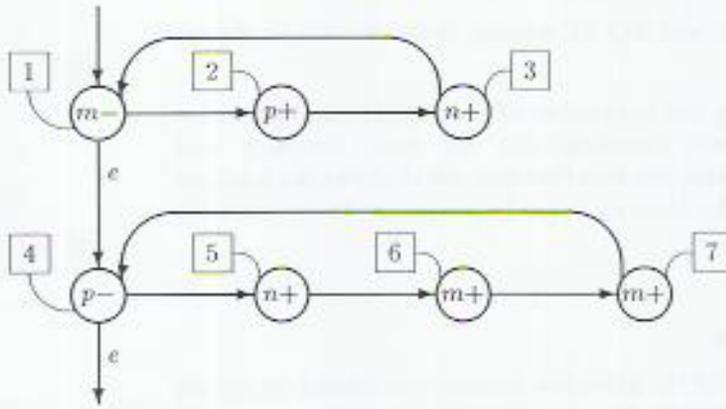


Mathematical Logic

Question 5 (Unit 2) - 10 marks

This question concerns the abacus machine with the following flow chart.



- (i) Write down the trace table for the computation of this machine when initially the contents of the registers are:

$$[m] = 1, [n] = 1, [p] = 2.$$

[4]

- (ii) Suppose that m, n, p initially contain respectively the first, second and third arguments of a function f and that the value of f is given by the content of register n when the computation halts.

- (a) Write down a formula which describes the rule of f .
 (b) How are the final contents of registers m and p related to the initial contents of the three registers?

[6]

Question 6 (Unit 2) - 10 marks

In both parts of this question you may use extra registers besides the ones mentioned, if you so wish.

- (i) Give in full the flow chart of an abacus machine program which has the effect shown in the following diagram.

$$\begin{array}{c}
 \downarrow \\
 \boxed{f([m]) \rightarrow m} \\
 \downarrow
 \end{array}
 \quad \text{where } f(x) = \begin{cases} x/3, & \text{if } x \text{ is exactly} \\ & \text{divisible by 3,} \\ 1, & \text{if the remainder when} \\ & x \text{ is divided by 3 is 1,} \\ 2, & \text{if the remainder when} \\ & x \text{ is divided by 3 is 2.} \end{cases}$$

[5]

- (ii) Give in full the flow chart of an abacus machine program which has the effect shown in the following diagram.

$$\begin{array}{c}
 \downarrow \\
 \boxed{\begin{array}{l} \mathcal{X} \mathcal{U} \\ \text{Min}([m], [n]) \rightarrow p \\ \mathcal{X} [n] \rightarrow m \\ 0 \rightarrow n \end{array}} \\
 \downarrow
 \end{array}
 \quad \text{where } \text{Min}(x, y) = \begin{cases} x, & \text{if } x \leq y, \\ y, & \text{if } y < x. \end{cases}$$

Assume that $[p] = 0$ initially.

[5]