Once integrated the first expression relates q to F and may be inverted to give $F(q)$: similarly the second expression gives $G(p)$. Clearly $F(z) = G(z)$. With these definitions of $F(q)$ and $G(p)$ the equations of motion become

$$\dot{q} = \alpha(G(p)) = \frac{\partial H}{\partial p}, \qquad \dot{p} = \beta(F(q)) = -\frac{\partial H}{\partial q},$$

so a Hamiltonian is

$$\begin{aligned} H(q,p) &= \int dp\, \alpha(G(p)) - \int dq\, \beta(F(q)) \\ &= \int dG\, \frac{\alpha(G)}{f(G)} - \int dF\, \frac{\beta(F)}{f(F)}. \end{aligned}$$

These two forms for $H(q,p)$ are equivalent but, in practice one may be more convenient than the other.

It is important to note this function, $H(q,p)$, it is *not* the Hamiltonian in the (x,y)-representation, which need not exist; in other words the function $H(q(x), p(y))$ of x and y is not a Hamiltonian of the original equations: the property of being Hamiltonian is coordinate-dependent.