Question 9 (Unit 7) — 10 marks

One of the following sentences is a theorem of Q, and one is not. Give a formal proof of the sentence that is a theorem of Q, and explain why the other sentence is not a theorem of Q. (You may assume that the axioms of Q are true in both the interpretations N^* and N^{**} given on pages 5 and 6 of the *Logic Handbook*.)

(i) $\forall x \forall y (x \cdot (y + \mathbf{0})) = ((x \cdot y) + (x \cdot \mathbf{0}))$

(ii) $\forall x \forall y ((y + \mathbf{0}) \cdot x) = ((y \cdot x) + (\mathbf{0} \cdot x))$ [10]

$x = \alpha \quad y = \beta$

$\alpha(\beta) = (\alpha \cdot \beta) + (\alpha \cdot 0)$

$x = \beta \cdot y = \alpha$

$\beta \cdot (\alpha) = \beta \cdot \alpha$

$x = y = \alpha$

$x = 0 \quad y = \alpha$

$\alpha \cdot 0 =$

$x = \alpha, y = \alpha$

$(\alpha + 0) \cdot \alpha = \alpha \cdot \alpha + 0 \cdot \alpha$

$\alpha \cdot \alpha = \alpha \cdot \alpha + 0 \cdot \alpha$

$\beta = \beta + \alpha$

$\beta =$

$x = \alpha, y = \beta$

$(\alpha + 0) \cdot \beta = (\alpha \cdot \beta) + (0 \cdot \alpha)$

$\alpha \cdot \beta = \beta + \alpha$

$\beta = \beta$

$x = \beta, y = \beta$

$(\beta + 0) \cdot \beta = (\beta \cdot \beta) + (0 \cdot \beta)$

$\beta \cdot \beta = \beta \cdot \beta + \beta$

$\alpha = \alpha + \beta$

$x = \beta \quad y = \alpha$

$(\beta + 0) \cdot \alpha = (\alpha \cdot \beta) + (0 \cdot \beta)$

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