(2)

$$= \frac{\varepsilon}{1+n\pi}\left(1+\frac{\varepsilon x_1}{1+n\pi}+\frac{\varepsilon^2 x_2}{1+n\pi}\right)^{-1}+O(\varepsilon^3)$$

$$= \frac{\varepsilon}{1+n\pi}\left(1-\frac{\varepsilon x_1}{1+n\pi}\right)+O(\varepsilon^3)$$

$$= \frac{\varepsilon}{1+n\pi}-\frac{\varepsilon^2 x_1}{(1+n\pi)^2}+O(\varepsilon^3)$$

This is true for all ε in an interval containing zero so we can match powers of ε to obtain

$$\varepsilon x_1+\varepsilon^2 x_2 = \frac{\varepsilon}{1+n\pi}-\frac{\varepsilon^2 x_1}{(1+n\pi)^2}$$

$$x_1 = \frac{1}{1+\pi n},\quad x_2 = -\frac{x_1}{(1+\pi n)^2} = -\frac{1}{(1+n\pi)^3}$$

$$x = n\pi + \frac{\varepsilon}{1+n\pi} - \frac{\varepsilon^2}{(1+n\pi)^3}$$ ✓

$\frac{9}{10}$

This is not the most straightforward expansion - see model soln.

b) Put $x = x_0+\varepsilon x_1+\varepsilon^2 x_2+O(\varepsilon^3)$

t dependency?

$$\frac{dx_0}{dt}+\varepsilon\frac{dx_1}{dt}+\varepsilon^2\frac{dx_2}{dt} = -\lambda x_0-\varepsilon\lambda x_1-\varepsilon^2\lambda x_2+O(\varepsilon^3)$$

$$+\varepsilon\left(x_0^3(t-a)+3\varepsilon x_0(t-a)x_1(t-a)\right)+O(\varepsilon^3)$$

Match powers of ε for 0, 1, 2 to obtain the three equations

$$\frac{dx_0}{dt} = -\lambda x_0 \quad ✓ \qquad (1)$$

$$\frac{dx_1}{dt} = -\lambda x_1 + x_0^3(t-a) \quad ✓ \qquad (2)$$

✓ good

$$\frac{dx_2}{dt} = -\lambda x_2 + 3x_0^2(t-a)x_1(t-a) \qquad (3)$$

$$(1) \Rightarrow \int_A^{x_0}\frac{dx_0}{x_0} = -\int_0^t \lambda\, dt$$

$$\ln(x_0/A) = -\lambda t$$

$$\therefore x_0 = Ae^{-\lambda t}$$

If $x_i=0$ at $t=0$ for $i=1,2,\ldots$ $x(0)=A$ is satisfied.