

c)  $[Q(\epsilon):Q] = 2m$ , where  $m$  is the minimum polynomial of  $\epsilon$  over  $Q$ .  $2m=4$ .  
 $Q(\epsilon)$  is a splitting field for  $t^4+1$ , so  $Q(\epsilon):Q$  is finite, separable (since  $Q$  has characteristic 0), normal (by The 8.4 of Stewart, since  $Q(\epsilon)$  is a splitting field for  $t^4+1$  over  $Q$ ).  $\therefore$  by Corollary 10.7,  $[Q(\epsilon):Q] = 2m = 4$ . The only groups of order 4 are  $C_4$  and the Klein group, both abelian.  $\therefore \Gamma(Q(\epsilon):Q)$  is abelian.  $(\Gamma(Q(\epsilon):Q) \cong V$  since every element self inverse).

by Part 1 of Fenn's Th.

$\frac{5}{6} \checkmark$

d)  $G$  has a subgroup which is abelian  $(\Gamma(\Sigma:Q(\epsilon)))$ . This subgroup is normal in  $G$ . We can form the quotient group  $\Gamma(Q(\epsilon):Q)$ , and we find this to be abelian.  $\therefore G$  is metabelian.

by (a)  $b_1(b)$   
 by (b)  $b_2(b)$   
 by (c)  $b_3(b)$   
 by (d)  $b_4(b)$   
 by (e)  $b_5(b)$   
 by (f)  $b_6(b)$   
 by (g)  $b_7(b)$   
 by (h)  $b_8(b)$   
 by (i)  $b_9(b)$   
 by (j)  $b_{10}(b)$   
 by (k)  $b_{11}(b)$   
 by (l)  $b_{12}(b)$   
 by (m)  $b_{13}(b)$   
 by (n)  $b_{14}(b)$   
 by (o)  $b_{15}(b)$   
 by (p)  $b_{16}(b)$   
 by (q)  $b_{17}(b)$   
 by (r)  $b_{18}(b)$   
 by (s)  $b_{19}(b)$   
 by (t)  $b_{20}(b)$   
 by (u)  $b_{21}(b)$   
 by (v)  $b_{22}(b)$   
 by (w)  $b_{23}(b)$   
 by (x)  $b_{24}(b)$   
 by (y)  $b_{25}(b)$   
 by (z)  $b_{26}(b)$   
 by (aa)  $b_{27}(b)$   
 by (ab)  $b_{28}(b)$   
 by (ac)  $b_{29}(b)$   
 by (ad)  $b_{30}(b)$   
 by (ae)  $b_{31}(b)$   
 by (af)  $b_{32}(b)$   
 by (ag)  $b_{33}(b)$   
 by (ah)  $b_{34}(b)$   
 by (ai)  $b_{35}(b)$   
 by (aj)  $b_{36}(b)$   
 by (ak)  $b_{37}(b)$   
 by (al)  $b_{38}(b)$   
 by (am)  $b_{39}(b)$   
 by (an)  $b_{40}(b)$   
 by (ao)  $b_{41}(b)$   
 by (ap)  $b_{42}(b)$   
 by (aq)  $b_{43}(b)$   
 by (ar)  $b_{44}(b)$   
 by (as)  $b_{45}(b)$   
 by (at)  $b_{46}(b)$   
 by (au)  $b_{47}(b)$   
 by (av)  $b_{48}(b)$   
 by (aw)  $b_{49}(b)$   
 by (ax)  $b_{50}(b)$   
 by (ay)  $b_{51}(b)$   
 by (az)  $b_{52}(b)$   
 by (ba)  $b_{53}(b)$   
 by (bb)  $b_{54}(b)$   
 by (bc)  $b_{55}(b)$   
 by (bd)  $b_{56}(b)$   
 by (be)  $b_{57}(b)$   
 by (bf)  $b_{58}(b)$   
 by (bg)  $b_{59}(b)$   
 by (bh)  $b_{60}(b)$   
 by (bi)  $b_{61}(b)$   
 by (bj)  $b_{62}(b)$   
 by (bk)  $b_{63}(b)$   
 by (bl)  $b_{64}(b)$   
 by (bm)  $b_{65}(b)$   
 by (bn)  $b_{66}(b)$   
 by (bo)  $b_{67}(b)$   
 by (bp)  $b_{68}(b)$   
 by (bq)  $b_{69}(b)$   
 by (br)  $b_{70}(b)$   
 by (bs)  $b_{71}(b)$   
 by (bt)  $b_{72}(b)$   
 by (bu)  $b_{73}(b)$   
 by (bv)  $b_{74}(b)$   
 by (bw)  $b_{75}(b)$   
 by (bx)  $b_{76}(b)$   
 by (by)  $b_{77}(b)$   
 by (bz)  $b_{78}(b)$   
 by (ca)  $b_{79}(b)$   
 by (cb)  $b_{80}(b)$   
 by (cc)  $b_{81}(b)$   
 by (cd)  $b_{82}(b)$   
 by (ce)  $b_{83}(b)$   
 by (cf)  $b_{84}(b)$   
 by (cg)  $b_{85}(b)$   
 by (ch)  $b_{86}(b)$   
 by (ci)  $b_{87}(b)$   
 by (cj)  $b_{88}(b)$   
 by (ck)  $b_{89}(b)$   
 by (cl)  $b_{90}(b)$   
 by (cm)  $b_{91}(b)$   
 by (cn)  $b_{92}(b)$   
 by (co)  $b_{93}(b)$   
 by (cp)  $b_{94}(b)$   
 by (cq)  $b_{95}(b)$   
 by (cr)  $b_{96}(b)$   
 by (cs)  $b_{97}(b)$   
 by (ct)  $b_{98}(b)$   
 by (cu)  $b_{99}(b)$   
 by (cv)  $b_{100}(b)$

Two freebies

3) i)  $f(0) = 0^2 + 0 + 2 = 2 \equiv 2 \pmod{3}$   
 $f(1) = 1^2 + 1 + 2 = 4 \equiv 1 \pmod{3}$   
 $f(2) = 2^2 + 2 + 2 = 8 \equiv 2 \pmod{3}$   
 $g(0) = 0^2 + 2 \times 0 + 2 = 2 \equiv 2 \pmod{3}$   
 $g(1) = 1^2 + 2 \times 1 + 2 = 5 \equiv 2 \pmod{3}$   
 $g(2) = 2^2 + 2 \times 2 + 2 = 10 \equiv 1 \pmod{3}$   
 $h(0) = 0^2 + 1 = 1 \equiv 1 \pmod{3}$   
 $h(1) = 1^2 + 1 = 2 \equiv 2 \pmod{3}$   
 $h(2) = 2^2 + 1 = 5 \equiv 2 \pmod{3}$

None of  $f, g, h$  have roots in  $\mathbb{Z}_3$ .  
 all of  $f, g, h$  are irreducible in  $\mathbb{Z}_3$ .  
 irreducible polynomials of degree 3.

See earlier argument about