ii) $z^2 - 4zw + 4w^2 - 3z + 1 = 0$

$4w^2 - 4zw + (z^2 - 3z + 1) = 0$

$$w = \frac{4z \pm \sqrt{(-4z)^2 - 4\cdot 4(z^2 - 3z + 1)}}{2\cdot 4}$$

$$= \frac{4z \pm \sqrt{16z^2 - 16z^2 + 48z - 16}}{8}$$

$$= \frac{4z \pm \sqrt{48z - 16}}{8}$$

$$= \frac{z \pm \sqrt{3z-1}}{2} \quad (\text{If } z = 1/3,\ w = 1/6)$$

$z = 1/3$ is a value of z for which w has only one value. As $\sqrt{3z-1}$ goes round a little circle centre 0 (ie z goes round a little circle centre $1/3$) it changes sign and has to go round twice to return to its original value. $(1/3, 1/6)$ has a disc like neighbourhood. ∴ $(1/3, 1/6)$ is a branch point.

3/3

6) i) $z^2 - 3zw + w^2 + 3z - 1 = 0$

(∞, w). Put $z = \frac{1}{\tilde{z}}$

$$\frac{1}{\tilde{z}^2} - \frac{3w}{\tilde{z}} + w^2 + \frac{3}{\tilde{z}} - 1 = 0$$

$$1 - 3w\tilde{z} + w^2\tilde{z}^2 + 3\tilde{z} - \tilde{z}^2 = 0$$

If $z = \infty$ $\tilde{z} = 0$ but $\tilde{z} = 0 \Rightarrow 1 = 0$ ∴ (∞, w) does not lie on the conic.

(z, ∞) Put $w = \frac{1}{\tilde{w}}$

$$z^2 - \frac{3z}{\tilde{w}} + \frac{1}{\tilde{w}^2} + 3z - 1 = 0$$