②

n divisible by 2 odd primes $p_i, p_j \geq 3 \Rightarrow$
$\phi(n)$ divisible by $(p_i - 1), (p_j - 1)$, which are both even since p_i, p_j are odd $\therefore \phi(n)$ is divisible by 4.

$\therefore \phi(n)$ not divisible by 4 implies
$\phi(n)$ odd or divisible by 2 but not 4
$\phi(n)$ odd $\Rightarrow n = 2^k$ but $\phi(2^k) = 2^{k-1}(2-1)$
$= 2^{k-1} \therefore k = 1.$

$\phi(n)$ even $\Rightarrow n = p, p \equiv 3 \bmod 4$ ie Explanation?
$p = 4k+3$ (there are an infinite no of primes of this form).
or $n = 2p$, $p = 4k+3$, since $\phi(n)$
$= \phi(2p) = \phi(2)\phi(p) = \phi(p)$ since $\phi(2) = 1$
Or $n = 2^k \Rightarrow \phi(2^k) = 2^{k-1}(2-1) = 2^{k-1}$
$\Rightarrow k = 2 \Rightarrow n = 4$

So: $\phi(n)$ not divisible by 4 implies 3/5
$n = 2, 4$, or
any prime of the form $4k+3$
or $2p$ where $p = 4k+3$ or p^s or $2p^s$ ⑤
or $n = 1$ by defn. $p = 4k+3$

3) i) $\gcd(1,16) = \gcd(3,16) = \gcd(5,16) = \gcd(7,16)$
$= \gcd(9,16) = \gcd(11,16) = \gcd(13,16) = \gcd(15,16) = 1$
which is square free since 1 is not prime

$\gcd(2,16) = \gcd(6,16) = \gcd(10,16) = \gcd(14,16) = 2$
which is prime and square free.
$\gcd(4,16) = \gcd(12,16) = 4 = 2^2$, not square free
$\gcd(8,16) = 8 = 2^2 \cdot 2$ not square free
$\gcd(16,16) = 16 = 4^2$, not square free.
Counting up the number of square free integers a, $1 \leq a \leq 16$, $\gcd(a,16)$ square free, we see $F(16) = 12$.