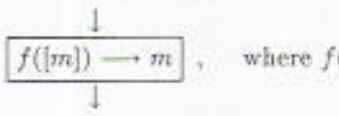
Question 7 (Unit 2) – 10 marks

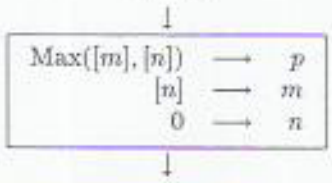
In both parts of this question you may use extra registers besides the ones mentioned, if you so wish.

(i) Give in full the flow chart of an abacus machine program which has the effect shown in the following diagram.



, where $f(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even,} \\ \frac{x+1}{2}, & \text{if } x \text{ is odd.} \end{cases}$ [5]

(ii) Give in full the flow chart of an abacus machine program which has the effect shown in the following diagram.



, where $\text{Max}(x, y) = \begin{cases} y, & \text{if } x \leq y; \\ x, & \text{if } y < x. \end{cases}$

Assume that $[p] = 0$ initially. [5]

Question 8 (Unit 3) – 12 marks

(i) Let h be the function

$$\text{Pr}[\text{Cn}[s, s], \text{Cn}[\exp, \text{id}_3^3, \text{Cn}[s, \text{id}_2^3]]],$$

where exp is the exponential function defined by $\exp(x, y) = x^y$. Determine the values of $h(4, 0)$ and $h(3, 2)$, writing down the stages of your calculation. [6]

(ii) Give a formal definition, using the operations of composition and primitive recursion in terms of the *basic* functions (i.e. the zero, successor and identity functions) and, if you wish, the sum function, which shows the function $h : N \times N \times N \longrightarrow N$ defined by $h(x_1, x_2, x_3) = x_1 + (x_2 \cdot x_3)$ to be primitive recursive. [6]

Questions 9 and 10 overleaf