

$$xxy^{-1}y^{-1}zz=1$$

Rename: x as a

y^{-1} as b

z as c

$\frac{3}{3}$ ✓ This gives $aabbcc=1$ as required

$$\text{ii) } ab^{-1}da^{-1}cdbe^{-1}=1$$

$$x = a^{-1}cd \Rightarrow d = c^{-1}ax$$

$$ab^{-1}c^{-1}axxb^{-1}=1$$

$$\text{Put } y = c^{-1}axxb^{-1} \Rightarrow c^{-1} = yb^{-1}x^{-1}x^{-1}a^{-1}$$

$$ab^{-1}yxb^{-1}x^{-1}x^{-1}a^{-1}=1$$

By trivial equivalence, this becomes

$$b^{-1}yxb^{-1}x^{-1}x^{-1}=1$$

$$\text{Put } u = b^{-1}yx \quad b^{-1} = uy^{-1}y^{-1}$$

$$uuy^{-1}y^{-1}x^{-1}x^{-1}=1$$

Rename: u as a

y^{-1} as b

x^{-1} as c

$\frac{3}{3}$ ✓

To give $aabbcc=1$ as required,

$$\text{iii) } a^{-1}c b a b c = 1$$

$$x = c b a b \Rightarrow c = x b^{-1} a^{-1} b^{-1}$$

$$a^{-1} x x b^{-1} a^{-1} b^{-1} = 1 \quad (\text{I could rename } b a^{-1} \text{ here})$$

$$\text{Put } y = x x b^{-1} a^{-1} \Rightarrow a^{-1} = b x^{-1} x^{-1} y$$

$$b x^{-1} x^{-1} y y b^{-1} = 1$$

By trivial equivalence this becomes

$$x^{-1} x^{-1} y y = 1$$

Rename: x^{-1} as a

y as b

$\frac{3}{3}$ ✓ To give $aabb=1$

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