vi) $\mathcal{V} = \{\emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}, B\}$

Check h^{-1} for each open set in turn.

$h^{-1}(\emptyset) = \emptyset \in \mathcal{T}$

$h^{-1}(\{b\}) = \emptyset \in \mathcal{T}$

$h^{-1}(\{c\}) = \{1\} \in \mathcal{T}$

✓ $h^{-1}(\{a,b\}) = \{2,3\} \notin \mathcal{T}$

3/3 ✓ $\therefore h$ is not $(\mathcal{T}, \mathcal{V})$ continuous

2) i) T1) $V_n = E \cup \{k \in \mathbb{Z} : |k| \geq n\}$

$V_1 = E \cup \{k \in \mathbb{Z} : |k| \geq 1\}$

$= \{\ldots -2, 0, 2 \ldots\} \cup \{\ldots -3, -2, -1, 1, 2, 3 \ldots\} = \mathbb{Z}$

✓ $\therefore \mathbb{Z} \in \mathcal{V}$ (also given by the question).

✓ $\emptyset \in \mathcal{V}$ (given by the question).

T2) $V_n \cap V_m = (E \cup \{k \in \mathbb{Z} : |k| \geq n\}) \cap (E \cup \{k \in \mathbb{Z} : |k| \geq m\})$

$= E \cup (\{k \in \mathbb{Z} : |k| \geq n\} \cap \{k \in \mathbb{Z} : |k| \geq m\})$

$= E \cup (\{k \in \mathbb{Z} : |k| \geq \mathrm{Max}\{n, m\}\})$

✓ $\therefore V_n \cap V_m = V_{\max\{n,m\}} \in \mathcal{V}$.

T3) $V_n \cup V_m = (E \cup \{k \in \mathbb{Z} : |k| \geq n\}) \cup (E \cup \{k \in \mathbb{Z} : |k| \geq m\})$

$= E \cup (\{k \in \mathbb{Z} : |k| \geq n\} \cup \{k \in \mathbb{Z} : |k| \geq m\})$

must take an arbitrary union, not just that of 2 elts of $\mathcal{V}$? $= E \cup \{k \in \mathbb{Z} : |k| \geq \mathrm{Min}\{n, m\}\} \in \mathcal{V}$.

4/5 T1, T2 and T3 are satisfied $\therefore \mathcal{V}$ is a topology on $\mathbb{Z}$

Let K be a non-empty set of positive integers & consider the collection $(V_n)_{n \in K}$. Let k be the smallest elt of K (k exists as $K \neq \emptyset$, $K \subset \mathbb{N}$). Then $\bigcup_{n \in K} V_n = V_k \in \mathcal{V}$

ii) The topology induced by $\mathcal{V}$ on H consists of all sets of the form $H \cap V_n$.

$5 \in H$ and V_n for $n \leq 5$ $\therefore 5 \notin H \cap V_n$, $n \geq 6$ and $3 \notin V_n$, $n \geq 6$ also, $E \subset V_n$ for all n.

✓ $\{-4, -2\} = \{-4, -2, 3, 5\} \cap V_n$, $n \geq 6$

$\therefore$ By the definition of subspace topologies $\{-4, -2\}$ is open in the topology induced by $\mathcal{V}$ on H.

$5 \in V_4, V_5$ but $3 \notin V_4, V_5$ and $E \subset V_4, V_5$

✓ $\therefore \{-4, -2, 5\} = \{-4, -2, 3, 5\} \cap V_4$