eg $d(1,0)=3 \neq 2 = d(0,1)$.

3/2 ✓ $\therefore$ M2 is not satisfied.

iii) $d(x,y)=|x-y|(|x-y|+2)$

M1 is satisfied since $x=y \Rightarrow |x-y|=0$
$\Rightarrow d(x,y)=0(0+2)=0$
$|x-y|(|x-y|+2)>0$ if $x \neq y$ since at least one of $x, y \neq 0$.

M2 is satisfied since
$d(x,y)=|x-y|(|x-y|+2)=|y-x|(|y-x|+2)=d(y,x)$
by the properties of addition multiplication, and the modulus function.

M3 is not satisfied:
$d(x,y)+d(y,z) \geq d(x,z)$
eg for $x=0, y=1, z=10$ (z = 2 [illegible])
$|0-1|(|0-1|+2)+|1-10|(|1-10|+2) \not\geq |0-10|(|0-10|+2)$
$3+9(9+2) \not\geq 10(10+2)$
$3+99=102 \not\geq 120$

3/2 ✓ a contradiction $\therefore$ M3 fails

4) i) $d(x,y)=\begin{cases} 0 & x=y \\ 2|x|+2|y|+|x-y| & \end{cases}$

M1: $d(x,y) \geq 0 \qquad d(x,y)=0 \Leftrightarrow x=y$
$x=y$ $d(x,y)=0$ by defn of d
$x \neq y \Rightarrow$ at least one of $x, y \neq 0$

✓ $\therefore d(x,y)>0$ since at least two of $2|x|, 2|y|, |x-y|$ are greater than 0.

M2: $d(x,y)=d(y,x)$
$2|x|+2|y|+|x-y|=2|y|+2|x|+|y-x|$
which is true since addition is commutative and $|x-y|=|y-x|$ by defn of modulus func.

✓

M3: $d(x,y)+d(y,z) \geq d(x,z)$
$2|x|+2|y|+|x-y|+2|y|+2|z|+|y-z| \geq 2|x|+2|z|+|x-z|$