

Question 9 (Unit 5) - 15 marks

- (i) Parts (a), (b) and (c) below are concerned with the following (contorted but correct) formal proof.

(1)	$(\theta \vee \neg \phi)$	Ass	1
(2)	$(\theta \rightarrow \neg \psi)$	Ass	2
(3)	$(\neg \theta \rightarrow \neg \phi)$	Taut, (1)	1
(4)	$((\theta \vee \neg \phi) \rightarrow (\psi \rightarrow \neg \phi))$	Ass	4
(5)	$(\psi \rightarrow \neg \phi)$	Taut, (2), (3)	1, 2
(6)	$((\theta \vee \neg \phi) \rightarrow (\neg \theta \rightarrow \neg \phi))$	CP, (3)	
(7)	$((\theta \vee \neg \phi) \rightarrow (\psi \rightarrow \neg \phi))$	CP, (5)	2
(8)	$((\theta \vee \neg \phi) \rightarrow (\psi \rightarrow \neg \phi))$	Taut, (4)	4

- (a) The assumption numbers are missing from the above proof. Write down what they should be for each line of the proof. [4]
- (b) State which formulas have been used where the Tautology rule has been applied (on lines (3), (5) and (8)). [3]
- (c) Show that the formula in your answer to part (b) which is used to obtain line (5) is indeed a tautology. [2]
- (ii) Write down a formal proof of the formula

$$((\phi \leftrightarrow \psi) \rightarrow (\neg \psi \rightarrow \neg \phi))$$

depending on no assumptions.

State which tautologies you have used whenever the Tautology rule has been applied in your proof. [6]