into the equation and collect the powers of ϵ^n, $n = 0, 1, \cdots$ to give

$$\begin{aligned}
\frac{dx_0}{dt} + \lambda x_0 &= 0, \\
\frac{dx_1}{dt} + \lambda x_1 &= x_0(t-a)^3, \\
\frac{dx_2}{dt} + \lambda x_2 &= 3x_0(t-a)^2 x_1(t-a).
\end{aligned}$$

The first equation gives $x_0(t) = Ae^{-\lambda t}$.

The second equation then gives

$$e^{-\lambda t}\frac{d}{dt}\left(e^{\lambda t}x_1\right) = A^3 e^{-3\lambda(t-a)}$$

so integration gives

$$\begin{aligned}
x_1(t) &= A^3 e^{-\lambda t} e^{3\lambda a} \int_0^t ds\, e^{-2\lambda s} \\
&= \frac{\left(Ae^{\lambda a}\right)^3}{2\lambda} e^{-\lambda t}\left(1 - e^{-2\lambda t}\right).
\end{aligned}$$

For the second term we have

$$e^{-\lambda t}\frac{d}{dt}\left(e^{\lambda t}x_2\right) = 3x_1(t-a)x_0(t-a)^2$$

which gives

$$e^{\lambda t}x_2(t) = \frac{3}{2\lambda}A^5 e^{3\lambda a}\int_0^t ds\,\left(e^{-2\lambda s}e^{3\lambda a} - e^{-4\lambda s}e^{5\lambda a}\right).$$

Hence

$$x_2(t) = \frac{3}{8\lambda^2}\left(Ae^{\lambda a}\right)^5 e^{-\lambda(t-a)}\left[2\left(1-e^{-2\lambda t}\right) - e^{2a\lambda}\left(1 - e^{-4\lambda t}\right)\right].$$