TMA M433 02

1)i) $f = (0\,1\,2\,3\,4)$
$g = (1\,4)(2\,3)$
$h = (1\,2\,4\,3)$

3/3

ii) $h^{-1} f h = (h(0)\ h(1)\ h(2)\ h(3)\ h(4))$
$= (0\,2\,4\,1\,3) = f^2$

$h^{-2} f h^2 = (h^2(0)\ h^2(1)\ h^2(2)\ h^2(3)\ h^2(4))$
$= (0\,4\,3\,2\,1) = f^{-1} = f^4$

$h^{-3} f h^3 = (h^3(0)\ h^3(1)\ h^3(2)\ h^3(3)\ h^3(4))$
$= (0\,3\,1\,4\,2) = f^3$

so f is conjugate to f^2, f^3, f^4

4/4

1342

iii) f is an element of order 5 (prime) hence generates a subgroup of G of order 5 (containing e, f, f^2, f^3, f^4). No element of order 4 is an element of $C_G(f)$, since all elements order 4 are conjugating elements for f and no conjugation by an element order 4 acts as the identity. $|C_G(f)| = 5$ (ie $C_G(f) = \langle f \rangle$)

Also, f is conjugate to f^2, f^3, f^4 and no others since $f^5 = e$ and these are the only 5 cycles) hence f is conjugate to 3 other elements $\therefore |f^G| = 4$

3/4

iv) G has order 20, since $20 = |C_G(f)| \times |f^G|$ = the order of G, and $5 = 1 + |f^G|$ (where 1 is the order of the conjugacy class containing the identity) and $5 \mid |G|$ $\therefore C_{(G)}(f)$ is a union of conjugacy classes and divides $|G|$, from iii) it is a subgroup $\therefore$ it is a normal subgroup

$H = \{e, (1\,2\,4\,3), (1\,4)(2\,3), (1\,3\,4\,2)\}$

$f^{-1} H f = \{e, (f(1)\ f(2)\ f(4)\ f(3)), (f(1)\ f(4))(f(2)\ f(3)), (f(1)\ f(3)\ f(4)\ f(2))\}$
$= \{e, (2\,3\,0\,4), (2\,0)(3\,4), (2\,4\,0\,3)\}$

6/6

ie $H \neq f^{-1} H f \Rightarrow H$ not normal.