Question 4 – 13 marks

For each positive integer n, let

$$A_n = \left[-\frac{1}{n}, \frac{1}{n}\right], \quad B_n = [-n, n].$$

(i) Sketch the subsets $A_1 \times B_1$, $A_2 \times B_2$ of $\mathbf{R}^2$. [2]

(ii) Let $x = (x_1, x_2) \in \mathbf{R}^2$.

(a) Prove that if $|x_2| > 1$, then

$$x \notin A_1 \times B_1.$$ [2]

(b) Prove that if $|x_1| > 0$, then there exists a positive integer m such that

$$x \notin A_m \times B_m.$$ [3]

(iii) Determine the subset

$$\bigcap_{n=1}^{\infty} (A_n \times B_n)$$

of $\mathbf{R}^2$. [6]

[Your answer should give the subset of $\mathbf{R}^2$ and explain why this set equals the original description as an intersection.]

Geometric Topology

Question 5 – 10 marks

(i) Write down the number of boundary curves of the following surfaces (made of thin paper). In each case, give a sketch to illustrate your answer.

(a)

[2]

(b)

[2]

(c)

[3]

(ii) Explain why no two of the three surfaces are homomorphic. [3]