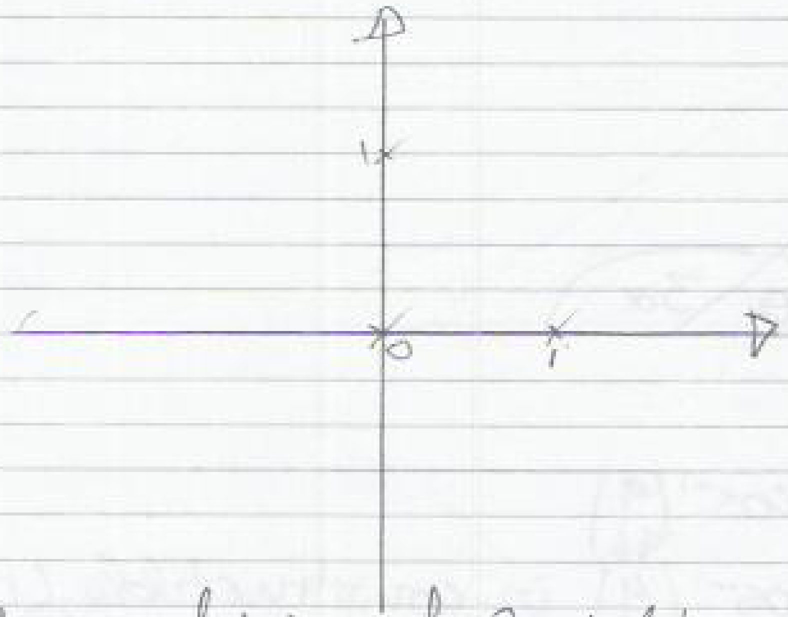
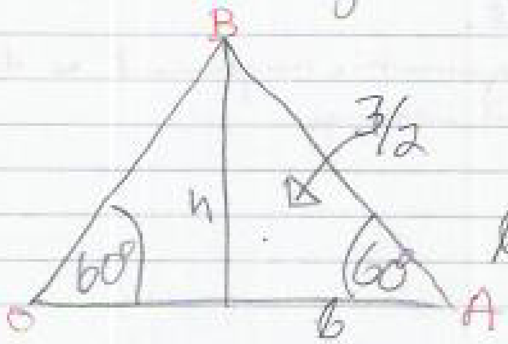
TMA M433 04

1) i)



Y ~~or~~. Area of triangle, $A = \frac{1}{2} bh$



$\frac{3}{2} = \frac{1}{2} bh$ ✓

$bh = 3$

but $\tan 60 = \frac{o}{a} = \frac{h}{b}$ ✓

$\therefore h = b\tan 60$

$\therefore b^2 \tan 60 = 3$

$b^2 \sqrt{3} = 3 \Rightarrow b^2 = \sqrt{3}$.

$\sqrt{3}$ is constructible (since $\sqrt{3}$ is a root of a polynomial degree 2 over Q) $\therefore \sqrt[4]{3}$ is constructible (from $P_0(\sqrt{3}, 0)$) since it is a zero of polynomial of degree 2 over $P_0(\sqrt{3}, 0)$ and $h = b\tan 60° = \sqrt[4]{3^3}$ is constructible. so you can construct the pt. $B = (b, h)$ from P_0 & hence A. (But it is easier to construct $A = (2\sqrt[4]{3}, 0)$, i.e. take arcs from O & A of length $2\sqrt[4]{3}$ to construct B.)

You have left a little detail out

7/9

ii) $3084 = 2^2 \times 3 \times 257$

3 and 257 are primes of form $2^{2^n} + 1$

7/7 ✓ $\therefore$ The said polygon is constructible. (Stewart theorem 17-11)

iii) $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta = \cos 3\theta + i\sin 3\theta$

$\cos^3\theta + 3i(1-\sin^2\theta)\sin\theta - 3\cos\theta(1-\cos^2\theta) - i\sin^3\theta = \cos 3\theta + i\sin 3\theta$

$4\cos^3\theta - 3\cos\theta + i(3\sin\theta - 4\sin^3\theta) = \cos 3\theta + i\sin 3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (1)

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ (2)

Put $3\theta = \pi/2$ in (1) Then

$\cos \pi/2 = 4\cos^3\theta - 3\cos\theta$