3

2) $f(x) = x^2 + 2x + 3$

$f(x) - f(1) = (x^2 + 2x + 3) - (1^2 + 2\times 1 + 3)$

$= (x^2 - 1) + 2(x-1) + (3-3)$

$= (x-1)(x+1+2)$

$= (x-1)(x+3)$

$\therefore |f(x) - f(1)| \leq |x-1||x+3|$

Suppose $|x-1| < 1$

Then $-1 < x-1 < 1 \Rightarrow 0 < x < 2$

$\therefore |x+3| < |2+3| \leq 5$

Choose $\delta = \text{Min}\{1, \varepsilon/5\}$

then $|x-1| < \delta$

$\Rightarrow |f(x) - f(1)| = |x-1||x+3|$

$< \varepsilon/5 \times 5 = \varepsilon.$

$\frac{5}{5}$ ie $|x-1| < \delta \Rightarrow |f(x) - f(1)| < \varepsilon$

as required

ii) $g(x) = x^3 + 2x^2 + 3x + 4$

$g(x) - g(1) = (x^3 + 2x^2 + 3x + 4) - (1^3 + 2\times 1^2 + 3\times 1 + 4)$

$= (x^3 - 1) + 2(x^2 - 1) + 3(x-1)$

$= (x-1)(x^2 + x + 1 + 2(x+1) + 3)$

$\therefore |g(x) - g(1)| \leq |x-1||x^2 + x + 1 + 2(x+1) + 3|$

Suppose $|x-1| < 1$

Then $-1 < x-1 < 1 \Rightarrow 0 < x < 2$

so $|x^2 + x + 1 + 2(x+1) + 3| < |2^2 + 2 + 1 + 2(2+1) + 3|$

$= 16$

Choose $\delta = \text{Min}\{1, \varepsilon/16\}$

Then $|x-1| < \delta$

$\Rightarrow |g(x) - g(1)| \leq |x-1||x^2 + x + 1 + 2(x+1) + 3|$

$< \varepsilon/16 \times 16 = \varepsilon$ as required

$\frac{6}{7}$

since $|x-1| < \delta \leq \varepsilon/16$, and $|x^2 + x + 1 + 2(x+1) + 3| < 16$

iii) For $x = 1$,

$f(x) = x^2 + 2x + 3 = 1^2 + 2\times 1 + 3 = 6 \neq 0$

From part i) $f(x)$ is continuous at