ii) The zeros of the polynomial t^6-5 are $\omega^i\gamma : i = 0, 1, \ldots, 5$

$\therefore t^6-5 = (t-\gamma)(t-\omega\gamma)(t-\omega^2\gamma)(t-\omega^3\gamma)(t-\omega^4\gamma)(t-\omega^5\gamma)$

$\therefore \gamma \in \mathbb{Q}(\omega,\gamma) \subseteq L$ and $\omega = \frac{\omega\gamma}{\gamma} \in \mathbb{Q}(\gamma,\omega) \subseteq L$

$\therefore \omega^i\gamma \in \mathbb{Q}(\omega,\gamma) \quad i = 1, 2, \ldots, 5$

Then by the defn of splitting field since t^6-5 can be expressed as a product of linear polynomials in $\mathbb{Q}(\gamma,\omega)$

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t^6-5 splits in $\mathbb{Q}(\omega,\gamma) \Rightarrow L = \mathbb{Q}(\omega,\gamma)$

~~[illegible]~~

vi) $[L:\mathbb{Q}] = [\mathbb{Q}(\gamma,\omega):\mathbb{Q}]$
$= [\mathbb{Q}(\gamma,\omega):\mathbb{Q}(\omega)][\mathbb{Q}(\omega):\mathbb{Q}]$

Better the other way, i.e. with $\mathbb{Q}(\gamma)$ as intermediate field. Then obvious that $\omega \notin \mathbb{Q}(\gamma) \subseteq \mathbb{R}$

$[\mathbb{Q}(\omega):\mathbb{Q}] = \partial m$ where m is minimum polynomial of m over $\mathbb{Q}$: $\partial m = 2$

✓ ($m = t^2+t+1$, for example)

$[\mathbb{Q}(\gamma,\omega):\mathbb{Q}(\omega)] = \partial s$, where s is minimum polynomial of γ over $\mathbb{Q}(\omega)$: $\partial s = 6$, namely $t^6-5 \in \mathbb{Q}(\omega)[t]$

* irreducible over $\mathbb{Q}$, but irreducible over $\mathbb{Q}(\omega)$? (you missing [illegible])

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$\therefore [L:\mathbb{Q}] = [\mathbb{Q}(\gamma,\omega):\mathbb{Q}]$
$= [\mathbb{Q}(\gamma,\omega):\mathbb{Q}(\omega)][\mathbb{Q}(\omega):\mathbb{Q}]$
$= 6 \times 2 = 12$

iv) $L:\mathbb{Q}$ is finite, separable and L is a splitting field for t^6-5 over $\mathbb{Q}$

$\therefore L:\mathbb{Q}$ is a normal extension

2/2 ✓

$\therefore$ By Corollary 10.7, $|\Gamma(L:\mathbb{Q})| = [L:\mathbb{Q}] = 12$

v) Each element of $\Gamma(L:\mathbb{Q})$ fixes each element of $\mathbb{Q}$. $L = \mathbb{Q}(\gamma,\omega)$, so if α_i is any element of $\Gamma(\mathbb{Q}(\gamma,\omega):\mathbb{Q})$ then $\alpha_i\left(\sum a_j\omega^j + \sum b_j\omega^j\gamma\right)$

$= \sum_j \alpha_i(a_j)\alpha_i(\omega^j) + \sum_j \alpha_i(b_j)\alpha_i(\omega^j)\alpha_i(\gamma)$ by homomorphism property

$= \sum a_j(\alpha_i(\omega))^j + \sum b_j(\alpha_i(\omega))^j\alpha_i(\gamma)$ by homomorphism property.