There 6 monic quadratic polynomials over $\mathbb{Z}_3$:

t^2+2t+1 which has a zero 2 (of multiplicity 2)
t^2+2t which has a zero 0 (and 1)
t^2+t+1 which has a zero 1 (of multiplicity 2)
The three polynomials above are all reducible (since they have zeros in $\mathbb{Z}_3$). so f, g, h are the only irreducible polynomials over $\mathbb{Z}_3$

ii) $m(t) = t^4+t+2$
$m(0) = 0^4+0+2 = 2 \pmod 3$
$m(1) = 1^4+1+2 = 4 \equiv 1 \pmod 3$
$m(2) = 2^4+2+2 = 20 = 2 \pmod 3$
m has no zeros in $\mathbb{Z}_3$ so it is irreducible over $\mathbb{Z}_3$

iii) Consider the Galois group of K over $\mathbb{Z}_3$. Since K is a finite field of order 3^4, K has order 4 over $\mathbb{Z}_3$ so we can consider K as a vector space over $\mathbb{Z}_3$ of dimension 4 so $[K:\mathbb{Z}_3]=4$, and there exists an irreducible polynomial of degree 4 over $\mathbb{Z}_3$ such that the zeros of f are $\alpha_1, \ldots, \alpha_4$ and $K = \mathbb{Z}_3(\alpha_1, \ldots, \alpha_4)$. But all finite fields are cyclic, generated by a single element $K = \mathbb{Z}_3(\alpha)$, and all finite fields of same order are isomorphic, so if α' is any other generating member of a field K' of order 81 (which will have prime subfield $\mathbb{Z}_3$) then $K' \cong K$ and $\mathbb{Z}_3(\alpha) \cong \mathbb{Z}_3(\alpha')$, so there exists an element $\alpha \in K$ such that $K \cong \mathbb{Z}_3(\alpha)$ for any field K of order 81. m also has degree 4 and m is irreducible so if α' is a zero of m' then $\mathbb{Z}_3(\alpha') \cong \mathbb{Z}_3(\alpha) = K$ and the case is proved.

iv) $m(\alpha^3+\alpha^2+\alpha)$
$= (\alpha^3+\alpha^2+\alpha)^4 + \alpha^3+\alpha^2+\alpha+2$
$= \alpha^4(\alpha^2+\alpha+1)^4 + \alpha^3+\alpha^2+\alpha+2$
$\alpha^4+\alpha+2 = 0 \Rightarrow \alpha^4 = -\alpha-2 = 2\alpha+1$
$m(\alpha^3+\alpha^2+\alpha) = (2\alpha+1)(\alpha^4+\alpha^2+1+2\alpha^3+2\alpha^2+2\alpha)^2+\alpha^3+\alpha^2+\alpha+2$
$= (2\alpha+1)(2\alpha+1+2\alpha^3+2\alpha+1)^2+\alpha^3+\alpha^2+\alpha+2$
$= (2\alpha+1)(2\alpha^3+\alpha+2)^2+\alpha^3+\alpha^2+\alpha+2$
$= (2\alpha+1)(\alpha^6+\alpha^4+\alpha^3+\alpha^2+\alpha+1)+\alpha^3+\alpha^2+\alpha+2$