8a) i) $z^2-3zw+w^2+3z-1=0$

$w^2-3zw+z^2+3z-1=0$

$w=\dfrac{3z\pm\sqrt{(-3z)^2-4\cdot1\cdot(z^2+3z-1)}}{2}$

$=\dfrac{3z\pm\sqrt{9z^2-4z^2-12z+4}}{2}$

$=\dfrac{3z\pm\sqrt{5z^2-12z+4}}{2}$

$=\dfrac{3z\pm\sqrt{(5z-2)(z-2)}}{2}$

$z=2\Rightarrow w=3$

$z=2/5\Rightarrow w=3/5$

Values of z giving only one soln for w are $z=2/5$, $z=2$

As z goes round a circle centre 2/5 $\sqrt{5z-2}$ changes sign, but $z-2$ goes round a circle centre 8/5, and does not change sign. ∴ The sign of $\sqrt{(5z-2)(z-2)}$ changes as z goes once round the little circle centre 2/5. (2/5, 3/5) is a branch point since the point (2/5, 3/5) has a disc like neighbourhood.

As $\sqrt{z-2}$ goes round the little circle centre 0, $\sqrt{5z-2}$ goes round the circle centre 2. $\sqrt{z-2}$ changes sign as z travels the circle centre 2, but $\sqrt{5z-2}$ does not. ∴ The sign of $\sqrt{(z-2)(5z-2)}$ changes as z goes round the circle centre 2, and z has to go round twice before it returns to its original value. ∴ the neighbourhood of (2,3) is a disc. ∴ (2,3) is a branch point.

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