

(9)

$$ii) acbfb^{-1}c^{-1}a^{-1}ef^{-1}d=1$$

Each edge which occurs twice occurs once in each direction  $\therefore$  surface is orientable, so we use the eqn for the Euler characteristic of orientable surfaces

$$\chi = 2 - 2p - r$$

$$2p = 2 - r - \chi$$

$$= 2 - 1 - (-1) = 2$$

$$p = 1$$

$\therefore$  The surface is equivalent to a torus with a hole, or  $S^1 \times T^2 \# D^2$

$$a b c d = 1$$

$$f a f^{-1} g h = 1 \Rightarrow h = g^{-1} f a^{-1} f^{-1}$$

$$g b h^{-1} = 1 \Rightarrow g b = h \Rightarrow b = g^{-1} h$$

$$k d k^{-1} c = 1 \Rightarrow c = k d^{-1} k^{-1}$$

The  $g$ 's will occur in the same sense after substituting the last three eqns into the 1st one:

$$a g^{-1} g^{-1} f a^{-1} f^{-1} k d k^{-1} d = 1$$

$\therefore$  the surface is non orientable so we use the eqn for the Euler characteristic of non-orientable surfaces.

$$\chi = 2 - q - r$$

$$q = 2 - \chi - r$$

$$= 2 - (-3) - 0 = 5$$

$\frac{3}{3} \checkmark$   $\therefore$  The surface is equivalent to a sphere with 5 crosscaps  $S \cong S^2 \# 5 \mathbb{R}P^2$  (or just  $5 \mathbb{R}P^2$ )

$$iii) ac b f b^{-1} c^{-1} a^{-1} e f^{-1} d = 1$$

No letters occur twice in the same sense, so we start by assembling