④

$x=1$, and $f(1)\neq 0$ ∴ from the result of SAQ 20 part(v) p60 (soln on p62)

$\frac{1}{f(1)}$ is continuous

and from the result of SAQ 20 part(iv) if two functions are continuous so is their product

∴ $g(x), \frac{1}{f(x)}$ cont. at $x \Rightarrow g(x)\times\frac{1}{f(x)}$ contin. at x

$h(x) = \frac{x^3+2x^2+3x+4}{x^2+2x+3} = \frac{g(x)}{f(x)} = g(x)\times\frac{1}{f(x)}$

3/2 which is a product $(g(x)\times {}^1/_{f(x)})$ of functions continuous at $x=1$, so is continuous at $x=1$

3) i) $d(x,y) = |x^2-y^2| = |x-y||x+y|$

d is not a metric on $\mathbb{R}$ because M1 is not satisfied

M1: $d(x,y)\geq 0$ $\quad d(x,y)=0$ iff $x=y$

1½/2 But $d(x,y)=0$ for $x=-y$ and $x=y$ [illegible]

M1 is not satisfied

ii) $d(x,y) = \begin{cases} 0 & \text{if } x=y \\ 3|x|+2|y| & \text{if } x\neq y \end{cases}$

$d(x,y)=0$ if $x=y$ by defn

$d(x,y)=3|x|+2|y|>0$ for $x\neq y$ since at least one of $x,y\neq 0$

∴ M1 is satisfied.

3/2 ✗ M1 is satisfied $d(x,y)=0$ for $x=y$, $d(x,y)>0$ [illegible]

M2: $d(x,y)=d(y,x)$

$3|x|+2|y| = 3|y|+2|x|$

This is only possible if $|x|=|y|$ (in which case anyway $d(x,y)=0$) or $x=-y$. It is not true in general