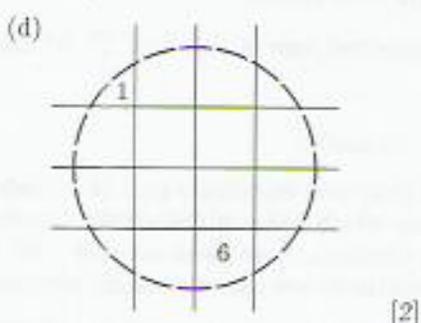
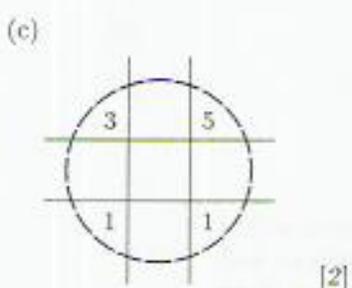
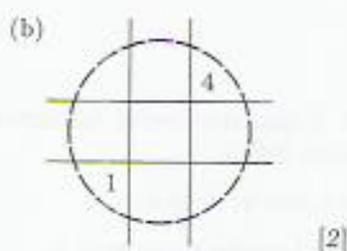
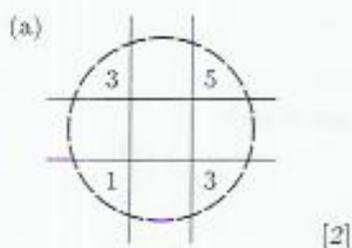
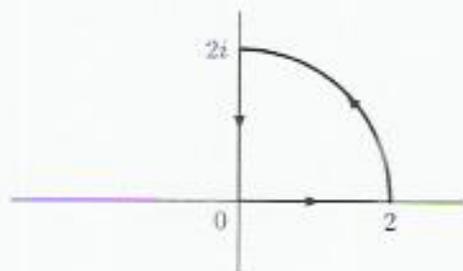


- (ii) With the same interpretation of diagrams as before, and, with the same restrictions concerning the dashed region, state which of the following can be part of a single closed curve with the given winding numbers, and which cannot. Accompany your answer with a sketch if such a curve is possible or a brief explanation why such a curve is impossible, as appropriate.



Question 9 (Unit 7) - 13 marks

- (i) Show that all zeros of $F(z) = z^6 + z + 1$ lie inside the circle $C_2 = \{z : |z| = 2\}$. [2]
 (ii) Show that $F(z)$ has no zeros inside the circle $C_{1/2} = \{z : |z| = \frac{1}{2}\}$. [2]
 (iii) Show that $F(z)$ has no real zeros. [1]
 (iv) Show that $F(z)$ has no purely imaginary zeros. [1]
 (v) By considering the image under F of the contour below, find out how many zeros of $F(z)$ have both real and imaginary parts positive.



(The contour consists of the real axis from 0 to 2, followed by the quarter-circle of radius 2, centre the origin, from 2 to $2i$, followed by the imaginary axis from $2i$ to 0.) [5]

- (vi) Hence determine the number of zeros of $F(z)$ in each quadrant. [2]