(5)

iii) $0 \in \{-1, 0, 1\}$ and $0 \in V_n$ for all n. The closure of $\{-1,0,1\}$ in $\mathcal{V}$ consists of $\{-1,0,1\}$ together with the set of all points n of $\mathbb{Z}$ such that every open set containing n contains a point of $\{-1,0,1\}$. But every V_n contains 0 $\therefore$ every n and $-n \in Cl(\{-1,0,1\})$
$\therefore Cl(\{-1,0,1\}) = \mathbb{Z}$ ✓

Alternatively $Cl(\{-1,0,1\})$ is the smallest closed set containing $\{-1,0,1\}$. But $0 \notin \mathbb{Z} - V_n$ for any n, and $0 \notin \mathbb{Z} - \mathbb{Z}$. On the other hand $\emptyset$ is open in $\mathcal{V}$ and $\{-1,0,1\} \subset \mathbb{Z} - \emptyset = \mathbb{Z}$. This is the only closed set containing $\{-1,0,1\}$ as a subset, hence is the smallest set.

3/2 ✓

iv) $\{\mathbb{Z}, \mathcal{V}\}$ is a Hausdorff space if distinct points of $\mathcal{V}$ are contained in disjoint open sets, but there are no disjoint open sets in $\{\mathbb{Z}, \mathcal{V}\}$. In particular $E \subset V_n$ for all n, so ... $-8, -6, -4, -2, 0, 2, 4, 6, 8$ ... which are all distinct points are all contained in the same open sets, which are therefore not disjoint, since $E \subset V_n \cap V_m$ for all n, m. Hence $\{\mathbb{Z}, \mathcal{V}\}$ is not a Hausdorff space.

3/2 ✓

Or use the fact that in a Hausdorff space finite subsets are closed ([illegible] p10 Notes(i))
Hence by (iii) $\{\mathbb{Z}, \mathcal{V}\}$ is not Hausdorff