(4)

$F(p^r) = p^r\left(1 - \frac{(r-1)}{p^r}\right)$

Since F is multiplicative if

$n = p_1^{r_1} p_2^{r_2} \ldots\ldots p_n^{r_n}$

$F(n) = F(p_1^{r_1}) F(p_2^{r_2}) \ldots\ldots F(p_n^{r_n})$

$= p_1^{r_1}\left(1 - \frac{(r_1-1)}{p_1^{r_1}}\right) p_2^{r_2}\left(1 - \frac{(r_2-1)}{p_2^{r_2}}\right) \ldots\ldots p_n^{r_n}\left(1 - \frac{(r_n-1)}{p_n^{r_n}}\right)$

$= p_1^{r_1} p_2^{r_2} \ldots p_n^{r_n}\left(1 - \frac{(r_1-1)}{p_1^{r_1}}\right)\left(1 - \frac{(r_2-1)}{p_2^{r_2}}\right) \ldots\ldots \left(1 - \frac{(r_n-1)}{p_n^{r_n}}\right)$

$= n \prod_{p^2 / n}\left(1 - \frac{(r_i-1)}{p_i^{r_i}}\right)$ ✗

0/1

(8)

4) i) $3x^2 - 11x + 9 \equiv 0 \pmod{17}$

Multiply through by 4·3

$36x^2 - 132x + 108 \equiv 0 \pmod{17}$

$(6x-11)^2 - 121 + 108 \equiv 0 \pmod{17}$

$(6x-11)^2 - 13 \equiv 0 \bmod 17$

$(6x-11)^2 \equiv 13 \bmod 17$

We must determine whether 13 is a quadratic residue of 17 ✓

a) Using Eulers criterion

$13^{(17-1)/2} \pmod{17} \equiv 13^8 \bmod 17 \equiv (169)^4 \bmod 17$

$\equiv (-1)^4 \bmod 17 \equiv 1$ ∴ the congruence has solns, since 13 a quadratic residue of 17 ✓

b) Using Gauss' Lemma.

Take the first $(17-1)/2 = 8$ members of the set of least +ve residues mod 17

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Multiply by 13

$S' = \{13, 26, 39, 52, 65, 78, 91, 104\}$

Reduce mod 17