mapped to elements of order 2.
The first element under the map has 3 choices of image. The second element has 2 choices, and the third element has only one choice. There are therefore $3\times2\times1=6$ possible isomorphisms.

$\frac{4}{4}$ ✓

iii)a) $G = S_3 \times C_2$
S_3 has a subgroup A_3, cyclic of order 3 and normal in S_3 (since it has index 2 in S_3). C_2 has a subgroup consisting of the identity ie it has order 1. By the properties of the external direct products of groups, $A_3 \times \{0\}$ is a subgroup of G.

see opp. for subgroup properties

✓

A general element of G has form (s, c), $s \in S_3$, $c \in C_2$ and

$(s, c) \circ (A_3, \{0\}) \circ (s^{-1}, c^{-1})$
$= (sA_3s^{-1}, c\{0\}c^{-1})$
$= (A_3, \{0\})$

Since A_3 normal in S_3 & $\{0\}$ normal in C_2, $(A_3, \{0\})$ is normal in G ✓

$\frac{G}{A_3\times\{0\}} = \frac{S_3}{A_3} \times C_2 \cong C_2 \times C_2$ ✓

$\therefore G/(A_3 \times \{0\})$ is isomorphic to the Klein group, hence not cyclic

$\frac{5}{5}$

b) $\frac{G}{A_3\times\{0\}} = \{A_3 \times \{0\}, (12)A_3 \times \{0\}, A_3 \times \{1\}, (12)A_3 \times \{1\}\}$

Klein group has trivial subgroup, 3 subgroups order 2 and the whole group ie subgroups of G/N & corresponding subgroups of G are:

1) $(A_3, \{0\})$ (corresponding to $A_3 \times \{0\}$)

2) $\{(A_3, \{0\}), (A_3, \{1\})\}$ (corresponding to $A_3 \times C_2$)

3) $\{(A_3, \{0\}), ((12)A_3, \{0\})\}$, ($S_3 \times \{0\}$)

4) $\{(A_3, \{0\}), ((12)A_3, \{1\})\}$ (isomorphic to S_3)