M821: TMA03 2001
Solution for question 1

Part i
A sketch of the graphs of the functions $y = 1/(1+x)$ and $y = \tan x$ is shown in the figure.

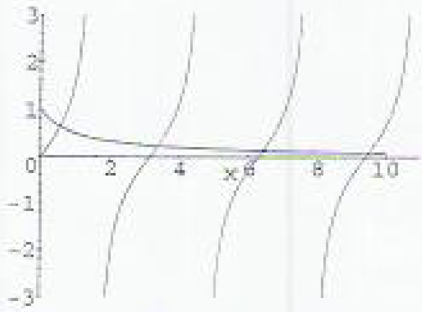

Figure 1 Graphs of $y = 1/(1+x)$ and $y = \tan x$.

From this figure we see that there will be positive solutions of the given equation near $x = n\pi$ where $n = 0, 1, \cdots$ if ϵ is sufficiently small. In order to find an approximation to the solution near $x = n\pi$ we first set $x = n\pi + y$, where y is a small quantity; since $\tan x = \tan y$ the equation becomes

$$\tan y = \frac{\epsilon}{1 + n\pi + y},$$

and since $\tan y \simeq y$ for small y this gives the first approximation as

$$y = \frac{\epsilon}{1 + n\pi} + O(\epsilon^2).$$

Now set $y = \epsilon y_1 + \epsilon^2 y_2$ where $y_1 = \frac{\epsilon}{1+n\pi}$ and use the expansion $\tan y = y + y^3/3 + O(y^5)$ to give

$$\frac{\epsilon}{1+n\pi} + \epsilon^2 y_2 = \frac{\epsilon}{1+n\pi}\left(1 - \frac{\epsilon y_1}{1+n\pi}\right) + O(\epsilon^3)$$

On equating the coefficients of the powers of ϵ^2 we obtain

$$y_2 = -\frac{y_1}{(1+n\pi)^2} = \frac{1}{(1+n\pi)^3}$$

and hence the required result.

Part ii
Put

$$x(t) = x_0(t) + \epsilon x_1(t) + \epsilon^2 x_2(t) + \cdots, \quad x_0(0) = A, \ x_k(0) = 0, \ k \geq 1.$$