In parts (iv), (v), (vi) and (vii), give a brief justification for each answer.

(iv) Is F isomorphic to $\mathbf{Q}$, the field of rational numbers? [2]

(v) Is F isomorphic to $\mathbf{Z}_9$? [2]

(vi) Find all the ideals of F. [2]

(vii) Is there a (ring) homomorphism from F onto $\mathbf{Z}_3$? [2]

(viii) Prove that the function $\phi : F \longrightarrow F$ defined by

$$\phi(a + bi) = a - bi$$

is an automorphism of the field F. [4]

(ix) Write down an automorphism of F different from ϕ. [1]

(x) Using Lagrange's Theorem, or otherwise, show that F has only two subfields. [3]

TMA M433 01 Part 2 — Cut-off date 21 April 1999

Questions 3, 4 and 5 below, on *Units 3, 4* and *5* of Block I, form the last part of Tutor-marked Assignment M433 01.

Your overall grade for TMA 01 will be based on the sum of your marks for all five questions in Parts 1 and 2.

Please send your answers to Questions 3, 4 and 5 to your tutor, who should have kept the PT3 for this assignment so that there is no need to send another. (If your tutor has returned your original PT3 by mistake with your answers to Questions 1 and 2, send it back with your answers to Questions 3, 4 and 5.) You will eventually receive your copy of the PT3, completed by your tutor, along with your answers to these questions.

Question 3 (Unit 3) – 20 marks

Let f and g be the polynomials in $\mathbf{Q}[t]$ defined by

$$f(t) = t^4 - t^3 + 2t^2 - 4t + 2,$$
$$g(t) = t - 1.$$

(i) Show that g divides f. [2]

(ii) Find a polynomial m in $\mathbf{Q}[t]$ such that

$$f = gm.$$ [2]

(iii) Prove that m is irreducible in $\mathbf{Q}[t]$. [2]

(iv) Find an hcf of m and g, and express this hcf in the form

$$um + vg$$

for suitable polynomials u and v in $\mathbf{Q}[t]$. [4]

(v) Find the multiplicative inverse of the coset $g + \langle m \rangle$ in the field $\mathbf{Q}[t]/\langle m \rangle$. [3]

(vi) Write down four distinct ideals of $\mathbf{Q}[t]$ which contain f. [4]

(vii) Prove that if I is an ideal of $\mathbf{Q}[t]$ such that $f \in I$, then I must be one of the ideals listed in your answer to part (vi). [3]