Better to write it in simpler notation e.g. write M^* for $\Gamma(L:M)$ ok

By Fundamental Theorem of Galois theory

$$\Gamma(M:K) \simeq \frac{\Gamma(L:K)}{\Gamma(L:M)}$$

By 2nd Isomorphism Theory with $\Gamma(L:M) = N$

$$\frac{\Gamma(L:M)\Gamma(L:N)}{\Gamma(L:M)} \simeq \frac{\Gamma(L:N)}{\Gamma(L:M) \cap \Gamma(L:N)}$$

$$\frac{\Gamma(L:K)}{\Gamma(L:M)} \simeq \frac{\Gamma(L:N)}{1} \simeq \Gamma(L:N)$$

Since $\Gamma(L:M)\Gamma(L:N) = G = \Gamma(L:K)$)

so $\Gamma(M:K) \simeq \dfrac{\Gamma(L:K)}{\Gamma(L:M)} \simeq \Gamma(L:N)$

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v) Must prove $\Gamma(L:M) \simeq \Gamma(N:K)$

$\Gamma(L:K) = \Gamma(L:N)\Gamma(N:K)$

By fundamental theorem of Galois theory

$$\Gamma(N:K) \simeq \frac{\Gamma(L:K)}{\Gamma(L:N)}$$

By 2nd Isomorphism theory with $\Gamma(L:N) = N$

$$\frac{\Gamma(L:N)\Gamma(L:M)}{\Gamma(L:N)} \simeq \frac{\Gamma(L:M)}{\Gamma(L:N) \cap \Gamma(L:M)}$$

$$\frac{\Gamma(L:K)}{\Gamma(L:N)} \simeq \frac{\Gamma(L:M)}{1} = \Gamma(L:M)$$

$$\therefore \Gamma(N:K) \simeq \frac{\Gamma(L:K)}{\Gamma(L:N)} \simeq \Gamma(L:M)$$

Now use the direct product theorem

$M^* = \Gamma(L:M)$ normal in G

$N^* = \Gamma(L:N)$ normal in G

$M^* \cap N^* = 1$

$G = M^* N^* \simeq N^* M^*$

$\therefore G \simeq \Gamma(L:N) \times \Gamma(L:M)$

$\simeq \Gamma(M:K) \times \Gamma(N:K)$ as required.

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