4) i) $\left(\frac{1}{n+1}, n\right) \in A$

$Cl\{(\frac{1}{n+1}, n)\} = \{x \in \mathbb{R} : x > 0\} - \{(\frac{1}{n+1}, n)\}$

$= \mathbb{R}^+ - \{(\frac{1}{n+1}, n)\}$

$\frac{1}{2}$ ✓ $= (0, \frac{1}{n+1}] \cup [n, \infty)$

So closed sets are $\emptyset, A$ + the sets $(0, \frac{1}{n+1}] \cup [N, \infty)$ $n = 1, 2, \ldots$

ii) x is a limit point of H iff every open set which includes x also includes a point of H other than x

$A_1 = (\frac{1}{1+1}, 1) = (\frac{1}{2}, 1)$

$A_2 = (\frac{1}{1+2}, 2) = (\frac{1}{3}, 2)$

$\frac{1}{2} \in A_2$ but $\frac{1}{2}$ is not a limit point

✓ of H since $2 \notin A_2$, and any point

✓ in $A_1 = (\frac{1}{2}, 1)$ is not a limit point of H since $\frac{1}{2}, 2 \in H$ but $\frac{1}{2}, 2 \notin (\frac{1}{2}, 1)$

$\therefore$ If x is a limit point of H we must have $x \le \frac{1}{2}$ or $x \geq 1$.

To see this, notice that $1 \in A_k$ and all A_k, $k \geq 2$? $\therefore 1$ is a limit point of H. If $x \in (1, 2)$ then x is a limit point of H since $(1, 2) \subset A_2$ and $\frac{1}{2} \notin (1, 2)$ $\therefore$ $\frac{1}{2} \in A_2 - (1, 2)$ $\therefore$ Every point in $(1, 2)$ is a limit point of H.

$\therefore \frac{1}{2} \in A_k$ i.e. another pt of H lies in $A_k \cap H$.

If $x \geq 2$, $x \in A_k$ for some $k > 2$

✓ and $H \subset A_k$ for $k > 2$ $\therefore$ x is a limit point of H.

Finally if $0 < x \le \frac{1}{2}$, then $x \in A_k$ for ~~all~~ some

$k \geq x^{-1}$ then $\frac{1}{2} \in A_k - x$ $\therefore$ x is a limit point of H ie every point of

$\frac{6}{7}$ $(0, \frac{1}{2})$ is a limit point of H.

b) $Cl(\{H\}) = \{H\} \cup$ set of limit points

$= \{\frac{1}{2}, 2\} \cup \{x \in \mathbb{R} : 0 < x \le \frac{1}{2} \text{ or } x \geq 1\}$