Question 12

In this question you may use any of the recursive functions (other than d), or results about them, given in the *Logic Handbook* without proving that they are recursive. You may also give your answers as informal definitions.

(i) Show that the function d defined by

$$d(x,y) = \begin{cases} 1, & \text{if } x \text{ is divisible by } y, \\ 0, & \text{otherwise,} \end{cases}$$

is primitive recursive. [5]

(ii) By summing the values of $d(x,y)$ for appropriate values of y, or otherwise, show that the function p defined by

$$p(x) = \begin{cases} 1, & \text{if } x \text{ is a prime number,} \\ 0, & \text{otherwise,} \end{cases}$$

is primitive recursive. [6]

Question 13

(i) Show that the following formula takes truth value 1 under all interpretations of its symbols.

$$(((x = y \vee \forall x\, x = y) \longrightarrow \exists x(x = y \vee \forall x\, x = y)) \\ \longrightarrow (\neg \exists x(x = y \vee \forall x\, x = y) \longrightarrow \neg x = y)) \quad [3]$$

(ii) The following is a correct (but contorted) proof from which the assumption numbers have been omitted.

(1)	$\exists x(\phi \longrightarrow \psi)$	Ass
(2)	$\forall x(\psi \longrightarrow (\chi \;\&\; \neg\chi))$	Ass
(3)	$(\phi \longrightarrow \psi)$	Ass
(4)	$(\psi \longrightarrow (\chi \;\&\; \neg\chi))$	UE, (2)
(5)	χ	Ass
(6)	$\neg\psi$	Taut, (4)
(7)	$\neg(\neg\chi \vee \phi)$	Taut, (3), (5), (6)
(8)	$(\chi \longrightarrow \neg(\neg\chi \vee \phi))$	CP, (7)
(9)	$\exists x(\chi \longrightarrow \neg(\neg\chi \vee \phi))$	EI, (8)
(10)	$\exists x(\chi \longrightarrow \neg(\neg\chi \vee \phi))$	EH, (9)

(a) Write down the assumptions in force on each line. [2½]

(b) Write down the tautology used on line (7). [½]

(c) For each of the following possible line (11)s, write down whether the proof would still be correct were the line to be added.

(A) 1(11) $(\phi \longrightarrow \psi)$ EH, (3)

(B) 3(11) $\exists x(\phi \longrightarrow \psi)$ EI, (3)

Answer YES or NO. [2]

(iii) (a) Give an example of formulas ϕ and ψ for which ψ is a tautological consequence of ϕ. [1½]

(b) Give an example of formulas ϕ and ψ for which ψ is a logical, but not a tautological, consequence of ϕ. [1½]

[No justification is required in either case.]