ii) $F(7)=7$ ✓ since 7 prime $\Rightarrow a \leq 7, \gcd(a,7)=1$ or 7
$F(8)=6$ ✓ since $\gcd(a,8)$ not square free only for $a=4$ or 8
$F(9)=8$ ✓ since $\gcd(a,9)$ not square free only for $a=3$ or 9
$F(11)=11$ ✓ since 11 prime $\Rightarrow a \leq 11, \gcd(a,11)=1$ or 11

iii) For any prime p, $F(p)=p$ ✓ since for $1 \leq a \leq p$ $\gcd(a,p)=1$ which is square free, or for $a=p$, $\gcd(a,p)=p$ which is a prime hence square free.

iv) $F(p^r)$. In the range $1 \leq a \leq p^r$ there are $r-1$ integers for which $\gcd(a,p^r)$ is divisible by p^2:
p square free since prime
p^2 not square free
p^3 not square free (divisible by p^2)
.....
p^r not square free (divisible by p^2)
ie every power of p from p^2 on is divisible by p^2, hence not square free. There are $r-1$ of these. $\therefore F(p^r)=p^r-(r-1)=p^r\left(1-\frac{(r-1)}{p^r}\right)$ x

what about $2p^2$ $3p^2$ $4p^2$ ....

1/3

v) $F(500)=F(2^2)F(5^3)$ ✓
$=(2^2-(2-1))(5^3-(3-1))$ x
$=(4-1)(125-2)$
$=3 \cdot 123=369$.
$F(540)=F(2^2)F(3^3)F(5)$ ✓
$=(2^2-(2-1))(3^3-(3-1))(5)$
$=(4-1)(25)(5)=375$ x

correct method