TMA 02 M435.

1) A bounded above $\Leftrightarrow$ there exists $\alpha \in \mathbb{R}$ such that $x \in A \Rightarrow x \leq \alpha$.
We can write this as A bounded above $\Leftrightarrow$ there exists $\alpha \in \mathbb{R}$ such that $x \in A \Rightarrow -x \geq -\alpha$
Thus if we define $B = \{-x : x \in A\}$
Then $-x \in B \Rightarrow -x \geq -\alpha$ so B is bounded below.
Since $\alpha = \sup A$, $-\alpha = \inf B$
$\therefore \inf B = -\sup A$.
Suppose $-\alpha + \varepsilon$, $\varepsilon > 0$, is also a lower bound for B. Then since $-x \geq -\alpha + \varepsilon$, $-x \in B$, $x \leq \alpha - \varepsilon$. But $x \in A$ and since α is an upper bound for A there exists, for $\varepsilon > 0$, $x \in A$ such that $\alpha - \varepsilon < x \leq \alpha$ so
$-\alpha \leq -x < -\alpha + \varepsilon$ $\therefore$ $-\alpha + \varepsilon$ is not a lower bound for B. $\therefore$
$\inf B = -\sup A$ and $\inf B$ is unique

①