

$$\text{iii) } w = z \pm \sqrt{1-z^3} = z \pm \sqrt{(1-z)(1+z+z^2)}$$

$$w_1 = z + \sqrt{(1-z)(1+z+z^2)}$$

$$w_2 = z - \sqrt{(1-z)(1+z+z^2)}$$



As z goes round the circle γ centre $\sqrt{1-z}$ goes round a little circle centre 0; it changes sign, but $\sqrt{1+z+z^2}$ goes round a little circle centre $\sqrt{3}$, and does not change sign. w_1 and w_2 become interchanged! z has to go round twice for w_1 and w_2 to return to their original values. The neighbourhood of $z=1$ is a disc. $z=1$ corresponds to a branch point $w(1) = 1+0=0$. This branch point is $(1, 1)$.

$$(z, w) = \left(\frac{-1+i\sqrt{3}}{2}, \frac{-1+i\sqrt{3}}{2} \right)$$

As z goes round the little circle centre $(-1+i\sqrt{3})/2$, $\sqrt{1+z+z^2}$ goes round the little circle centre 0; it changes sign but $\sqrt{1-z}$ goes round a little circle centre $\sqrt{1 - \left(\frac{-1+i\sqrt{3}}{2} \right)} = \sqrt{\frac{3-i\sqrt{3}}{2}}$ and this does not change sign.