7

So the fixed subfield of M^*N^* is K

M^*N^* is a subgroup of $G \Rightarrow M^*N^* \subseteq G$

Must prove $G \subseteq M^*N^*$

Suppose $g \in G$ Then $g(k) = k$ for all $k \in K$ and if $l \in L - K$ then since $G^+ = K$ there exists $g \in G$ such that $g(l) \neq l$

We know that $mn(k) = k$ for all m, n, k

So suppose $M^*N^* \neq G$. Since $M^*N^* \subseteq G$, $G^+ \subseteq (M^*N^*)^+$ since every element which fixes $(M^*N^*)^+$ fixes K. But $(M^*N^*)^+ = K$, So if there exists some $l \in L - K$ such that

5/7 $l = mn(l)$ for all $m, n \in M^*N^*$ then $(M^*N^*)^+ \neq K$

$\therefore$ By contradiction, $G = M^*N^*$

✓ or $M^*N^{*} = (M^*N^*)^{+*}$ (F.T. Part 2) $= K^* = \Gamma(L, K) = G$.

replace argument from top of page

iii) $M^* = \Gamma(L:M) \Rightarrow$ if $x \in M^*$, $x(m) = m$ for all $m \in M$

$N^* = \Gamma(L:N) \Rightarrow$ if $y \in N^*$, $y(n) = n$ for all $n \in N$

① Put $z \in M^* \cap N^*$, then $z \in M^* \Rightarrow z(m) = m$ for all $m \in M$

$z \in N^* \Rightarrow z(n) = n$ for all $n \in N$

$\therefore M^* \cap N^*$ fixes $(M^* \cap N^*)^+$ hence fixes MN

normal (HB P29)

we place instead of MN

$MN \subseteq L$ and M^*N^* a subgroup of G implies that MN is a subfield of L^*, but the only subfield of L containing M, N is L $\therefore MN = L$

by Galois correspondence.

what is MN? It's certainly not a field!

$\therefore (M^* \cap N^*)^+ = L$

Carrying on from ① above

$z \in M^* \cap N^* \Rightarrow z \in M^* \Rightarrow z(m) = m$ for all $m \in M$

$z \in N^* \quad z(n) = n$ for all $n \in N$

$\therefore z(h) = h$ for all $h \in M \cap N$

but $M \cap N = K$

$\therefore z(k) = k$ for all $k \in K$

and if $k \in L - K$ then there exists $g \in G$ st $g(k) \neq k$ by defn of fixed field.

6/7 $\therefore z = 1$ and $M^* \cap N^* = 1$

iv) $G = M^*N^*$ can be written $\Gamma(L:K) = \Gamma(L:M)\Gamma(L:N)$

but also