suppose $m = n \neq e$

$$M \cap N = K$$

If $m \in M^*$ $\quad m(M \cap N) = m(K)$

$$m(M) \cap m(N) = K$$

$$M \cap m(N) = K$$

This is false
It is in general possible for $M \cap N = \emptyset$
but $m(M) \cap m(N) \neq \emptyset$
(I know this is not the case here, but you can add a $M \cap N$ & get a more general argument)

From set theory.

$$B - \cap A_i = \cup (B - A_i)$$

$$M \cap N - (M \cap m(N)) = K - K = \emptyset$$

$$(M \cap N - M) \cup (M \cap N - m(N)) = \emptyset$$

$$\emptyset \quad \cup (M \cap N - m(N)) = \emptyset$$

$$\therefore m(N) \subseteq M \cap N = K$$

similary $n(M) \subseteq M \cap N = K$

$$\therefore m(N) = n(M) \subseteq K$$

which are contradictions since they imply that M^* maps every element of N^* to the fixed field K of G, and N^* maps M to field K. but N^* and M^* contain only elements that are automorphisms of $L : K$