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ii) $h(x_1, x_2, s(x_3)) = x_2 \cdot (x_1 + x_3 + 1)$
$= x_2 \cdot (x_1 + x_3) + x_2$
$= h(x_1, x_2, x_3) + x_2$

$\therefore g(x_1, x_2, x_3, h(x_1, x_2, x_3))$
$= Cn[\text{Sum } id_4^4, id_2^4]$

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$h(x_1, x_2, 0) = x_2 \cdot (x_1 + 0) = x_1 \cdot x_2 = \sum_1^{x_2} x_1 = f_1(x_1, x_2)$

$f(x_1, x_2) = \underbrace{x_1 + x_1 + x_1 + \ldots + x_1}_{x_2 + \text{times}}$

$h = Pr[Cn[f\ id_1^2, id_2^2], Cn[\text{sum}\ id_4^4\ id_2^4]](x_1, x_2, x_3, h(x_1, x_2, x_3))$

~~Also~~ see opp. for what I now think is the correct soln.

8) i) $f(x) = \begin{cases} 1 & x \text{ is even} \\ 0 & x \text{ is odd} \end{cases}$

If x is even $s(x)$ is odd
If x is odd, $s(x)$ is even
$\therefore f(s(x)) = \overline{sg}(f(x))$
Where $\overline{sg}$ is the signum function defined in problem 13, p65 of units 1-4 and BJ p85

$f(0) = s(z) = 1$
$f(s(x)) = \overline{sg}(f(x))$ } $f = Pr[Cn[s, z], Cn[\overline{sg}\ id_1^2]](x, f(x))$

no need to be formal, but if you are - see my remarks on fn/image

hence $f(x)$ is primitive recursive

ii) $g(x) = \begin{cases} x & \text{if } x \text{ is even} \\ 0 & \text{if } x \text{ is odd} \end{cases}$

If x is even, $g(x) = x$, $f(x) = 1$
x is odd $g(x) = 0$, $f(x) = 0$
hence $g(x) = x \cdot f(x)$

units 1-4 p65 prob 12

$g(x)$ is a product of primitive recursive functions, hence is primitive recursive.

BJ p85 Defn by cases

Also $g(s(x)) = s(x) \cdot f(s(x))$
$= s(x) \cdot \overline{sg}(f(x))$
And $g(0) = 0 \cdot f(0) = 0$

$g = Pr[z, Cn[\text{Prod}\ Cn[s\ id_1^2], Cn[\overline{sg}\ Cn[f\ id_1^2]]]](x, g(x))$