④

$$\therefore x(t) = Ae^{-\lambda t} + \frac{\varepsilon}{2\lambda}(Ae^{\lambda a})^3 e^{-\lambda t}(1-e^{-2\lambda t})$$
$$+\frac{3\varepsilon^2}{8\lambda^2}(Ae^{a\lambda})^5 e^{-\lambda(t-a)}(2(1-e^{-2\lambda t})-e^{2a\lambda}(1-e^{-4\lambda t}))$$
$$+O(\varepsilon^3)$$

(20)

Total (29)

2) a) $y = \sum_{k=0}^{\infty} \varepsilon^k Y_k$

$$\frac{dy}{dx} = \sum_{k=0}^{\infty} \varepsilon^k \frac{d}{dx}(Y_k(x,\eta))$$
$$= \sum_{k=0}^{\infty} \varepsilon^k \frac{\partial Y_k}{\partial x} + \varepsilon^k \frac{\partial Y_k}{\partial \eta}\frac{\partial \eta}{\partial x}$$

Please write $\frac{\partial \eta}{\partial x}$ and not $\frac{\partial \eta}{\partial x}$ as you do everywhere here

$$\eta = x/\varepsilon \Rightarrow \frac{\partial \eta}{\partial x} = \varepsilon^{-1}$$

$$\therefore \frac{dy}{dx} = \sum_{k=0}^{\infty} \varepsilon^k \frac{\partial Y_k}{\partial x} + \varepsilon^{k-1}\frac{\partial Y_k}{\partial \eta}$$
$$\frac{d^2y}{dx^2} = \sum_{k=0}^{\infty} \varepsilon^k \frac{d}{dx}\left(\frac{\partial Y_k}{\partial x}\right) + \varepsilon^{k-1}\frac{d}{dx}\left(\frac{\partial Y_k}{\partial \eta}\right)$$
$$= \sum_{k=0}^{\infty} \varepsilon^k \frac{\partial^2 Y_k}{\partial x^2} + \sum_{k=0}^{\infty} \varepsilon^k \frac{\partial^2 Y_k}{\partial x \partial \eta}\frac{\partial \eta}{\partial x} + \varepsilon^{k-1}\frac{\partial^2 Y_k}{\partial x \partial \eta}$$
$$+ \varepsilon^{k-1}\frac{\partial^2 Y_k}{\partial \eta^2}\frac{\partial \eta}{\partial x}$$
$$= \sum_{k=0}^{\infty} \varepsilon^k \frac{\partial^2 Y_k}{\partial x^2} + 2\varepsilon^{k-1}\frac{\partial^2 Y_k}{\partial x \partial \eta} + \varepsilon^{k-2}\frac{\partial^2 Y_k}{\partial \eta^2}$$

$\varepsilon \frac{d^2y}{dx^2} + \frac{dy}{dx} + (xy)^2 = 0$ becomes

$$\sum_{k=0}^{\infty} \varepsilon^{k+1}\frac{\partial^2 Y_k}{\partial x^2} + 2\varepsilon^k \frac{\partial^2 Y_k}{\partial x \partial \eta} + \varepsilon^{k-1}\frac{\partial^2 Y_k}{\partial \eta^2}$$
$$+ \sum_{k=0}^{\infty} \varepsilon^k \frac{\partial Y_k}{\partial x} + \varepsilon^{k-1}\frac{\partial Y_k}{\partial \eta} + \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} x^2 \varepsilon^{k+j} Y_k Y_j$$
$$= \frac{1}{\varepsilon}\left(\frac{\partial^2 Y_0}{\partial \eta^2} + \frac{\partial Y_0}{\partial \eta}\right) + \left(2\frac{\partial^2 Y_0}{\partial x \partial \eta} + \frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_0}{\partial x} + \frac{\partial Y_1}{\partial \eta}\right.$$
$$\left. + x^2 Y_0^2\right) + O(\varepsilon)$$