Question 4 (Unit 5) – 15 marks

Let $A = \{x \in \mathbf{R} : x > 0\}$ and let $\mathscr{T}$ be the topology on A which consists of $\varnothing$, A and the intervals $\left(\frac{1}{n+1}, n\right)$ for all positive integers n.

[You need *not* verify that $\mathscr{T}$ is a topology on A.]

(i) Write down the closed sets of $(A, \mathscr{T})$. [2]

(ii) Let $H = \{\frac{1}{2}, 2\}$.

(a) Show that if $x \in A$, then x is a limit point of H if and only if $x < \frac{1}{2}$ or $x \geq 1$.

(b) Find the closure of H in $(A, \mathscr{T})$. [9]

(iii) Write down the closure of $K = \{\frac{1}{4}, \frac{3}{2}\}$ in $(A, \mathscr{T})$, and hence determine the set of limit points of K. [4]

Geometric Topology

Question 5 (Unit 4) – 11 marks

(i) Using the technique of inserting vertices, determine χ, the Euler characteristic, and β, the number of boundary curves, for each of the surfaces described by the systems of edge equations below.

(a) $acbfb^{-1}c^{-1}a^{-1}ef^{-1}d = 1$.

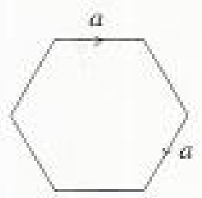
(b) $abcd = 1$; $faf^{-1}gh = 1$; $gbh^{-1} = 1$; $kdk^{-1}c = 1$. [4]

(ii) For each of the surfaces in part (i), determine whether or not the surface is orientable, and write it in connected sum form. [3]

(iii) For each of the systems of equations in part (i), use Lemmas 1, 2 and 3 of **B** (pages 66–8) and the results of Steps 1 to 4 of the reduction procedure (**B** pages 70–73) to reduce it to canonical form. Justify each step of your working. [4]

Question 6 (Unit 4) – 14 marks

This question concerns surfaces without boundary which can be obtained from a topological hexagon with a fixed pair of opposite edges identified as shown.



One such surface is the following.