \frac{C}{3x^3}$$

$$- \frac{1}{3x^3}\,O\left(\frac{1}{x}\right) + O\left(\frac{1}{x^3}\right)$$