

Question 8 (Unit 5) - 14 marks

Each of the following conics is non-degenerate.

(i) $z^2 - 2zw + w^2 - 3z + 1 = 0$

(ii) $2z^2 - 2zw + w^2 - 3z + 1 = 0$

For each of these conics, do the following.

- (a) Working from first principles, locate all finite branch points, if any. [5]
 - (b) Determine all infinite points on the conic. [5]
 - (c) Using the fact that a conic has either two branch points or none, write down which points (if any) are branch points and which points (if any) are pinch points. [2]
 - (d) Use the main theorem (Unit 5, page 30) to confirm your answers. [2]
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Questions 1 to 5 are on Metric and Topological Spaces.
Questions 6 to 9 are on Geometric Topology.

Metric and Topological Spaces

Question 1 (Unit 6) - 6 marks

Prove that the subspace

$$H = \{(x_1, x_2) \in \mathbf{R}^2 : x_2 \geq |x_1|\}$$

of \mathbf{R}^2 with the usual topology is not compact, by writing down an appropriate open cover of H and showing that it does not have a finite subcover. [6]

Question 2 (Unit 6) - 12 marks

Let $A = [0, 1] \times [0, 1]$ and let \mathcal{F} be the topology on A which has all sets of the form $[a, 1] \times [0, b]$ for $(a, b) \in A$ as a basis. [You need not verify that this collection of sets is a synthetic basis.]

- (i) Show that $\{A, \mathcal{F}\}$ is a compact space. [5]
- (ii) Let $B = \{(x_1, x_2) \in A : x_1 = \frac{1}{2}\}$.
 - (a) Show that B is not closed in $\{A, \mathcal{F}\}$.
 - (b) Show that the subspace B of $\{A, \mathcal{F}\}$ is compact. [5]
- (iii) Deduce that $\{A, \mathcal{F}\}$ is not a Hausdorff space. [2]

Question 3 (Unit 6) - 10 marks

Let

$$A = \{(x_1, x_2) \in \mathbf{R}^2 : |x_1| \leq 2, x_1^2 + x_2^2 \geq 1\}$$

and let B and C be the subsets of A defined as follows:

$$B = \{(x_1, x_2) \in A : |x_2| < 3\}$$

and $C = \{(x_1, x_2) \in A : |x_2| \leq 2\}$.

For each of the subsets A , B and C of \mathbf{R}^2 with its usual topology, state whether or not it is: (a) bounded; (b) closed; (c) compact. For each subset, justify your answer to (c). [10]