Question 8

(i) As the cycle in the ICF of $\sqrt{15}$ has even length, each even convergent yields a solution of this Pell's equation. The convergents begin

$$\frac{3}{1}, \frac{4}{1}, \frac{27}{7}, \frac{31}{8}, \frac{213}{55}, \frac{244}{63}, \cdots$$

and so the solutions are $x = 4$ and $y = 1$; $x = 31$ and $y = 8$; $x = 244$ and $y = 63$.

(ii) Solutions are given by $x = 2mn$, $y = m^2 - n^2$, $z = m^2 + n^2$, where m and n are relatively prime positive integers with opposite parity and $m > n$. As $x = 60$ we solve $mn = 30$. There are four solutions:

$m = 30, n = 1$ giving the triple $(60, 889, 901)$;

$m = 15, n = 2$ giving the triple $(60, 221, 229)$;

$m = 10, n = 3$ giving the triple $(60, 91, 109)$;

$m = 6, n = 5$ giving the triple $(60, 11, 61)$.