

(iii) Devise and give the flow graph of a Turing machine which, when started scanning the leftmost symbol of a string s , as defined above, halts:

(a) scanning a single 1, on an otherwise blank tape, if s contains both a 1 and a 2;

(b) on a blank tape, otherwise. [5]

(iv) Repeat part (ii) for your machine in part (iii). [1]

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Questions 1 to 5 are on Number Theory.

Questions 6 to 10 are on Mathematical Logic.

Number Theory

Question 1 (Unit 3) - 7 marks

(i) Solve the linear congruences

(a) $3x \equiv 11 \pmod{19}$;

(b) $12x \equiv 9 \pmod{7}$;

(c) $6x \equiv 9 \pmod{33}$. [4]

(ii) Determine the least positive integer which simultaneously satisfies all three of the congruences in part (i). [3]

Question 2 (Unit 3) - 6 marks

The International Standard Book Number (ISBN) is a ten-digit code. For example, *Unit 3* of this course has ISBN 0749264446. The first nine digits indicate the language (0 stands for english), publisher's code (7492) and book number assigned by the publisher (6444). The final digit, 6 in this example, is a check digit used to verify the correctness of the number, and is used particularly in connection with electronic transmission of information. The check digit may be any of the digits 0 to 9 inclusive, or may be X which represents 10.

In general, if the ISBN is the string

$$a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10}$$

the check digit a_{10} is determined so that the congruence

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7 + 8a_8 + 9a_9 + 10a_{10} \equiv 0 \pmod{11}$$

holds. If this congruence does not hold, the corresponding string of digits is said to be *non-ISBN*.

(i) Determine the check digits for the books whose ISBNs begin with nine digits as shown below.

(a) 035762486_

(b) 203417812_ [2]

(ii) A supposed ISBN has one digit wrong. For example the ISBN of *Unit 3* might be recorded as 0749284446. Prove that such a string, with exactly one digit wrong, must be non-ISBN. [2]

(iii) A supposed ISBN has two of its digits transposed. For example the *Unit 3* ISBN might be recorded as 0746294446 (where the fourth digit, 9, and the sixth digit, 6, have been transposed). Prove that such a string, with two distinct digits transposed, must be non-ISBN. [2]