TMA M433 03

1) i) $\{1, \sigma\tau\}$ (I shall call this subgroup R)

ii) Suppose $\sigma(x) = x$ for all $x \in L$.
Then $1(x) = x$
$\tau(x) = \tau(x)$
$\sigma(x) = x$
$\tau\sigma(x) = \tau(x) = \sigma\tau(x)$
(Since $\sigma\tau = \tau\sigma$)
$\therefore G \simeq C_2 \neq V$
$\therefore$ There exists $x \in L$ st $\sigma(x) \neq x$

iii) If G^+ is the fixed field of G, then
$[L : G^+] = |G| = 4$ ①
Likewise, $[L : S^+] = |S| = 2$ ②
By the tower law, from ①
$[L : G^+] = [L : S^+][S^+ : G^+]$
$\therefore [S^+ : G^+] = [L : G^+] / [L : S^+] = 2$
ie $[S^+ : G^+] = 2 \Rightarrow S$ contains an element not in G^+ $\therefore S^+ \neq G^+$

iv) Suppose $x \in S^+ \cap T^+$, then $x \in S^+$
$x \in T^+$
then $\sigma(x) = x$ since $x \in S^+$
$\tau(x) = x$ since $x \in T^+$
and $\sigma\tau(x) = \sigma(x) = x = \tau(x) = \sigma\tau(x)$
$\therefore x \in G^+ \Rightarrow S^+ \cap T^+ \subseteq G^+$

Suppose $x \in G^+$, then
$\sigma(x) = x$
$\tau(x) = x$
$\sigma\tau(x) = x = \tau\sigma(x)$
$1(x) = x$
$\therefore G^+ \subseteq S^+ \cap T^+$

so $x \in S^+$
$x \in T^+$

$\therefore G^+ = S^+ \cap T^+$

v) Suppose $S^+ = T^+$ then $G^+ = S^+ \cap T^+ = S^+$ from iv
but from iii) $S^+ \neq G^+$ $\therefore S^+ \neq T^+$

1/1 ✓ 2/2 ✓ 5/5 ✓ 6/6 ✓ 2/2 ✓