

Question 7 (Unit 3) - 9 marks

Establish the following combinatorial equivalences.

- (i) $abacbc = 1 \rightarrow aabbcc = 1$ [3]
 (ii) $ab^{-1}da^{-1}cdbcc^{-1} = 1 \rightarrow aabbcc = 1$ [3]
 (iii) $a^{-1}cbabc = 1 \rightarrow aabb = 1$ [3]

Question 8 (Unit 3) - 16 marks

- (i) For each of the following edge equations, draw a polygon with edge identifications having the given equation as its edge equation.
 (a) $aba^{-1}cb^{-1}c^{-1} = 1$
 (b) $abaccb = 1$ [2]
 (ii) One of the above represents a surface which is a torus. Write down which one, and:
 (a) show that its equation is combinatorially equivalent to $xyx^{-1}y^{-1} = 1$; [2]
 (b) give a sketch (or succession of sketches) starting with your polygon in part (i) to show how joining identified edges in pairs produces a torus. [4]
 (iii) One of the above represents a surface which is a Klein bottle. Identify which one, and:
 (a) show that its equation is combinatorially equivalent to $xyyy = 1$; [4]
 (b) mark a Möbius band on your sketch in part (i). [4]

