$(*) = \cos\theta(4\cos^2\theta - 3)$

If $3\theta = \pi/2 = 90^\circ$ then $\theta = 30^\circ$ is constructible from P_0 since $\cos\theta$ is a zero of a polynomial of degree 2 over P_0, and $\cos\theta = \frac{\sqrt{3}}{2}$. From $P_0(\cos 30^\circ, 0)$. We need to construct $(\cos 10^\circ, 0)$. We can do this if $\cos 10^\circ$ is a zero of a polynomial of degree 2 over $P_0(\cos 30^\circ, 0)$. Use (1) with $30^\circ = 3\theta$

✓

$$\cos 30 = 4\cos^3\theta - 3\cos\theta$$

$$\frac{\sqrt{3}}{2} = 4\cos^3\theta - 3\cos\theta$$

$$8\cos^3\theta - 6\cos\theta - \sqrt{3} = 0$$

Put $\cos\theta = t\sqrt{3}$, then

$$8 \times 3t^3\sqrt{3} - 6t\sqrt{3} - \sqrt{3} = 0$$

$24t^3 - 6t - 1 = 0$, Call this $m(t)$ but this is irreducible by [illegible] with $q = 3$

And this must be reducible for 10° to be constructible. Reduce mod 5

$4t^3 + 4t + 4 = 0$

✓

$t=0$	$4\times 0^3 + 4\times 0 + 4 = 4 \pmod 5$
$t=1$	$4\times 1^3 + 4\times 1 + 4 = 2 \pmod 5$
$t=2$	$4\times 2^3 + 4\times 2 + 4 = 4 \pmod 5$
$t=3$	$4\times 3^3 + 4\times 3 + 4 = 4 \pmod 5$
$t=4$	$4\times 4^3 + 4\times 4 + 4 = 1 \pmod 5$

7/7

So the polynomial has no zeros in $\mathbb{Z}_5$, hence no zeros in $\mathbb{Q}$, hence not reducible (If it is reducible it must have a linear factor, ✓ since if $\partial m = 3$, $\partial p + \partial q = \partial m$ and $1+2=3$ or $2+1=3$ so m has linear factor.) Any zero therefore is a zero of an irreducible polynomial of degree 3 over $\mathbb{Q}$, hence 10° is not constructible in $P_0(\cos 30^\circ, 0)$, or P_0.

d) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Put $\theta = \alpha$, $\cos 3\alpha = 4 \times \left(\frac{3}{4}\right)^3 - 3 \times \frac{3}{4}$

$$= \frac{27}{16} - \frac{9}{4} = \frac{27}{16} - \frac{36}{16} = \frac{-9}{16}$$

So, we can construct the angle 3α. We can bisect this angle to get the angle $3\alpha/2$, and bisect again to get the angle $3\alpha/4$. This angle is therefore constructible. (see opposite)

7/7 ✓

ii) zeros of $t^7 - 1$ have the form

$$e^{2\pi i n/7} = \cos\left(\frac{2\pi}{7}n\right) + i\sin\left(\frac{2\pi n}{7}\right) \in \Sigma, n \in \mathbb{N}$$