You need to clear all the fraction before reducing - ie multiply by 17!s - or present your argument for s|16! first. 14

Multiply by 17! and reduce (mod 17)

$$1 + 16! + 17!\frac{r}{s} = 17! \equiv 0 \pmod{17}$$

Every term in the brackets on previous page has denominator which divides 16! ∴ the denominator of sum also divides 16! so $16!/s$ is an integer (if r/s has all common terms cancelled), and $17!\,r/s \equiv 0 \pmod{17}$

$$\therefore 1 + 16! \equiv 0 \pmod{17}$$

$$16! \equiv -1 \pmod{17}$$

1.

iii) Note 823 is prime. Follow Wolstenholmes Th

iii) $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{823} = \frac{r}{823s}$ ← you can quote this & use the result

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{822} = \frac{r}{823s} - \frac{1}{823}$$

$$= \frac{1}{823}\frac{(r-s)}{s}$$

LHS is $\sum_{k=1}^{822} \frac{1}{k} = \frac{1}{822!}\sum_{k=1}^{822}\frac{822!}{k}$

The sum is the sum of the products of the numbers 1, 2, 3, ..., 822 taken 821 at a time, and is also equal to the coefficient of $-x$, S_{821}, in the polynomial

$g(x) = (x-1)(x-2)\dots(x-822)$

When we expand this we find the coefficient of x^k is the k^{th} elementary symmetric polynomial $(-1)^k S_k$

$$g(x) = x^{822} - S_1 x^{821} + S_2 x^{820} - \dots + (-1)^{821} S_{821} x + (-1)^{822}(822)!$$

$$g(823) = 822! = 823^{822} - S_1 823^{821} + \dots (-1)^{821} S_{821}(823) + 822!$$