c) $[\mathbb{Q}(\varepsilon):\mathbb{Q}] = \partial m$, where m is the minimum polynomial of ε over $\mathbb{Q}$. $\partial m = 4$.
$\mathbb{Q}(\varepsilon)$ is a splitting field for t^4+1, so $\mathbb{Q}(\varepsilon):\mathbb{Q}$ is finite, separable (since $\mathbb{Q}$ has characteristic zero), normal (by Thm 8.4 of Stewart, since $\mathbb{Q}(\varepsilon)$ is a splitting field for t^4+1 over $\mathbb{Q}$) $\therefore$ by Corollary 10.7, $|\Gamma(\mathbb{Q}(\varepsilon):\mathbb{Q})| = \partial m = 4$. The ~~only~~ groups of order 4 are C_4 and the Klein group, both abelian $\therefore \Gamma(\mathbb{Q}(\varepsilon):\mathbb{Q})$ is abelian ($\Gamma(\mathbb{Q}(\varepsilon):\mathbb{Q}) \cong V$ since every element self inverse).

by Fund. Th^m^ of Galois Th^y^

$\frac{5}{6}$ ✓

d) G has a subgroup which is abelian ($\Gamma(\Sigma:\mathbb{Q}(\varepsilon))$ by (b)). This subgroup is normal in G, $\therefore$ we can form the quotient group $\cong \Gamma(\mathbb{Q}(\varepsilon):\mathbb{Q})$, and we find this to be abelian $\therefore$ G is metabelian.

by (a) or by the F.T.G.T. 4(c)

$\frac{4}{6}$

Two freebies

3) i) $f(0) = 0^2+0+2 = 2 \equiv 2 \pmod 3$
$f(1) = 1^2+1+2 = 4 \equiv 1 \pmod 3$
$f(2) = 2^2+2+2 = 8 \equiv 2 \pmod 3$

✓

$g(0) = 0^2+2\times0+2 = 2 \equiv 2 \pmod 3$
$g(1) = 1^2+2\times1+2 = 5 \equiv 2 \pmod 3$
$g(2) = 2^2+2\times2+2 = 10 \equiv 1 \pmod 3$

✓

$h(0) = 0^2+1 = 1 \equiv 1 \pmod 3$
$h(1) = 1^2+1 = 2 \equiv 2 \pmod 3$
$h(2) = 2^2+1 = 5 \equiv 2 \pmod 3$

✓

None of f, g, h have zeros in $\mathbb{Z}_3$ $\therefore$ all of f, g, h are irreducible in ~~in~~ $\mathbb{Z}_3$. ✓ See earlier argument about irreducible polynomials of degree 3.