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6) $f(x,y)=\begin{cases} x^3y & \text{if } x+y=60 \text{ and } x>19 \\ 2^y & \text{if } x+2y\geq 105 \\ 7 & \text{otherwise} \end{cases}$

$g_1 = prod(exp(x,3),y)$

$g_2 = exp(2,y)$

$g_3 = 7\ (= s(s(s(s(s(s(s(z))))))))$

$c_1 = sg(prod(dif(x,19), Eq(sum(x,y),60)))$

$c_2 = \overline{sg}(dif(105, sum(x, prod(2,y))))$

$c_3 = \overline{sg}(sum(c_1, c_2))$

Then $f(x,y) = g_1\cdot c_1 + g_2\cdot c_2 + g_3\cdot c_3$ — You should prove this not just claim it

and f is the sum of products of primitive recursive fns hence f primitive recursive

We must check for collectively exhaustive and mutually exclusive conditions.

$c_1: x+y=60$ and $x>19 \Rightarrow 20\leq x\leq 60$
$0\leq y\leq 40$

$c_2: x+2y\geq 105$

Use c_1 to check mutually exclusive

$x+2y = x+2(60-x) = 120-x < 105$

since c_1 implies $x\geq 20$ ∴ c_1 and c_2 are mutually exclusive.

c_1 and c_3 are mutually exclusive and c_2 and c_3 are mutually exclusive by defn of otherwise.

c_1, c_2 and c_3 have the relationship if not c_1 and not c_2 then c_3 ∴ c_1, c_2 and c_3 are collectively exhaustive

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