

5) i)  $P a Q c R b S f S b^{-1} R c^{-1} Q a^{-1} P e S f^{-1} S d P = 1$  (8)

One edge equation: 1 face

4 vertices (P, Q, R, S)

6 edges (a, c, b, f, e, d)

$$X = F - E + V$$

$$= 1 - 6 + 4 = -1$$

We can form the boundary edges into cyclic sets as follows

$$P: (da) = (a^{-1}e)$$

involving boundary edges d, e

$$Q: (ac) = (c^{-1}a^{-1})$$

$$R: (cb) = (b^{-1}c^{-1})$$

$$S: (bf) = (fb^{-1}) = (ef^{-1}) = (f^{-1}d)$$

involving boundary edges d, e

Thus the only cyclic set is de and this is the <sup>only</sup> boundary curve of the

surface there is one boundary curve:  $B=1$

i) b)  $P a P b P c P d P$

$P f P a P f^{-1} P g P h P$

$P g P b P h^{-1} P$

$P k P d P k^{-1} P c P$

Four edge equations: four faces

1 vertex P

8 edges (a, b, c, d, f, g, h, k)

$$X = F - E + V$$

$$= 4 - 8 + 1 = -3$$

We can form the boundary edges into cyclic sets as follows

$$P: (ab) = (bc) = (cd) = (da) = (fa) = (af^{-1})$$

$$= (f^{-1}g) = (gh) = (h^{-1}f) = (gb) = (bh^{-1}) = (h^{-1}g)$$

$$= (kd) = (dk^{-1}) = (k^{-1}c) = (ck)$$

Each edge appears twice: there are no boundary curves

ii)