Question 7 (Unit 3) – 12 marks

(i) Let h be the function

$$\mathrm{Pr}[\mathrm{Cn}[s,s], \mathrm{Cn}[\mathrm{dif}, \mathrm{Cn}[s, \mathrm{id}_3^2], \mathrm{id}_2^3]],$$

where dif is the difference function of Example 8, page 64 of *Unit 3*.

Determine the values of $h(4,0)$ and $h(3,2)$, writing down the stages of your calculation. [6]

(ii) Give a formal definition, using the operations of composition and primitive recursion in terms of the *basic* functions (i.e. the zero, successor and identity functions) and, if you wish, the sum function, which shows the function $h : N \times N \times N \longrightarrow N$ defined by $h(x_1, x_2, x_3) = (x_1 \cdot x_2) + x_3$ to be primitive recursive. [6]

Question 8 (Unit 3) – 12 marks

In this question you may use any of the primitive recursive functions or results about such functions discussed in Unit 3. You may present your arguments using either formal or informal definitions of the functions involved. Please give your tutor a reference to any result you use.

(i) Show that the function f defined by

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is even,} \\ 0, & \text{if } x \text{ is odd,} \end{cases}$$

is primitive recursive. [3]

(ii) Using the function of part (i), or otherwise, show that the function g defined by

$$g(x) = \begin{cases} x, & \text{if } x \text{ is even,} \\ 0, & \text{if } x \text{ is odd,} \end{cases}$$

is primitive recursive. [3]

(iii) Using the result of part (ii), or otherwise, show that the function h defined by

$h(x) =$ the sum of all the even numbers less than x

is primitive recursive. [3]

(iv) Show that the function k defined by

$k(x) =$ the sum of all the even numbers less than or equal to x

is primitive recursive. [3]

Question 9 (Unit 3) – 6 marks

The function f is defined by

$$f(x,y) = (x \dot{-} y^2) + (y^2 \dot{-} x).$$

(i) Compute the values of $\mathrm{Mn}[f](1)$ and $\mathrm{Mn}[f](4)$. [2]

(ii) Explain why $\mathrm{Mn}[f](3)$ is not defined. [1]

(iii) For which natural numbers x is it the case that $\mathrm{Mn}[f](x)$ is defined? Give a brief justification of your answer. [3]