$z^2\tilde{w}^2 - 3z\tilde{w} + 1 + 3z\tilde{w}^2 - \tilde{w}^2 = 0$

$1 = 0$

$\therefore$ (z,∞) does not lie on the conic.

(∞,∞) Put $(z,w) = \left(\frac{1}{\tilde{z}}, \frac{1}{\tilde{w}}\right)$

$\frac{1}{\tilde{z}^2} - \frac{3}{\tilde{z}\tilde{w}} + \frac{1}{\tilde{w}^2} + \frac{3}{\tilde{z}} - 1 = 0$

$\tilde{w}^2 - 3\tilde{z}\tilde{w} + \tilde{z}^2 + 3\tilde{z}\tilde{w}^2 - \tilde{z}^2\tilde{w}^2 = 0$ i.e. $0 = 0$

3/3

$\therefore$ (∞,∞) lies on the conic. Put $\tilde{z} = 0$ then $\tilde{w}^2 = 0$ so (∞,∞) has multiplicity 2.

ii) $z^2 - 4zw + 4w^2 - 3z + 1 = 0$

(∞, w) Put $z = \frac{1}{\tilde{z}}$

$\frac{1}{\tilde{z}^2} - \frac{4w}{\tilde{z}} + 4w^2 - \frac{3}{\tilde{z}} + 1 = 0$

$1 - 4w\tilde{z} + 4w^2\tilde{z}^2 - 3\tilde{z} + \tilde{z}^2 = 0$

$\tilde{z} = 0$ since $z = \infty$

but $1 \neq 0$ $\therefore$ (∞, w) does not lie on the conic.

(z,∞) Put $w = \frac{1}{\tilde{w}}$

$z^2 - \frac{4z}{\tilde{w}} + \frac{4}{\tilde{w}^2} - 3z + 1 = 0$

$z^2\tilde{w}^2 - 4z\tilde{w} + 4 - 3z\tilde{w}^2 + \tilde{w}^2 = 0$

$\tilde{w} = 0$ since $w = \infty$ but $\tilde{w} = 0 \Rightarrow 4 = 0$

$\therefore$ (z,∞) does not lie on the conic

(∞,∞). Put $z = \frac{1}{\tilde{z}}$, $w = \frac{1}{\tilde{w}}$

$\frac{1}{\tilde{z}^2} - \frac{4}{\tilde{z}\tilde{w}} + \frac{4}{\tilde{w}^2} - \frac{3}{\tilde{z}} + 1 = 0$