$\Rightarrow a = 0$ or 2 but then $x_0 = 0$ if $a=0$ for all t.

$\therefore a = 2$. If $a=2$ $\omega_1 = 3\alpha^2/8 \times 2^2 = 3\alpha^2/2$

With these values the equation for x_1 becomes.

$$x_1'' + x_1' = \frac{a^3}{4}\sin 3\tau - \frac{\alpha^2 a^3}{4}\cos 3\tau$$
$$= 2\sin 3\tau - 2\alpha^2 \cos 3\tau$$

Put $x_1 = A\sin 3\tau + B\cos 3\tau$

$$x_1'' = 9A\sin 3\tau - 9B\cos 3\tau$$
$$x_1'' + x_1 = -8A\sin 3\tau - 8B\cos 3\tau$$
$$= 2\sin 3\tau - 2\alpha^2 \cos 3\tau$$

So $A = -\frac{2}{8} <= -\frac{1}{4}$

$$B = \frac{\alpha^2}{4}$$

the particular soln

$$x_p = -\frac{1}{4}\sin 3\tau + \frac{\alpha^2}{4}\cos 3\tau$$

Homogeneous soln is

$$x_1 = C\cos\tau + D\sin\tau$$

So general soln is

$$x_1 = C\cos\tau + D\sin\tau - \frac{1}{4}\sin 3\tau + \frac{\alpha^2}{4}\cos 3\tau$$

We need $x_1(0) = x_1'(0) = 0$

$$0 = C + \frac{\alpha^2}{4} \Rightarrow C = -\alpha^2/4$$

$$0 = D - 3/4 \Rightarrow D = 3/4$$

$$\therefore x_1 = \frac{\alpha^2}{4}(-\cos\tau + \cos 3\tau) + \frac{3}{4}(3\sin\tau - \sin 3\tau)$$

$$= \frac{1}{4}((\sin 3\tau - 3\sin\tau) + \alpha^2(\cos\tau - \cos 3\tau))$$

$$= \frac{1}{4}((\sin 3\tau - 3\sin\tau) - \alpha^2(\cos 3\tau - \cos\tau))$$

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