(i) Classify the surface obtained from the second hexagon above. Show your working and write your answer in connected sum form. [3]

(ii) In the first topological hexagon, the unlabelled edges are to be identified in pairs to produce a surface without boundary. Show that whatever pairing of the edges is chosen, the Euler characteristic χ of the resulting surface cannot be less than -1. [2]

[Hint: what is the smallest number of vertices after identification of edges?]

(iii) Explain why any surface obtained in this way must be non-orientable. Deduce that the largest number of vertices obtainable after identification of edges must be less than 4, and write down the values of the Euler characteristic χ consistent with the information you have obtained so far. [3]

(iv) For each value of the Euler characteristic χ obtained in your answer to part (iii), show that a surface can be constructed as in part (ii) above by writing down the edge equation of a hexagon with its edges labelled to correspond to the surface. (Remember that the edges marked a are assumed to have been marked in advance.) [3]

(v) Write each surface obtained in part (iv) above as a connected sum. [3]

Question 7 (Unit 5) – 10 marks

Consider the complex locus

$$w^2 - 2wz + (z^3 + z^2 - 1) = 0.$$

(i) Determine all infinite points on the locus. [3]

(ii) Find the three (finite) values of z which have only one solution (z, w) on the locus. [Hint: $z = 1$ is a one value.] [3]

(iii) Write down the general solutions w_1, w_2 for the two points lying above the point z on the locus, in terms of z and the three solutions t_1, t_2 and t_3 which you found in part (ii). Arguing as in Frames 18–20 of the tape section, show that the three points corresponding to t_1, t_2 and t_3 are branch points of the locus. [4]

Question 8 (Unit 5) – 15 marks

Each of the following conics is non-degenerate.

(i) $z^2 - 3zw + w^2 + 3z - 1 = 0$

(ii) $z^2 - 4zw + 4w^2 - 3z + 1 = 0$

For each of these conics, do the following.

(a) Working from first principles, locate all finite branch points, if any. [6]

(b) Determine all infinite points on the conic. [6]

(c) Using the fact that a conic has either two branch points or none, write down which infinite points (if any) are branch points and which (if any) are pinch points. [2]

(d) Use the main theorem (Unit 5, page 30) to confirm your answers. [1]
