(9)

7) i) $c(x,y,z) = \begin{cases} 1 & \text{if } yz \le x \\ 0 & \text{otherwise} \end{cases}$

$g_1 = s(z) (=1)$

$g_2 = z$

$c_1 = \overline{sg}(dif(prod(y,z),x))$ ← this is $c(x,y,z)$

$c_2 = \overline{sg}(c_1)$

But if $c_2 = 1$, $g_2 = 0$ so we only need to define g_1 and c_1, since if not c_1, then z.

$c(x,y,z) = prod(s(z), \overline{sg}(dif(prod(y,z),x))$

$= \overline{sg}(dif(prod(y,z),x))$

so c is a composition of primitive recursive functions, hence primitive recursive.

Is this right too? yes

$c_1 = sg(dif(s(x), prod(y,z)))$

so $c(x,y,z) = sg(dif(s(x), prod(y,z)))$

ii) quot(x,y) is the no. of z s greater than zero and less than or equal to x for which $yz \le x$ ie $x - yz \ge 0$

ie $quot(x,y) = \sum_{z=1}^{x} \overline{sg}(dif(prod(y,z),x))$

and why is this p r? refer to Ex 3 p 79

then if $prod(y,z) > x$

$dif(prod(y,z),x) > 0 \Rightarrow \overline{sg}(dif(prod(y,z),x)) = 0$

and if $prod(y,z) < x$

$dif(prod(y,z),x) = 0 \Rightarrow \overline{sg}(dif(prod(y,z),x)) = 1$

if $prod(y,z) = x$

$dif(prod(y,z),x) = 0 \Rightarrow \overline{sg}(dif(prod(y,z),x)) = 1$

(8)

Example $quot(5,2) = \overline{sg}(dif(prod(2,1),5))$
$+ \overline{sg}(dif(prod(2,2),5)) + \overline{sg}(dif(prod(2,3),5))$
$+ \overline{sg}(dif(prod(2,4),5)) + \overline{sg}(prod(2,5),5))$
$= \overline{sg}(dif(2,5)) + \overline{sg}(dif(4,5)) + \overline{sg}(dif(6,5))$
$+ \overline{sg}(dif(8,5)) + \overline{sg}(dif(10,5))$
$= 1 + 1 + 0 + 0 + 0 = 2$ which is correct PTO ?

You didnt finish it