Question 9 (Unit 5) – 15 marks

(i) Parts (a), (b) and (c) below are concerned with the following (contorted but correct) formal proof.

(1)	$(-\theta \,\&\, \phi)$	Ass	1
(2)	$(-\psi \vee \theta)$	Ass	2
(3)	$(-\theta \rightarrow -\psi)$	Taut, (2)	2
(4)	$((-\psi \vee \theta) \rightarrow (-\theta \rightarrow -\psi))$	CP, (3)	
(5)	$-\psi$	Taut, (1), (3)	1,2
(6)	$((-\theta \,\&\, \phi) \rightarrow -\psi)$	Ass	6
(7)	$((-\theta \,\&\, \phi) \rightarrow -\psi)$	Taut, (6)	6
(8)	$((-\theta \,\&\, \phi) \rightarrow -\psi)$	CP, (5)	2

(a) The assumption numbers are missing from the above proof. Write down what they should be for each line of the proof. [4]

(b) State which formulas have been used where the Tautology Rule has been applied (on lines (3), (5) and (7)). [3]

(c) Show that the formula in your answer to part (b) which is used to obtain line (5) is indeed a tautology. [2]

(ii) Write down a formal proof of the formula

$$((\phi \longleftrightarrow -\psi) \rightarrow (-\phi \rightarrow \psi))$$

depending on no assumptions. State which tautologies you have used whenever the Tautology Rule has been applied in your proof. [6]

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Questions 1 to 5 are on Number Theory.
Questions 6 to 9 are on Mathematical Logic.

Number Theory

Question 1 (Unit 7) – 14 marks

(i) Determine the simple continued fraction of $\frac{121}{84}$ and hence obtain the general solution of the linear Diophantine equation

$$121x - 84y = 3.$$ [4]

(ii) (a) Determine the irrational number α which has the periodic continued fraction $[2; 4, \overline{1,3}]$. [5]

(b) Write down the convergents $C_0, C_1, C_2, \ldots, C_6$ of α and, without using a calculator, find the least value of i for which $|\alpha - C_i| < 0.0005$, justifying your answer. [*Hint*: To show that a convergent lacks the wanted accuracy you may find SAQ 17 helpful.] [5]