$\{c\} \cap \{b, c\} = \{c\} \in \tau_1$

$\{a, b\} \cap \{b, c\} = \{b\} \in \tau$

These cover all possibilities $\therefore$ T2 is satisfied.

T3) If $S \in \tau$ $\emptyset \cup S = S \in \tau$

$B \cup S = B \in \tau$

$\{b\} \cup \{c\} = \{b\} \cup \{b, c\} = \{b, c\} \in \tau_1$

$\{b\} \cup \{a, b\} = \{a, b\} \in \tau_1$

$\{c\} \cup \{a, b\} = \{b, c\} \cup \{a, b\} = B \in \tau_1$

T3 is satisfied T1, T2 and T3 are satisfied $\therefore$ $\{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, B\} = \tau_1 = \tau$ is a topology. $B \subset \tau$ (and any other topology containing B as a proper subset must contain $\{a\}$ or $\{a, c\}$ as members, and so must be the discrete topology).

3/3 ✓

iv) If f is continuous, for every set S open in $\mathcal{V}$, $f^{-1}(S)$ is open in $\mathcal{T}$.

$f^{-1}(\emptyset) = \emptyset \in \mathcal{T}$

$f^{-1}(\{b\}) = \{1, 2\} \in \mathcal{T}$

$f^{-1}(\{c\}) = \emptyset \in \mathcal{T}$

$f^{-1}(\{a, b\}) = \{1, 2, 3\} \in \mathcal{T}$

$f^{-1}(\{b, c\}) = \{1, 2\} \in \mathcal{T}$

$f^{-1}(\{B\}) = A \in \mathcal{T}$

$\therefore$ f is continuous since for every set S open in $\mathcal{V}$, $f^{-1}(S)$ is open in $\mathcal{T}$.

3/3 ✓

v) $\{b\} \in \mathcal{V}$ but $f^{-1}(\{b\}) = \{2, 3\}$ is not open in $\mathcal{T}$ $\therefore$ g is not $(\mathcal{T}, \mathcal{V})$ continuous

2/2 ✓