so this equation becomes

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} + x^2 B^2 e^{-2\eta} + \left(2x^2 AB - \frac{dB}{dx}\right) e^{-\eta} + \frac{dA}{dx} + x^2 A^2 = 0. \tag{5}$$

Part iii
In solving this equation x is considered to be a constant, so the particular integral contains three parts: one each from the exponential terms, $e^{-2\eta}$ and $e^{-\eta}$, and one from the last term. Now the equation

$$\frac{d^2 f}{d\eta^2} + \frac{df}{d\eta} = \alpha + \beta e^{-\eta}$$

where α and β are constants, has the general solution

$$f(\eta) = \alpha\eta - \beta(1+\eta)e^{-\eta} + C_1 + C_2 e^{-\eta}$$

where C_1 and C_2 are constants. Since $\eta = x/\epsilon$, $f \sim x/\epsilon$ unless $\alpha = \beta = 0$. Thus unless $\alpha = \beta = 0$, $\epsilon Y_1 \sim Y_0$; but in writing the solution in the form of equation 1 it is assumed that $|\epsilon Y_1| \ll |Y_0|$ and for this to be true we need $\alpha = \beta = 0$ or

$$\frac{dA}{dx} = -x^2 A^2 \tag{6}$$

$$\frac{dB}{dx} = 2x^2 AB. \tag{7}$$

Then Y_1 satisfies the equation

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} = -x^2 B^2 e^{-2\eta}. \tag{8}$$

Part iv
Equation 6 for $A(x)$ can be solved to give

$$A(x) = \frac{3}{\alpha + x^3},$$

for some constant α. On dividing equation 6 by equation 7 we obtain

$$\frac{dA}{dB} = -\frac{A}{2B} \quad \Longrightarrow \quad B(x) = \frac{c}{A(x)^2} = \frac{c}{9}(\alpha + x^3)^2,$$

for some constant c.

Finally, a particular integral of equation 8 is

$$Y_1(x,\eta) = -\frac{1}{2}x^2 B^2 e^{-2\eta} = -\frac{x^2 c^2}{162}(\alpha + x^3)^4 e^{-2\eta}.$$

There is also a contribution $C_1(x) + C_2(x)e^{-\eta}$ having the same form as Y_0 which we ignore. If the calculation were to be carried though to $O(\epsilon^2)$ then it would