Since each of the maps τ^j and σ^i are $\mathbb{Q}$ automorphisms, so is their composite $\sigma^i\tau^j$. Further, all the $\sigma^i\tau^j$ are distinct, for suppose

$$\sigma^{i_1}\tau^{j_1} = \sigma^{i_2}\tau^{j_2}$$

Then

$$\sigma^{i_1}\tau^{j_1}(\gamma) = \sigma^{i_2}\tau^{j_2}(\gamma)$$

$$\sigma^{i_1}(\gamma) = \sigma^{i_2}(\gamma) \Rightarrow i_1 = i_2$$

Similarly $\sigma^{i_1}\tau^{j_1}(\omega) = \sigma^{i_2}\tau^{j_2}(\omega)$ implies

$$\sigma^{i_1}(\tau^{j_1}(\omega)) = \sigma^{i_2}(\tau^{j_2}(\omega))$$

$$\tau^{j_1}(\omega) = \tau^{j_2}(\omega)$$

$$\Rightarrow j_1 = j_2$$

✓

But from v) each element of $\Gamma(L:\mathbb{Q})$ is completely determined by its effect on γ and ω, so all the $\sigma^i\tau^j$ are ~~al~~ distinct ✓

By Theorem 10.6 of Stewart they are all the $\mathbb{Q}$ automorphisms ~~of~~ L ✓

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