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$+O(\varepsilon^2)$

$$0=(x_0''+x_0)+\varepsilon(2\omega_1 x_0''+x_1''+(x_0^2-1)x_0'+x_1+\alpha^2x_0^3)$$

Setting coefficients of powers of ε to zero gives, since ε can take any value in an interval around zero, ✓

$x_0''+x_0=0 \Rightarrow x=a\cos\tau$. ✓ good (5)

(using that $x_0(0)=a$, $\dot{x}_0(0)=0$)

$$2\omega_1 x_0''+x_1''+(x_0^2-1)x_0'+x_1+\alpha^2x_0^3=0$$

$$x_1''+x_1=-2\omega_1x_0''-(x_0^2-1)x_0'-\alpha^2x_0^3$$

$$=2\omega_1 a\cos\tau+(a^2\cos^2\tau-1)a\sin\tau-\alpha^2a^3\cos^3\tau.$$

$$=2\omega_1 a\cos\tau+a^3(1-\sin^2\tau)\sin\tau-a\sin\tau-\frac{\alpha^2a^3}{4}\cos3\tau\ \frac{3\alpha^2a^3}{4}\cos\tau$$

$$=2\omega_1 a\cos\tau+a^3\sin\tau-\frac{3a^3}{4}\sin\tau+\frac{a^3}{4}\sin3\tau-a\sin\tau-\frac{\alpha^2a^3}{4}\cos3\tau-\frac{3\alpha^2a^3}{4}\cos\tau$$

$$=\cos\tau\,(2\omega_1 a-3\alpha^2a^3/4)+\sin\tau\,(a^3-a-3a^3/4)+a^3/4\sin3\tau-\alpha^2a^3/4\cos3\tau.$$

If periodic soln exists, the above coefficients of $\cos\tau$, $\sin\tau$ must both be zero. ✓ good

o/w SECULAR terms arise (like $t\cos t$, for example)

ie $2\omega_1 a-\frac{3\alpha^2a^3}{4}=0$

$$\omega_1=\frac{3\alpha^2a^2}{8}$$

$$a^3-a-\frac{3a^3}{4}=\frac{a^3}{4}-a=a\left(\frac{a^2}{4}-1\right)=0$$