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a) Since g is irreducible and unseparable, the characteristic of K is greater than zero. The irreducible polynomial g is inseparable iff $g = k_0 + k_1 t^p + k_2 t^{2p} + \ldots + k_r t^{rp}$, where p is the characteristic of K

but g is cubic $\therefore$ $p = 3$ is the characteristic of K is 3.

$\frac{3}{3}$ ✓

b) $g = k_0 + k_1 t^p + k_2 t^{2p} + \ldots k_r t^{rp}$

but g is cubic $(p=3)$.

$\therefore g = k_0 + k_1 t^3$

and monic $\therefore k_1 = 1$

$g = k_0 + t^3$

γ is a zero $\Rightarrow t - \gamma \mid t^3 + k_0$

$$
\begin{array}{r l}
 & t^2 + \gamma t + \gamma^2 \\
t - \gamma \,\big) & t^3 + 0 \cdot t^2 + 0 \cdot t + k \\
 & t^3 - \gamma t^2 \\
 & \quad \gamma t^2 \\
 & \quad \gamma t^2 - \gamma^2 t \\
 & \qquad \gamma^2 t + k \\
 & \qquad \gamma^2 t - \gamma^3 \\
 & \qquad\quad k + \gamma^3
\end{array}
$$

Easier to say:
If γ is a root then
$g(\gamma) = 0$
so $\gamma^3 + k_0 = 0$
so $k_0 = -\gamma^3$
& $g(t) = t^3 - \gamma^3$

$g = t^3 + k_0 = (t^2 + \gamma t + \gamma^2)(t - \gamma) + (k + \gamma^3)$

Since $g(\gamma) = 0$, we must have $k + \gamma^3 = 0$

$\therefore k = -\gamma^3 \Rightarrow g(t) = t^3 - \gamma^3$

$\frac{3}{3}$ ✓

c) $g(t) = t^3 - \gamma^3$
$= t^3 - 3\gamma t^2 + 3\gamma^2 t - \gamma^3$ in $K[t]$
$= (t - \gamma)^3$

$\frac{3}{3}$ ✓

$\Rightarrow$ all zeros of g are the same

d) $\gamma \in L$ and γ is the multiple (and only) zero of $g \Rightarrow L$ contains all the zeros of $g \Rightarrow L$ contains a splitting field for g over K

$\frac{3}{3}$ ✓