②

ii) $|x_2| < x_1$

then $x_1 > 0$ since we can write the above eqtn $x_1 > |x_2| \geq 0$

$$0 \leq \frac{|x_2|}{x_1} < 1$$

We must prove that if $|x_2| < x_1$ there is an n such that $|x_2| \leq \frac{n}{n+1} x_1$

Then by the rational/irrational number density theories, there exists a number $1-\varepsilon$, where $\varepsilon > 0$ such that

$$0 < \frac{|x_2|}{x_1} \leq 1 - \varepsilon < 1$$

Since the set of natural numbers is not bounded, for $\varepsilon > 0$ there exists an $n \in \mathbb{N}$ such that $n > 1/\varepsilon$

then $n+1 > 1/\varepsilon \Rightarrow \varepsilon > 1/(n+1)$

$$0 < |x_2|/x_1 \leq 1 - 1/(n+1) < 1$$

$$0 < \frac{|x_2|}{x_1} \leq \frac{n+1-1}{n+1} = \frac{n}{n+1} < 1$$

$$\therefore \quad 0 < \frac{|x_2|}{x_1} \leq \frac{n}{n+1}$$

$$0 < |x_2| \leq \frac{n x_1}{n+1}$$

and $(x_1, x_2) \in A_m$ for $m+1 > \frac{1}{\varepsilon}$

Note: if $x_1 = x_2 = 0$ then $0 \leq \frac{n}{n+1} \times 0 = 0$

$\therefore (0,0) \in A_n$ for all n.

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