7

$$\ln(BA^2) = C \Rightarrow BA^2 = e^c = K$$

$$\therefore B = K/A^2$$ ✓

d) ① $\Rightarrow \int A^{-2}\,dA = \int -x^2\,dx$

$$-A^{-1} = -x^3/3 + C = \frac{3C - x^3}{3}$$

$$A = \frac{3}{(-3C) + x^3} = \frac{3}{\alpha + x^3}, \quad \alpha = -3C$$

From c) $B = \frac{K(\alpha + x^3)^2}{9}$. Assume that

when $\varepsilon = 0$ $Y_1 = 0$ and $y = A(x) + B(x)$
then $y(0,0) = A(0) + B(0) = 0$ from
the condition given in a)
so

$$\frac{3}{\alpha} + \frac{K\alpha^2}{9} = 0 \Rightarrow K = -\frac{27}{\alpha^3}$$

then $Y_0 = \frac{3}{\alpha + x^3} - \frac{27}{\alpha^3} \cdot \frac{(\alpha + x^3)^2}{9} e^{-\eta}$

$$= \frac{3}{\alpha + x^3} - \frac{3(\alpha + x^3)^2}{\alpha^3} e^{-x/\varepsilon}.$$

Sub. for $B(x)$ into ③

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} = -x^2 B^2 e^{-2\eta}$$

$$\frac{\partial}{\partial \eta}\left(\frac{\partial Y_1}{\partial \eta} + Y_1\right) = -\frac{9x^2}{\alpha^6}(\alpha + x^3)^4 e^{-2\eta}.$$

$$\int \frac{\partial}{\partial \eta}\left(\frac{\partial Y_1}{\partial \eta} + Y_1\right) d\eta = \int -\frac{9x^2}{\alpha^6}(\alpha + x^3)^4 e^{-2\eta}\,d\eta$$

$$\frac{\partial Y_1}{\partial \eta} + Y_1 = +\frac{9x^2}{2\alpha^6}(\alpha + x^3)^4 e^{-2\eta}$$

$$\frac{\partial}{\partial \eta}(Y_1 e^{\eta}) = +\frac{9x^2}{2\alpha^6}(\alpha + x^3)^4 e^{-\eta}$$