Question 7 (Unit 4) — 10 marks

(i) Show that the function c of three arguments defined by

$$c(x, y, z) = \begin{cases} 1, & \text{if } yz \leq x, \\ 0, & \text{otherwise,} \end{cases}$$

is primitive recursive. [3]

(ii) This part of the question concerns the result of the Euclidean Algorithm, namely that: given any two integers x, y with $y > 0$, there exist unique integers q and r such that

$$x = qy + r, \quad \text{with } 0 \leq r < y;$$

the number q is called the *quotient* on division of x by y.

Show that the function quot : $N \times P \longrightarrow N$ defined by

$$\text{quot}(x, y) = \text{the quotient on division of } x \text{ by } y$$

is primitive recursive.

[The domain of quot is taken as $N \times P$ to avoid $y = 0$. *Hint*: How is the number of zs less than or equal to x for which $yz \leq x$ related to quot(x, y)?] [7]

Question 8 (Unit 4) — 5 marks

(i) Draw the flow graph of the Turing machine M given by the following tables of values of $a(x, y)$ and $q(x, y)$.

$a(x,y)$		$y=0$	$y=1$
x	1	0	3
	2	3	2
	3	1	1
	4	0	1
	5	1	2

$a(x, y) = y$ otherwise

$q(x,y)$		$y=0$	$y=1$
x	1	0	3
	2	1	2
	3	5	0
	4	0	0
	5	2	3

$q(x, y) = 0$ otherwise

[2]

$q_1 1 R q_3$, $q_2 0 R q_2$
$q_2 1 L q_2$ $q_3 0 1 q_5$
$q_5 0 1 q_2$ $q_5 1 L q_3$

(ii) At a certain stage in M's computation on a certain input, the left number and right number describing M's tape configuration are 13 and 46 respectively, and M is in state 2.

(a) Produce a diagram of M's tape at this stage.

(b) Calculate the left and right numbers after the next step in M's computation. [3]

Question 9 (Unit 5) — 10 marks

(i) For each of the following strings of symbols, state whether or not the string is a formula. (Simply answer YES or NO.)

(a) $(\forall y\ y = \mathbf{0}' \vee y = (\mathbf{0} + t + \mathbf{0}'))$

(b) $\forall z((x \cdot y) = z \rightarrow \exists z(x \cdot y) = (x \cdot y))$

(c) $\exists z \forall t(x + (\mathbf{0} + (y \cdot \mathbf{0}')))$ [3]

(ii) In addition, for each string in part (i) above which is *not* a formula, explain briefly why it is not a formula. [2]

(iii) By choosing suitable subformulas, show that the formula

$((\forall x\ x = \mathbf{0}\ \&\ \exists x(x = \mathbf{0} \rightarrow \forall x\ x = \mathbf{0})) \rightarrow (x = \mathbf{0} \vee (\forall x\ x = \mathbf{0} \longleftrightarrow -x = \mathbf{0})))$

has truth value 1 under all interpretations of the symbols used in the formula. [5]