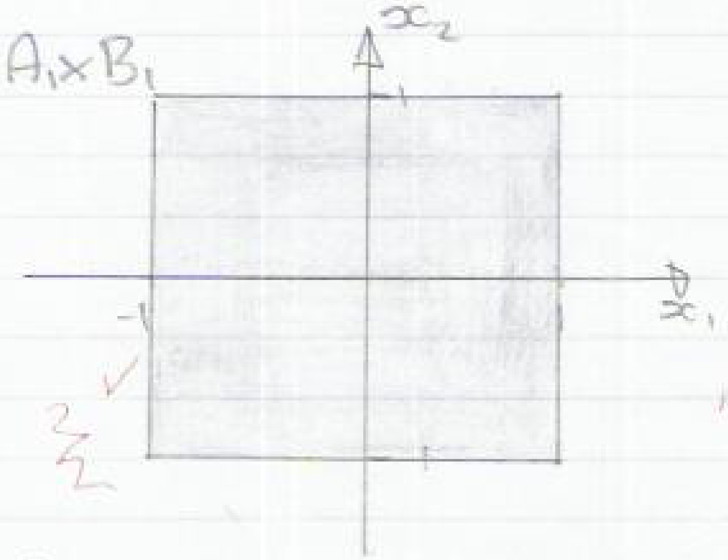
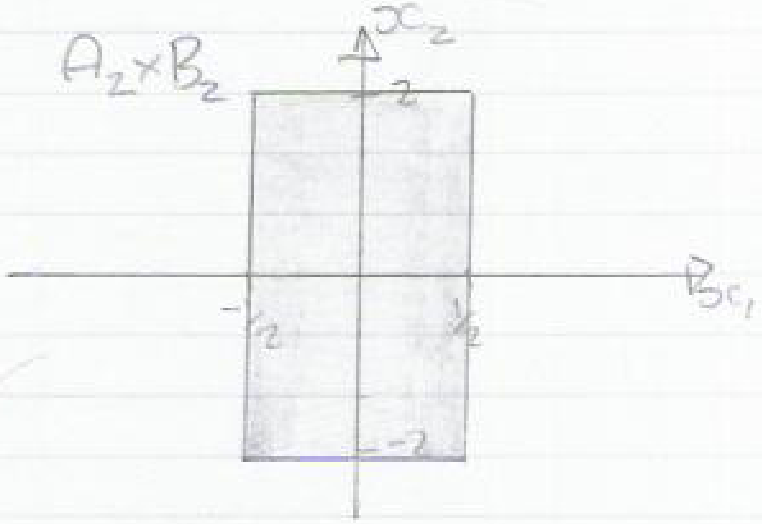
(7)

4) i) $A_1 = [-1, 1]$ $\quad B_1 = [-1, 1]$

$A_2 = [-\frac{1}{2}, \frac{1}{2}]$ $\quad B_2 = [-2, 2]$





ii) $|x_2| > 1$ then either $x_2 < -1$ or $x_2 > 1$
then $x_2 \notin [-1, 1] = B_1$
$\therefore (x_1, x_2) \notin A_1 \times B_1$ if $|x_2| > 1$ since $x_2 \notin B_1$

b) $|x_1| > 0$ then, ~~there~~ there exists $\varepsilon > 0$ such that $|x_1| = \varepsilon > 0$.
$\therefore$ The set of natural numbers is unbounded
so for $\varepsilon > 0$ there exists $m \in \mathbb{N}$ such $m > 1/\varepsilon$, $\varepsilon > 1/m > 0$
then $|x_1| = \varepsilon > \frac{1}{m} > 0 \Rightarrow x_1 \notin \left[\frac{-1}{m}, \frac{1}{m}\right]$ for $m > \frac{1}{\varepsilon}$, $\varepsilon = |x_1| > 0$

$\therefore (x_1, x_2) \notin A_m \times B_m$ for $m > \frac{1}{\varepsilon} = \frac{1}{|x_1|}$
as required

iii) Put $C = \bigcap_{n=1}^{\infty} \left[-\frac{1}{n}, \frac{1}{n}\right] \times [-n, n]$

$D = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 = 0, -1 \leq x_2 \leq 1\}$

We prove $C = D$
$(x_1, x_2) \notin D$ if $x_1 \neq 0$ ie $|x_1| > 0$ or $|x_2| > 1$
By i) and ii b) there exists an m for which if $|x_1| > 0$, $(x_1, x_2) \notin A_m \times B_m$