Geometric Topology

Question 6 (Unit 6) – 11 marks

(i) For each of the following coverings, give a diagram to indicate that the covering can occur:

(a) $2T^2 \xrightarrow{2} T^2$, with a total branch deficit of $d = 2$;

(b) $7T^2 \xrightarrow{6} 2T^2$, unbranched. [4]

(ii) Give a diagram to indicate that the following unbranched covering can occur: $(k+1)T^2 \xrightarrow{k} 2T^2$ (an unbranched k-fold covering of a torus with 2 holes by a torus with $k+1$ holes). [3]

(iii) Deduce that there are branched coverings of the form

$$(k+1)T^2 \xrightarrow{2k} T^2,$$

and calculate the total branch deficit of d in this case. [4]

Question 7 (Unit 6) – 12 marks

A three-sheeted covering $\tilde{S}$ of T^2 with edge equation $aba^{-1}b^{-1} = 1$ is represented by the following system of edge equations:

$$a_1 b_1 a_3^{-1} b_1^{-1} = 1,$$
$$a_2 b_2 a_2^{-1} b_3^{-1} = 1,$$
$$a_3 b_3 a_1^{-1} b_2^{-1} = 1.$$

(i) Determine which points of T^2 (if any) are covered by branch points, and in each such case determine the branch deficit. [3]

(ii) Determine the Euler characteristic $\chi(\tilde{S})$ of $\tilde{S}$. [3]

(iii) Determine the number $\beta(\tilde{S})$ of boundary curves of $\tilde{S}$. [2]

(iv) Is $\tilde{S}$ orientable? Justify your answer. [2]

(v) Write down $\tilde{S}$ as a connected sum of copies of T^2, D^2 and/or $\mathbb{RP}^2$. [2]