3) i) $M:K$ a normal extension implies
✓ $M^* = \Gamma(L:M)$ is a normal subgroup of $G$. Similarly for $N$ (by Fundamental Galois Thm)

Closure $\therefore M^*, N^*$ both normal subgroups of $G$. Suppose $x \in M^*N^*$, $y \in M^*N^*$
Then $x = m_1 n_1$ for some $m_1 \in M^*$, $n_1 \in N^*$
and $y = m_2 n_2$ for some $m_2 \in M^*$, $n_2 \in N^*$
$xy = m_1 n_1 m_2 n_2 = m_1 (n_1 m_2 n_1^{-1}) n_1 n_2$
$M^*$ is a normal subgroup of $G$
$\therefore n_1 m_2 n_1^{-1} \in M^* \Rightarrow n_1 m_2 n_1^{-1} = m_2'$ say.
$\therefore m_1 n_1 m_2 n_2 = m_1 m_2' n_1 n_2 \in M^*N^*$
$\therefore$ Closure axiom satisfied.

Identity $e \in M^*, N^*$ since $M^*, N^*$ are subgroups of $G$ $\therefore e \in M^*N^*$

Inverses Must show that if $mn \in M^*N^*$
$(mn)^{-1} \in M^*N^*$
$(mn)^{-1} = n^{-1}m^{-1} = m^{-1}(mn^{-1}m^{-1})$
$N^*$ is a normal subgroup of $G$
$\Rightarrow mn^{-1}m^{-1} \in N^*$. Put $mn^{-1}m^{-1} = n'$ say.
then $(mn)^{-1} = m^{-1}n' \in M^*N^*$
since $m^{-1} \in M^*$, $n' \in N^*$

$M^* = \Gamma(L:M)$
$N^* = \Gamma(L:N)$

$\frac{3}{3}$ ✓ Subgroup axioms satisfied $\therefore M^*N^*$ is a subgroup of $G$.

Just quote Result 6-3.3 (HB p21) for last part of argument

ii) Suppose $(M^*N^*)^+ = P$, then $mn(p) = p$ for all $m \in M^*$, $n \in N^*$, $p \in P$, and $m \neq n^{-1}$ for any $m \in M^*$, $n \in N^*$ (See opp. for a proof that $m \neq n^{-1}$ for any $m \in M^*$, $n \in N^*$)

m and case [illegible] [illegible] hold for [illegible] $m \neq n^{-1}$ [illegible] so $m(p) = p$ [illegible] This doesn't need [illegible] opposite

since $m(p) = p$ and $n(p) = p$ for all $m, n, p \in P$, we have that
$P \subseteq (M^*)^+$ (since any element of $P$ fixed by $M^*N^*$ is fixed by $M^*$)
and $P \subseteq (N^*)^+$ similarly
$\therefore P \subseteq (M^*)^+ \cap (N^*)^+ = M \cap N = K$
But $K \subseteq P$ since $K$ is fixed by $G$ and so by every subgroup of $G$
$\therefore P \subseteq K$, $K \subseteq P$
✓ $\therefore P = K = M \cap N$