

$$w_1 = z + \sqrt{1-z} \sqrt{1+z+z^2}$$

$$w_2 = z - \sqrt{1-z} \sqrt{1+z+z^2}$$

\therefore Since $\sqrt{1+z+z^2}$ changes sign but $\sqrt{1-z}$ does not, $\sqrt{1-z} \sqrt{1+z+z^2}$ change sign so w_1 and w_2 become interchanged. z has to go around the circle twice before w_1 and w_2 return to their original values. \therefore The neighbourhood of $\left(\frac{-1+i\sqrt{3}}{2}, \frac{-1+i\sqrt{3}}{2}\right)$ is a disc.

✓ this point is a branch point

$$(z, w) = \left(\frac{-1-i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right)$$

The argument for this point is precisely the same as above.

✓ This point too has a disc like neighbourhood on the locus. \therefore is a branch point.

$\frac{9}{4}$