Question 9 (Unit 7) – 15 marks

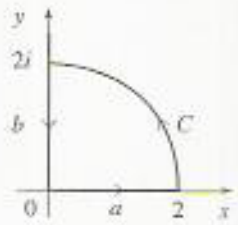
(i) Show that all seven zeros of $F(z) = z^7 + z^5 + z^3 + 4$ lie inside the circle $C_2 = \{z : |z| = 2\}$. [2]

(ii) Show that $F(z)$ has no zeros inside the circle $C_1 = \{z : |z| = 1\}$. [2]

(iii) Show that $F(z)$ has exactly one real zero, and state whether it lies in the interval $]-2, -1[$ or in the interval $]1, 2[$. [2]

(iv) Show that $F(z)$ has no purely imaginary zeros. [2]

(v) By considering the image under F of the contour below, find out how many zeros of $F(z)$ have both real and imaginary parts positive.



(The contour consists of a, the real axis from 0 to 2, followed by C, the quarter-circle of radius 2 centre the origin, from 2 to $2i$, followed by b, the imaginary axis from $2i$ to 0.) You may find it helpful to sketch the images under F of the pieces a, b and C, in that order. [5]

(vi) Hence determine the number of zeros of $F(z)$ in each quadrant. [2]

$F = 128e^{7i\theta} + 32e^{5\theta i} + 8e^{3i\theta} + 4$

$= 128\cos 7\theta + 32\cos 5\theta + 8\cos 3\theta + 4$

$128\sin 7\theta + 32\sin 5\theta + 8\sin 3\theta$

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