Question 10

(i) Let h be the function

$$\mathrm{Pr}[\mathrm{Cn}[s,s], \mathrm{Cn}[\mathrm{dif}, \mathrm{Cn}[s, \mathrm{id}_3^3], \mathrm{id}_2^3]],$$

where dif is the function defined by $\mathrm{dif}(x,y) = \begin{cases} x - y, & \text{if } x \geq y, \\ 0, & \text{if } x < y. \end{cases}$

Compute the values of

(a) $h(3,0)$,

(b) $h(3,2)$. [4]

(ii) In this part you may present your arguments using either formal or informal definitions.

(a) Show that the product function is primitve recursive by defining it in terms of the initial functions. [3]

(b) Show that the function

$$f(x_1, x_2, x_3) = x_2 \cdot x_1^{x_3}$$

is primitive recursive by defining it in terms of the initial functions and, if you wish, the sum and product functions. [2]

(iii) Write down the pairs (x_1, x_2) of natural numbers for which $\mathrm{Mn}[f](x_1, x_2)$ is defined, where f is the function defined by

$$f(x_1, x_2, y) = (x_2 \dot{-} (x_1 + y)) + (8 \dot{-} x_1) \cdot y.$$ [2]

Question 11

(i) A Turing machine has a configuration with left number 37 and right number 61. Find the left and right numbers of the configuration which results when the scanning head is moved one square to the right, and draw the resulting configuration. [3]

(ii) In parts (a) and (b) below, subject to the proviso in part (a), you may use any of the recursive functions, or results about them, given in the *Logic Handbook* without proving that they are recursive.

(a) Show that the condition '$x = y$' on pairs (x, y) of natural numbers is primitive recursive. You may *not* use the function Eq in the *Logic Handbook* without proving that it is primitive recursive. [4]

(b) Show that the function f defined by

$$f(x,y) = \begin{cases} x^2 y, & \text{if } 3x = 4y, \\ x + 2y, & \text{if } x = 2y + 1, \\ 4, & \text{otherwise,} \end{cases}$$

is primitive recursive. [4]