

Question 4 - 13 marks

For each positive integer n , let

$$A_n = \left[0, 1 + \frac{1}{n}\right], \quad B_n = \left[-\frac{1}{n}, \frac{1}{n}\right].$$

- (i) Sketch the subsets $A_1 \times B_1, A_2 \times B_2$ of \mathbb{R}^2 . [2]
- (ii) Show that if $x = (x_1, x_2)$ and either $x_1 > 1$ or $x_2 \neq 0$, then there is some positive integer n such that $x \notin A_n \times B_n$. [6]
- (iii) Determine the subset

$$\bigcap_{n=1}^{\infty} (A_n \times B_n)$$

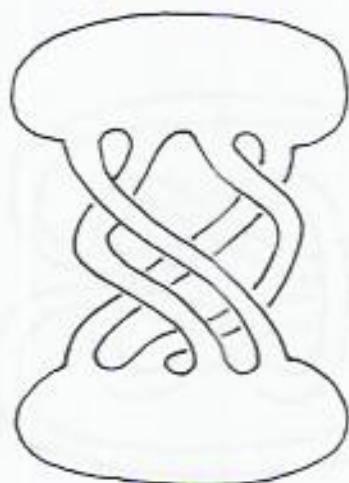
of \mathbb{R}^2 . [5]

Geometric Topology

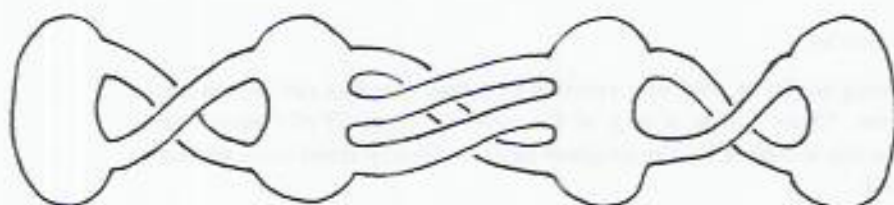
Question 5 - 5 marks

- (i) Write down the number of boundary curves of the following two surfaces (made of thin paper). In each case, give a sketch to illustrate your answer.

(a)



(b)



[4]

- (ii) Explain why the two surfaces above are not homeomorphic. [1]