i) $5n^2-1=N$ divisible by p

$\Rightarrow 5n^2-1\equiv 0 \mod p \Rightarrow 5n^2\equiv 1 \mod p$

so $(5/p)(n^2/p)=(1/p)$

but $(n^2/p)=(1/p)=1 \therefore (5/p)\cdot 1 = 1 \Rightarrow (5/p)=1$

You'll never know how much agony this question caused me.

ii) $(5/p)=1=(p/5)$ (by LQR: $5\equiv 1 \pmod 4$) $\therefore$ p is a quadratic residue of 5. Any no. can be expressed as

$5k$
$5k+1$
$5k+2$
$5k+3$
$5k+4$

Taken modulo 5, these become

0
1
2
3
4

5 has $(5-1)/2=2$ quadratic residues:

1 since $1=1^2 \pmod 5$
4 since $4=2^2 \pmod 5$

and

$(5k+3)^2=5(5k^2+6k+1)+4=4 \pmod 5$
$(5k+4)^2=5(5k^2+8k+3)+1\equiv 1 \pmod 5$

$\therefore$ p has one of the forms $5k+1$ or $5k+4$

iii) Suppose that all the primes which divide p have form $5k+1$ (since N is odd from part ii), all the primes which divide N have form $5k+1$ or $5k+4$). The product of any two of these primes will be of the form $5k+1$.

$(5m+1)(5n+1)=5(5nm+n+m)+1$

and the product of any no. of primes