$= \{-4,-2,3,5\} \cap V_5$. (4)

By the definition of subspace topology $\{-4,-2,5\}$ is open in the topology induced by $\mathcal{V}$ on H. ✓

b) $V_1 \cap H = V_2 \cap H = V_3 \cap H = H$ since $H \subset V_1, V_2, V_3$

$V_4 \cap H = V_5 \cap H = \{-4,-2,5\}$ from ii) a)

$V_6 \cap H = V_7 \cap H = V_n \cap H,\ n \geq 6 = \{-4,-2\}$

$\emptyset \cap H = \emptyset$

$\mathcal{V} \cap H = \{\emptyset, \{-4,-2\}, \{-4,-2,5\}, \{-4,-2,3,5\}\}$

$= \{\emptyset, \{-4,-2\}, \{-4,-2,5\}, H\}$

6/6 ✓ // = topology induced by $\mathcal{V}$ on H

3) i) $V_2 = \{\ldots -3,-2,0,2,3\ldots\}$

$\mathbb{Z} - V_2 = \mathbb{Z} - \{\ldots -3,-2,0,2,3\ldots\}$

$= \{-1,1\}$

2/2 ✓ V_2 is open in $\{\mathbb{Z},\mathcal{V}\}$, $\mathbb{Z} - V_2 = \{-1,1\}$ is closed in $\{\mathbb{Z},\mathcal{V}\}$

ii) $\mathbb{Z} - V_n = \mathbb{Z} - (E \cup \{k \in \mathbb{Z} : |k| \geq n\})$

$= (\mathbb{Z} - E) \cap (\mathbb{Z} - \{k \in \mathbb{Z} : |k| \geq n\})$

$= O \cap \{k \in \mathbb{Z} : |k| < n\}$

Where O denotes the subset of $\mathbb{Z}$ of odd integers. Thus closed sets in $\mathbb{Z} - V_n$ take the form ✓

$\mathbb{Z} - V_n = \{-(n-1), -(n-3) \ldots -1, 1, \ldots n-3, n-1\}$, n even

$\mathbb{Z} - V_n = \{-(n-2), -(n-4) \ldots -1, 1, \ldots n-4, n-2\}$, n odd

3/4 Also $\emptyset$ & $\mathbb{Z}$