

$$\tilde{w}^2 - 4\tilde{z}\tilde{w} + 4\tilde{z}^2 - 3\tilde{z}\tilde{w}^2 + \tilde{z}\tilde{w}^3 = 0$$

$\tilde{w}, \tilde{z} = 0 \Rightarrow 0 = 0 \therefore (\infty, \infty)$ does lie on the conic. The equation reduces to $\tilde{w}^2 = 0$ if $\tilde{z} = 0$. (∞, ∞) has multiplicity 2.

c) i) From a) i), $\tilde{z}^2 - 3\tilde{z}\tilde{w} + \tilde{w}^2 + 3\tilde{z} - 1 = 0$ has two ^{finite} branch points: $(\frac{2}{5}, \frac{3}{5})$ and $(2, 3)$. (∞, ∞) is a pinch point, since a curve has either two branch points or none.

From a) ii) $\tilde{z}^2 - 4\tilde{z}\tilde{w} + 4\tilde{w}^2 - 3\tilde{z} + 1 = 0$ has the finite branch point $(\frac{1}{3}, \frac{1}{6})$. Since the conic has two branch points (∞, ∞) is a branch point.

d) i) $\tilde{z}^2 - 3\tilde{z}\tilde{w} + \tilde{w}^2 + 3\tilde{z} - 1 = 0$
 $b^2 - 4ac = (-3)^2 - 4 \cdot 1 \cdot 1 = 5 \neq 0$. The main theorem says this locus is a pinched sphere containing two points corresponding to (∞, ∞) , which are pinch points.

ii) $\tilde{z}^2 - 4\tilde{z}\tilde{w} + \tilde{w}^2 - 3\tilde{z} + 1 = 0$
 $b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 4 = 0$. The main theorem says this locus is homeomorphic to the \mathbb{C}^* -sphere, and that one of the branch points is (∞, ∞) .

Both these answers agree with c) i) & c) ii)