

Question 4 (Unit 12) - 25 marks

The *soluble length*, $l(G)$, of a soluble group G is the *smallest* (non-negative) integer n such that G has a finite series of subgroups

$$1 = G_0 \subseteq G_1 \subseteq \dots \subseteq G_n = G$$

such that

$$(1) G_i \triangleleft G_{i+1} \quad \text{and} \quad (2) G_{i+1}/G_i \text{ is abelian, for } i = 0, 1, \dots, n-1.$$

- (i) Find $l(C_6)$ and $l(S_3)$. [4]
- (ii) A soluble group G has a proper non-trivial normal subgroup N . By considering Theorem 13.2 and its proof, or otherwise, prove that
- (a) $l(N) \leq l(G)$, [5]
- (b) $l(G) \leq l(N) + l(G/N)$. [6]
- (iii) Give examples to show that amongst soluble groups G and proper non-trivial normal subgroups N :
- (a) both cases $l(N) < l(G)$ and $l(N) = l(G)$ occur; [4]
- (b) both cases $l(G) < l(N) + l(G/N)$ and $l(G) = l(N) + l(G/N)$ occur. [6]

TMA M433 04

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On this assignment you should answer only TWO questions; both of these will be used for assessment purposes.

Question 1 (Unit 13) - 50 marks

Determine whether or not each of the following may be constructed using just a straight-edge and compasses. Justify your answer in each case.

- (i) A cube with volume seven times that of a given cube. [9]
- (ii) An equilateral triangle equal in area to a given square. [9]
- (iii) An angle of $\pi/170$ radians. [9]
- (iv) An acute angle α such that $\cos(\alpha) = \frac{9}{16}$. [9]
- (v) The point with coordinates $(0, x)$, where
- (a) $x = \sqrt[4]{2}$, [7]
- (b) $x = \sqrt[3]{1 + \sqrt{7}}$. [7]

The points $(0, 0)$, $(1, 0)$ and $(0, 1)$ are given.

Question 2 (Unit 14) - 50 marks

- (i) Show that the following polynomials are not soluble by radicals over \mathbb{Q} .
- (a) $t^5 - 4t^2 + 2$ [15]
- (b) $t^7 - 10t^5 + 15t + 5$ [20]
- (Hint: Differentiate more than once if necessary.)
- (ii) Find a quintic polynomial which is irreducible over \mathbb{Q} but is soluble by radicals over \mathbb{Q} , fully justifying your answer. [15]