

Question 9 (Unit 3) – 12 marks

In this question you may use any of the primitive recursive functions or results about such functions discussed in Unit 3. You may present your arguments using either formal or informal definitions of the functions involved. Please give your tutor a reference to any result you use.

- (i) Show that the function f defined by

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is even,} \\ 0, & \text{if } x \text{ is odd,} \end{cases}$$

is primitive recursive.

[3]

- (ii) Using the function of part (i), or otherwise, show that the function g defined by

$$g(x) = \begin{cases} x^2, & \text{if } x \text{ is even,} \\ 0, & \text{if } x \text{ is odd,} \end{cases}$$

is primitive recursive.

[3]

- (iii) Using the result of part (ii), or otherwise, show that the function h defined by

$h(x)$ = the sum of the squares of all the even numbers less than x

is primitive recursive.

[3]

- (iv) Show that the function k defined by

$k(x)$ = the sum of the squares of all the even numbers
less than or equal to x

is primitive recursive.

[3]

Question 10 (Unit 3) – 6 marks

The function f is defined by

$$f(x, y) = (3x \div 5y) + (5y \div 3x).$$

- (i) Compute the values of $\text{Mn}[f](5)$ and $\text{Mn}[f](15)$. [2]

- (ii) Explain why $\text{Mn}[f](2)$ is not defined. [1]

- (iii) For which natural numbers x is it the case that $\text{Mn}[f](x)$ is defined?
Give a brief justification of your answer. [3]
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