

The handles

$$acbf b^{-1} c^{-1} a^{-1} e f^{-1} d = 1$$

Use lemma 2 to move  $a^{-1} e$  from in front of  $f^{-1}$  to behind  $f$

$$acbf a^{-1} e b^{-1} c^{-1} f^{-1} d$$

Use lemma 2 to move  $a^{-1} e b^{-1}$  from in front of  $c^{-1}$  to behind  $c$

$$aca^{-1} e b^{-1} b f c^{-1} f^{-1} d$$

Use lemma 2 to move  $a^{-1} e b^{-1} b$  from in front of  $f$  to behind  $f^{-1}$

$$ac f c^{-1} f^{-1} a^{-1} e b^{-1} b d = 1$$

Assemble the cuffs

Cancel  $b^{-1} b$  to give

$$ac f c^{-1} f^{-1} a^{-1} e d = 1$$

Replace  $ed$  by  $g$

$$ac f c^{-1} f^{-1} a^{-1} g = 1$$

Compose on the left with  $a^{-1}$

$$c f c^{-1} f^{-1} a^{-1} g = a^{-1}$$

And on the right with  $a$

$$c f c^{-1} f^{-1} a^{-1} g a = 1$$

Which is the equation of a torus with a hole.

2  
2 ✓

From part i) the edge equations representing the surface form the single edge equation

$$ag' a^{-1} f a^{-1} f^{-1} K d K^{-1} d = 1$$

Assemble the crosscaps

Using lemma 1, move  $K^{-1}$  from in front of one  $d$  to behind the other.

$$ag' a^{-1} f a^{-1} f^{-1} K K d d = 1$$

Using lemma 2 move  $K K d d$  from behind  $f^{-1}$  to in front of  $f$