vi) $G = \{1, \sigma, \tau, \sigma\tau\}$ has 5 distinct subgroups:

$$I = \{1\}$$
$$S = \{1, \sigma\}$$
$$T = \{1, \tau\}$$
$$R = \{1, \sigma\tau\}$$
$$G = \{1, \sigma, \tau, \sigma\tau\}$$

Each subgroup of G is a finite subgroup of the group of automorphisms of the field L, so we can associate it with a subfield of L (Th^m 9.4). The subfields so defined are

$$L$$
$$S^+$$
$$T^+$$
$$R^+$$
$$G^+$$

This is shown [$K : K_0$] which have 4 the same [illegible] groups [illegible]

3/5

Some of these inequalities have already been proved in (iii) & (v) & the others follow analogously (use $L = \{1\}^\dagger$)

No two subgroups of G are equal
∴ no two subfields of L are equal.

vi) a) $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ has subfields

$$\mathbb{Q}, \mathbb{Q}(\sqrt{2}), \mathbb{Q}(\sqrt{3}), \mathbb{Q}(\sqrt{6}), \mathbb{Q}(\sqrt{2}, \sqrt{3})$$

2/2 ✓

b) $\mathbb{Q}(\sqrt{2}, \sqrt[4]{3}) = \mathbb{Q}(\sqrt{2}, \sqrt[4]{3}, \sqrt{3})$
which has subfields

$$\mathbb{Q}, \mathbb{Q}(\sqrt{2}), \mathbb{Q}(\sqrt[4]{3}), \mathbb{Q}(\sqrt{3}), \mathbb{Q}(\sqrt{2}, \sqrt[4]{3})$$
$$\mathbb{Q}(\sqrt{2}, \sqrt{3})$$

2/2 ✓ ie $\mathbb{Q}(\sqrt{2}, \sqrt[4]{3})$ has 6 subfields at least

2) i) $(t^6 - 1) = (t-1)(t^2+t+1)(t^2-t+1)(t+1)$
The zeros of t^6-1 not in $\mathbb{Q}$ are the zeros of

$$t^2+t+1 = 0 \quad ①$$
$$t^2-t+1 = 0 \quad ②$$

Take ① $\quad t_1 = \dfrac{-1 \pm i\sqrt{3}}{2}$

$$t_1 = \frac{-1+i\sqrt{3}}{2} = \omega, \quad t_2 = \frac{-1-i\sqrt{3}}{2} = -1-\omega$$

From ② $\quad t_3 = \dfrac{1 \pm i\sqrt{3}}{2}$

$$t_3 = \frac{1+i\sqrt{3}}{2} = 1+\omega, \quad t_4 = \frac{1-i\sqrt{3}}{2} = -\omega$$

3/3 ✓ ∴ All zeros of t^6-1 are in $\mathbb{Q}(\omega)$