of form $5k+1$ is itself of the form $5k+1$, hence if N is the product of primes all of form $5k+1$ it would itself be of this form, which its not the case. $\therefore$ N has a prime divisor of the form $5k+4$.

iv) Suppose only finitely many primes of form $5k+4$; $p_1, p_2, p_3, \ldots, p_n$ say, where p_n is the largest. Consider the number

$N = 5(p_n!)^2 - 1$. not necessarily!!

$\boxed{p_n \text{ is even}}$, so $5(p_n!)^2$ is even and $5(p_n!)^2 - 1$ is odd. $\therefore$ from iii) N has a prime divisor of the form $5k+4$ and by hypothesis of the finite no. of such primes it must be one of $p_1, p_2, p_3, \ldots, p_n$, say p_i $i \le n$.

Write $5(p_n!) - N = 1$, then since $p_i \mid p_n!$ and $p_i \mid N$ $p_i \mid (5(p_n!) - N)$ ie $p_i \mid 1$ which is a contradiction since $p_i > 1$

$\therefore$ p_i is not any of $p_1, p_2, p_3, \ldots, p_n$ but p_i is of the form $5k+4$ $\therefore$ there are infinitely many primes of this form.

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