

(5)

$$S'' = \{13, 9, 5, 1, 14, 10, 6, 2\}$$

$$= \{1, 2, 5, 6, 9, 10, 13, 14\}$$

Four elements of  $S''$  are greater than  $(17-1)/2 = 8$  (these are 9, 10, 13, 14)  
 $\therefore (-1)^4 = 1 \therefore 13$  is a quadratic residue of 17, so the congruence has soln. ✓

$$\begin{aligned} \text{ii) } (30/71) &= (2/71) \cdot (3/71) \cdot (5/71) \quad (\text{Th 2.1}) \\ &= 1 \cdot (3/71) \cdot (5/71) \quad \text{since } 71 \equiv 7 \pmod{8} \quad (\text{Theorem 3.2}) \\ &= -(71/3) (71/5) \quad \text{using LQR since } 3, 71 \equiv 3 \pmod{4} \\ &\quad 5 \equiv 1 \pmod{4} \\ &= -(2/3) (1/5) \quad (\text{Using Th 3.2, } (2/3) \equiv -1 \text{ since } 3 \equiv 3 \pmod{8}) \\ &= (1/5) = 1 \quad \checkmark \text{ since 1 is a quadratic residue of any no} \end{aligned}$$

$$\begin{aligned} (99/37) &= (9/37) (11/37) = (3^2/37) \cdot (11/37) \quad (\text{Th 2.1}) \\ &= (11/37) \quad (\text{Th 2.1}) \\ &= (37/11) \quad (\text{LQR since } 37 \equiv 1 \pmod{4}) \\ &= (4/11) \quad (\text{Th 2.1}) \\ &= (2^2/11) = 1 \quad \checkmark \quad (\text{Th 2.1}). \end{aligned}$$

$$\begin{aligned} (211/251) &= -(251/211) \quad (\text{LQR since } 251 \equiv 211 \equiv 3 \pmod{4}) \\ &= -(40/211) \quad (\text{Th 2.1}) \\ &= -(2/211) (4/211) (5/211) \quad (\text{Th 2.1}) \\ &= -(2/211) (5/211) \quad (\text{Th 2.1}) \\ &= (5/211) \quad (\text{Th 3.2 since } 211 \equiv 3 \pmod{8}) \\ &= (211/5) \quad (\text{LQR since } 5 \equiv 1 \pmod{4}) \\ &= (1/5) = 1 \quad \checkmark \quad (\text{Theorem 2.1}) \end{aligned}$$

14