Similar to Prob11 p65 units 1-4 + prob12 p65

iii) $h(x) = \sum_{n<x} g(n)$

$h(s(x)) = \sum_{n<x+1} g(n) = \sum_{n<x} g(n) + g(x)$

but refer to prob11 p65 for justification

Which is a sum of primitive recursive functions

Define $h(0) = 0 = z$

Then h is primitive recursive

$\cancel{h = Pr[z, Cn[sum\ Cn[g, id_1^2]\ id_2^2]](x, h(x))}$

can't have h in the defn. of h(x)

or

$h(s(x)) = h(x) + x \cdot f(x)$

$\cancel{h = Pr[z, Cn[sum\ Cn[prod\ id_1^2\ Cn[f, id_1^2], id_2^2](x, h(x))}$

iv) $k(x) = x \cdot f(x) + h(x)$ $= g(x) + h(x)$.

units 1-4 prob12 p65 for sum

which is a sum of primitive recursive fns, hence primitive recursive, and

$k(s(x)) = s(x) \cdot f(s(x)) + k(x)$

Define $k(0) = 0 = z$

$k = Pr[z, Cn[sum\ Cn[prod\ Cn[s\ id_1^2], Cn[f\ Cn[s\ id_1^2]]], id_2^2](x, k(x))$

I can also write

$k(x) = x \cdot f(x) + h(x)$

$= g(x) + h(x)$

which is a sum of primitive recursive functions, hence is primitive recursive

$k(s(x)) = g(s(x)) + h(s(x))$

$= g(s(x)) + k(x)$

$k = Pr[z, Cn[sum\ Cn[g\ Cn[s\ id_1^2]], id_2^2]](x, k(x))$