

Question 10 (Unit 5) 15 marks

- (i) Parts (a), (b) and (c) below are concerned with the following (contorted but correct) formal proof.

1	(1) $\neg(\theta \& \neg\psi)$	Ass	$\neg(\theta \& \neg\psi) \rightarrow (\neg\psi \rightarrow \neg\theta)$
1	(2) $(\neg\psi \rightarrow \neg\theta)$	Taut, (1)	
3	(3) $(\neg\phi \vee \theta)$	Ass	$(\neg\psi \rightarrow \neg\theta) \& (\neg\phi \vee \theta) \rightarrow (\phi \rightarrow \psi)$
5	(4) $(\neg(\theta \& \neg\psi) \rightarrow (\neg\psi \rightarrow \neg\theta))$	CP, (2)	
5	(5) $((\neg\phi \vee \theta) \rightarrow (\theta \rightarrow \psi))$	Ass	$((\neg\phi \vee \theta) \rightarrow (\theta \rightarrow \psi))$
3	(6) $(\phi \rightarrow \psi)$	Taut, (2), (3)	
5	(7) $((\neg\phi \vee \theta) \rightarrow (\phi \rightarrow \psi))$	Taut, (5)	$\rightarrow ((\neg\phi \vee \theta) \rightarrow (\phi \rightarrow \psi))$
1	(8) $((\neg\phi \vee \theta) \rightarrow (\phi \rightarrow \psi))$	CP, (6)	

- (a) The assumption numbers are missing from the above proof. Write down what they should be for each line of the proof. [4]
- (b) State which formulas have been used where the Tautology Rule has been applied (on lines (2), (6) and (7)). [3]
- (c) Show that the formula in your answer to part (b) which is used to obtain line (6) is indeed a tautology. [2]
- (b) Write down a formal proof of the formula

$$((\neg\phi \leftrightarrow \psi) \rightarrow (\phi \rightarrow \neg\psi))$$

depending on no assumptions. State which tautologies you have used whenever the Tautology Rule has been applied in your proof. [6]