

$$\int \frac{\partial}{\partial \eta} (Y_1 e^\eta) = \int + \frac{9x^2}{2\alpha^6} (\alpha + x^3)^4 e^{-\eta} d\eta$$

$$Y_1 e^\eta = - \frac{9x^2}{2\alpha^6} (\alpha + x^3)^4 e^{-\eta}$$

$$Y_1 = - \frac{9x^2}{2\alpha^6} (\alpha + x^3)^4 e^{-2\eta}$$

$$y(\epsilon, x) = \frac{3}{\alpha + x^3} - \frac{3}{\alpha^3} (\alpha + x^3)^2 e^{-x/\epsilon}$$

$$- \frac{9}{2\alpha^6} \epsilon x^2 (\alpha + x^3)^4 e^{-2x/\epsilon} + O(\epsilon^2)$$

Why did you not need to fit initial conditions to the general soln here?? See model for this.

3) a) when $\epsilon=0$ equation reduces to $\ddot{x} + x = 0$ which has a soln $x = a \cos t$ with angular frequency $\omega=1$. Assume that perturbed soln has angular frequency close to 1 and put $\omega = 1 + \epsilon\omega_1 + O(\epsilon^2)$, put also that perturbed soln is close to unperturbed soln so put $x = x_0 + \epsilon x_1 + O(\epsilon^2)$, $x_0 = a \cos \tau$, with $\tau = \omega t$.

$$\frac{dx}{dt} = \frac{\partial x}{\partial \tau} \frac{\partial \tau}{\partial t} = \omega x'$$

$$\frac{d^2 x}{dt^2} = \omega x'' \text{ so modified eqn becomes.}$$

$$\omega^2 x'' + \epsilon(x^2 - 1)\omega x' + x + \epsilon \alpha^2 x^3 = 0$$

$$(1 + 2\epsilon\omega_1)(x_0'' + \epsilon x_1'') + \epsilon(x_0^2 + \epsilon x_1^2 - 1)(1 + \epsilon\omega_1)(x_0' + \epsilon x_1') + x_0 + \epsilon x_1 + \epsilon \alpha^2 (x_0 + \epsilon x_1)^3 + O(\epsilon^2) = 0$$

$$+ x_0 + \epsilon x_1 + \epsilon \alpha^2 (x_0 + \epsilon x_1)^3 + O(\epsilon^2) = 0$$

$$(1 + 2\epsilon\omega_1)(x_0'' + \epsilon x_1'') + \epsilon(x_0^2 - 1)x_0' + x_0 + \epsilon x_1 + \epsilon \alpha^2 x_0^3 + O(\epsilon^2) = 0$$