⑥

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} + (xB)^2 e^{-2\eta} + \left(2x^2AB - \frac{\partial B}{\partial x}\right)e^{-\eta}$$
$$+\frac{\partial A}{\partial x} + x^2A^2 = 0$$

The soln for Y_0 contains a term multiplied by $e^{-\eta} = e^{-x/\varepsilon}$. Assume that as $\varepsilon \to 0$ Y_1 tends smoothly to zero faster than any term in Y_0, and so faster than any function of x times $e^{-\eta}$. Hence in the limit as $\varepsilon \to 0$ we can isolate the terms in the above equation according to whether they are multiplied by $e^{-\eta}$, by no exponential of $-\eta$, and we can pair the $e^{-2\eta}$ term with the differential of Y_1, since if $Y_1 \to 0$ faster than $e^{-\eta}$, so will $\frac{\partial Y_1}{\partial \eta}$ and $\frac{\partial^2 Y_1}{\partial \eta^2}$. We obtain the three equations

No, you need to constrain the general soln of a 2nd order d.e. to make Y_1 small enough. See model

$$\frac{dA}{dx} + x^2A^2 = 0 \Rightarrow \frac{dA}{dx} = -x^2A^2 \qquad ①$$

$$2x^2AB - \frac{dB}{dx} = 0 \Rightarrow \frac{dB}{dx} = 2ABx^2 \qquad ②$$

$$\frac{\partial^2 Y_1}{\partial \eta^2} + \frac{\partial Y_1}{\partial \eta} = -x^2B^2 e^{-2\eta} \qquad ③$$

$\frac{5}{10}$

$$① \div ② \Rightarrow \frac{dA/dx}{dB/dx} = \frac{-A}{2B} = \frac{dA}{dB}$$

$$\int \frac{dB}{B} = \int \frac{-2\,dA}{A}$$

$$\ln(B) = -2\ln A + C$$

$$\ln B + 2\ln A = C$$