Please make sure that the assignment number is correctly entered on your PT3 form as

$$\boxed{\text{M821 03}}.$$

Question 1 – 30 marks

(a) By sketching the graphs of $y = \tan x$ and $y = 1/(1+x)$ for $x \geq 0$ show that the equation

$$\tan x = \frac{\epsilon}{1+x}, \quad 0 < \epsilon \ll 1,$$

has roots near $x = n\pi$, $n = 0, 1, 2, \cdots$.

Use perturbation theory to show that the root near $n\pi$ is

$$x = n\pi + \frac{\epsilon}{1+n\pi} - \frac{\epsilon^2}{(1+n\pi)^3} + O(\epsilon^3).$$ [10]

(b) Consider the differential-delay equation

$$\frac{dx}{dt} = -\lambda x + \epsilon x(t-a)^3, \quad |\epsilon| \ll 1,$$

where λ and a are positive constants and the notation $x(t-a)$ means the function $x(t)$ evaluated at $t-a$.

Use perturbation theory to show that an approximate solution satisfying the condition $x(0) = A$ is

$$\begin{aligned} x(t) \;=\; & Ae^{-\lambda t} + \frac{\epsilon}{2\lambda}\left(Ae^{\lambda a}\right)^3 e^{-\lambda t}\left(1 - e^{-2\lambda t}\right) \\ & + \frac{3\epsilon^2}{8\lambda^2}\left(Ae^{a\lambda}\right)^5 e^{-\lambda(t-a)}\left\{2\left(1-e^{-2\lambda t}\right) - e^{2a\lambda}\left(1-e^{-4\lambda t}\right)\right\}. \end{aligned}$$

[20]