

be necessary to retain this term and find expressions for $C_1(x)$ and $C_2(x)$: in that case it is also necessary to introduce another variable $\eta_2 = x/\epsilon^2$.

Thus an approximate general solution is

$$y(x) = \frac{3}{\alpha + x^3} + \frac{c}{9}(\alpha + x^3)^2 e^{-x/\epsilon} - \epsilon \frac{x^2 c^2}{162} (\alpha + x^3)^4 e^{-2x/\epsilon}.$$

But $y(0, \epsilon) = 0$, so $c = -27\alpha^{-3}$ and the required solution is

$$y(x) = \frac{3}{\alpha + x^3} - \frac{3}{\alpha^3}(\alpha + x^3)^2 e^{-x/\epsilon} - \frac{9\epsilon x^2}{2\alpha^6}(\alpha + x^3)^4 e^{-2x/\epsilon}.$$