②

v) N is normal and the cosets of N are the hN where h∈H. The cosets are all disjoint and |H| = 4 so with each distinct coset having 5 disjoint members there are 20 distinct elements. But |G| = 20 ∴ G = HN

2/3

vi) $G/N \simeq HN/N$ (since G = HN)

$\simeq H/(H \cap N)$ (2nd Isomorphism Theorem)

$\simeq H/e$ (since N is cyclic, order 5 & H has no elements order 5)

$\simeq C_4$

5/5

Alternatively $H = \langle (1243) \rangle \Rightarrow H \simeq C_4$

I suspect you were really meant to use the cosets you have discovered in (v)

2) $h = a^{2k} \Rightarrow h \in H$

subgroup conditions see app

$a^i a^{2k} a^{-i} = a^{2k} \in H$

$a^i b a^{2k} (a^i b)^{-1} = a^i b a^{2k} b a^{-i} = a^{i-2k} b b a^{-i}$

$= a^{i-2k} a^{-i} = a^{-2k} \in H$. (includes case of b = 0)

∴ H is normal in G

✓ $k = a^{6k} \Rightarrow k \in K$

$a^i a^{6k} a^{-i} = a^{6k} \in K$

$a^i b a^{6k} (a^i b)^{-1} = a^i b a^{6k} b^{-1} a^{-i} = a^i a^{-6k} b b^{-1} a^{-i}$

$= a^i a^{-6k} a^{-i} = a^{-6k} \in K$ $= a^{6k}$

5/5 ✓ ∴ K normal in G

✓ b) $a^{12} = e$ and $H = \langle a^2 \rangle \Rightarrow |H| = 6$

✓ $a^{12} = e$ and $K = \langle a^6 \rangle \Rightarrow |K| = 2$

✓ H normal in G ⇒ |G/H| = |G|/|H| = 24/6 = 4 since H has 4 cosets in G

✓ K normal in G ⇒ |G/K| = |G|/|K| = 24/2 = 12 Since K has 12 cosets in G

(Since the order of a quotient group is the number of cosets of the normal group; by Lagrange's theorem $|\mathrm{Orb}\,N| = \frac{|G|}{|\mathrm{Stab}\,N|} = \frac{|G|}{|N|}$. Orb N is the number of cosets, all disjoint,