ii) X bounded above and below
$\Rightarrow \inf X \leq x \leq \sup X$ for all $x \in X$
$Y = \{|x| : x \in X\}$ then $|x| \geq 0$ for all x by defn of the modulus function
$\therefore Y$ is bounded below by 0 (though 0 is not necessarily the greatest lower bound) $\therefore \inf Y \geq 0$.
$|x| \in Y \Rightarrow |x| \geq \inf Y$ by defn of infimum $\therefore -|x| \leq \inf Y \leq |x|$.
and $\inf X \leq x \leq \sup X$

Suppose $0 \leq \inf X \leq \sup X$
Then $0 \leq \inf X \leq x = |x| \leq \sup X$
$0 \leq \inf Y = \inf X = |\inf X| = \min\{|\inf X|, |\sup X|\}$

Suppose $\inf X \leq \sup X \leq 0$
Then $\inf x \leq x = -|x| \leq \sup X \leq 0$ for all $x \in X$
$0 \leq -\sup X = |\sup X| \leq -x = |x| \leq -\inf X = |\inf X|$
$0 \leq \inf Y = |\sup X| = \min\{|\inf X|, |\sup X|\}$
since $\inf X \leq \sup X \leq 0$
$\Rightarrow 0 \leq |\sup X| \leq |\inf X|$ see opp

Suppose $\inf X < 0 < \sup X$
Then if $0 \in X$ since $|x| \geq 0$ for all $x \in X$ $\inf Y = 0$. If $0 \notin X$ then $|x| > 0$ for all $x \in X$ $\therefore \inf Y > 0$ $\therefore \inf Y \geq 0$ if $\inf X \leq 0 \leq \sup X$
In these three cases all possibilities have been covered $\therefore$ we can say

$0 \leq \inf Y \leq \min\{|\inf X|, |\sup X|\}$.
as required.

You have not proved the case for $\inf X < 0 < \sup X$. There exists $x_0 \in X$ with $0 \leq x_0 \leq \sup X$ so $\inf Y \leq x_0 \leq \sup X = |\sup X|$. Also there exists $x_1 \in X$ with $\inf X \leq x_1 \leq 0$ so $-x_1 \in Y$ [illegible] $\inf Y \leq -x_1 \leq -\inf X = |\inf X|$ so $\inf Y \leq \min\{|\inf X|, |\sup X|\}$

$\frac{5}{7}$

(It is much simpler to obtain the upper bound for Y
since $\inf X \leq x \leq \sup X$
$-\max\{|\inf X|, |\sup X|\} \leq x \leq \max\{|\inf X|, |\sup X|\}$
$\therefore 0 \leq |x| \leq \sup Y = \max\{|\inf X|, |\sup X|\}$)