$4|y|+|x-y|+|y-z| \geqslant |x-z|$

$4|y|+|x-y|+|y-z| \geqslant \underbrace{|x-y|+|y-z| \geqslant |x-z|}_{\text{triangle inequality}}$

$\because |y| \geq 0$

(you can use the Δle inequality)

$|x-y|+|y-z| \geqslant |(x-y)+(y-z)|$

$(|x-y|+|y-z|)^2 \geqslant |(x-y)+(y-z)|^2$

$(x-y)^2+2|x-y||y-z|+(y-z)^2 \geqslant (x-y)^2+2(x-y)(y-z)+(y-z)^2$

$\therefore 2|x-y||y-z| \geqslant 2(x-y)(y-z)$

as required $\therefore$ M3 is satisfied

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ii) a) $B_1(0) = \{x \in \mathbb{R} : d(x,0) < 1\}$

ie x such that

$2|x|+2|0|+|x-0|<1$

$3|x| < 1 \Rightarrow |x| < 1/3$

2/2 ✓ $\therefore -1/3 < x < 1/3$

$\therefore B_1(0) = \{x : -1/3 < x < 1/3\}$

b) $B_1(1) = \{x \in \mathbb{R} : d(x,1) < 1\}$

ie x such that

$2|x|+2|1|+|x-1|<1$

$2|x|+|x-1| < -1$

Which is impossible to satisfy.

1/2 ✓ $\therefore B_1(1) = \{1\}$ since $d(1,1)=0$ by defn

iii) $B_r(a) = \{x \in \mathbb{R} : d(x,a) < r\}$

ie x such that

$2|x|+2|a|+|x-a| < r$. $r>0$

If $r < 2|a|$

$2|x|+|x-a| < r-2|a| < 0$

There are no values of x which satisfy this apart from the single point a, since $d(a,a)=0<r$ by defn of d.

2/2 ✓ $\therefore B_{2|a|}(a) = \{a\}$

($\therefore r>0$ exists such that $B_r(a) = \{a\}$)