(5)

The equation is satisfied for all ε in an interval, by assumption so equating coefficients of powers of ε to zero gives

$$\frac{\partial^2 Y_0}{\partial \eta^2} + \frac{\partial Y_0}{\partial \eta} = 0 \quad (1)$$

(3)

$$2\frac{\partial^2 Y_0}{\partial x \partial \eta} + \frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_0}{\partial x} + \frac{\partial Y_1}{\partial \eta} + x^2 Y_0^2 = 0 \quad (2)$$

(1) $\frac{\partial}{\partial \eta}\left(\frac{\partial Y_0}{\partial \eta} + Y_0\right) = 0$

$$\int \frac{\partial}{\partial \eta}\left(\frac{\partial Y_0}{\partial \eta} + Y_0\right) d\eta = \int 0 \, d\eta$$

$$\frac{\partial Y_0}{\partial \eta} + Y_0 = A(x)$$

$$\int \frac{\partial}{\partial \eta}\left(Y_0 e^{\eta}\right) d\eta = \int A(x) e^{\eta} d\eta$$

$$Y_0 e^{\eta} = A(x) e^{\eta} + B(x)$$

$$\Rightarrow Y_0(x,\eta) = A(x) + B(x) e^{-\eta}.$$

Sub into (2).

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} + 2\frac{\partial^2 Y_0}{\partial x \partial \eta} + \frac{\partial Y_0}{\partial x} + x^2 Y_0^2 = 0$$

$$\frac{\partial Y_0}{\partial \eta} = -B(x) e^{-\eta} \Rightarrow \frac{\partial^2 Y_0}{\partial x \partial \eta} = -\frac{\partial B}{\partial x} e^{-\eta}$$

$$\frac{\partial Y_0}{\partial x} = \frac{\partial A}{\partial x} + \frac{\partial B}{\partial x} e^{-\eta}$$

$$x^2 Y_0^2 = x^2 A^2 + 2x^2 A B e^{-\eta} + x^2 B^2 e^{-2\eta}$$

Then we have

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} - 2\frac{\partial B}{\partial x} e^{-\eta} + \frac{\partial A}{\partial x} + \frac{\partial B}{\partial x} e^{-\eta} + x^2 A^2$$

$$+ 2x^2 A B e^{-\eta} + x^2 B^2 e^{-2\eta} = 0$$

(3)