TMA 03 M435 (1)

1) $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{1,3\}, A\}$.

T1) $\emptyset, A \in \mathcal{T}$ $\therefore$ T1 satisfied. ✓

T2) $\forall S \in \mathcal{T}$, $\emptyset \cap S = \emptyset \in \mathcal{T}$

$A \cap S = S \in \mathcal{T}$

$\{1\} \cap \{1\} = \{1\} \cap \{1,2\} = \{1\} \cap \{1,3\} = \{1\} \in \mathcal{T}$

$\{1,2\} \cap \{1,3\} = \{1\} \in \mathcal{T}$

✓ Those cover all possibilities $\therefore$ T2 is satisfied.

T3) $\forall S \in \mathcal{T}$, $\emptyset \cup S = S \in \mathcal{T}$

$A \cup S = A \in \mathcal{T}$

$\left.\begin{array}{l}\{1\} \cup \{1,2\} = \{1,2\} \\ \{1\} \cup \{1,3\} = \{1,3\}\end{array}\right\} \in \mathcal{T}$

$\frac{2}{2}$ ✓

$\{1,2\} \cup \{1,3\} = \{1,2,3\} = A$

Those cover all possibilities $\therefore$ T3 is satisfied. T1, T2 and T3 are satisfied $\therefore \mathcal{T}$ is a topology on A.

ii) $\{b\}, \{c\} \in \mathcal{B}$ but $\{b\} \cup \{c\} = \{b,c\} \notin \mathcal{B}$ $\therefore$ T3 is not satisfied. $\frac{2}{2}$ ✓

$\therefore \mathcal{B}$ is not a topology on B

iii) We must have $\{b,c\} \in \mathcal{T}$, since $\{b\}, \{c\} \in \mathcal{B}$.

Check if

$\mathcal{T}_1 = \{\emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}, B\}$ is a topology on B

T1) $\emptyset, B \in \mathcal{T}_1$

T2) $\forall S \in \mathcal{B}$ $\emptyset \cap S = \emptyset \in \mathcal{T}_1$

$B \cap S = S \in \mathcal{T}_1$

$\{b\} \cap \{c\} = \emptyset \in \mathcal{B}$

$\{b\} \cap \{a,b\} = \{b\} \cap \{b,c\} = \{b\} \in \mathcal{T}_1$

$\{c\} \cap \{a,b\} = \emptyset \in \mathcal{T}_1$