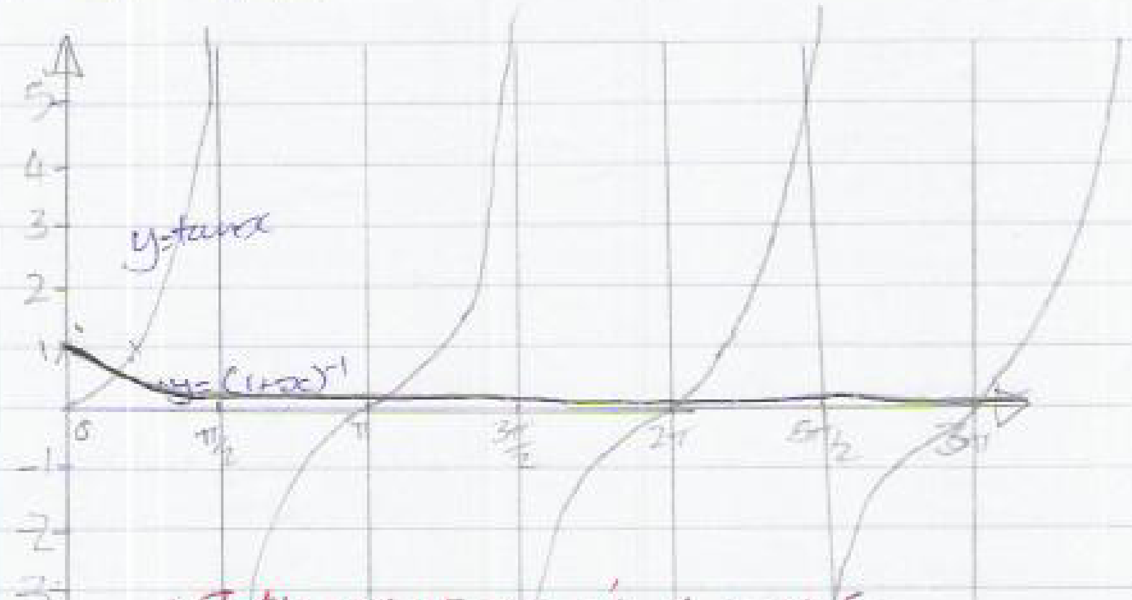
M821 TMA 03

1) a) When $\varepsilon = 0$ the equation is $\tan x = 0$, with solns $x = n\pi$, $n = 0, \pm 1, \pm 2$



(-1) Justify soln at $n\pi$ apply for small ε.

If $\varepsilon \neq 0$, expand the sides of the equation in a Taylor series about $x = n\pi$. Put $x = n\pi + \varepsilon x_1 + \varepsilon^2 x_2 + O(\varepsilon^3)$

$(\tan\theta)' = \sec^2\theta$

$(\sec^2\theta)' = (1/\cos^2\theta)' = \dfrac{0 + 2\sin\theta\cos\theta}{\cos^4\theta}$

$= 2\tan\theta\sec^2\theta$

$\tan(n\pi + \varepsilon x_1 + \varepsilon^2 x_2)$

$= \tan(n\pi) + (\varepsilon x_1 + \varepsilon^2 x_2)(\sec^2 n\pi)$

$+ \frac{1}{2}(\varepsilon x_1 + \varepsilon^2 x_2)^2(2\tan \pi n \sec^2 \pi n) + O(\varepsilon^3)$

$= \varepsilon x_1 + \varepsilon^2 x_2 + O(\varepsilon^3)$.

$\dfrac{\varepsilon}{1+x} = \dfrac{\varepsilon}{1 + n\pi + \varepsilon x_1 + \varepsilon^2 x_2} + O(\varepsilon^3)$

$= \dfrac{\varepsilon}{(1+n\pi)(1 + \varepsilon x_1/_{1+n\pi} + \varepsilon^2 x_2/_{1+n\pi})} + O(\varepsilon^3)$