9, 11 are coprime we have
$x \equiv 5 \pmod{11}$
$9x \equiv 1 \pmod{17}$
$1 \equiv 18 \pmod{17} \Rightarrow 9x \equiv 18 \pmod{17}$
Divide through by 9, then since 9, 17 are coprime we have
$x \equiv 2 \bmod 17$
Then the smallest value of x satisfying all three congruences is given by
$x_0 \equiv 3 \times 3 \times 187 + 5 \times 7 \times 99 + 2 \times 6 \times 77 \bmod (7 \times 11 \times 17)$
$\equiv 1683 + 4165 + 924 \pmod{1309}$
$\equiv 6772 \pmod{1309}$
$x_0 \equiv 227 \pmod{1309}$ ∴ smallest value of x satisfying all 3 congruences simultaneously $= 227$
(Check: $5 \times 227 \pmod 7 \equiv 1135 \pmod 7 \equiv 1 \bmod 7$
$7(227-2) \pmod{11} \equiv 1575 \pmod{11} \equiv 2 \pmod{11}$
$6 \times 227 \pmod{17} \equiv 1362 \pmod{17} \equiv 2 \pmod{17}$)

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2) i) Prove by induction.
$10 \equiv 10 \equiv 4 \pmod 6$
∴ True for $k=1$ so we have the basis for the induction. Suppose True for k, then try for $k+1$
$10^k \equiv 4 \pmod 6$
$10^{k+1} \equiv 4 \times 10 \pmod 6$
$10^{k+1} \equiv 6 \times 6 + 4 \pmod 6 \equiv 4 \pmod 6$
Hence true for $k=1$, and if true for $n=k$, then true for $n=k+1$. Hence inductively true

ii) We must prove $m \equiv n \pmod 6 \Rightarrow 10^m \equiv 10^n \pmod 7$
Assume $m \geq n$, and write $m = n + 6k$
then $10^m \equiv 10^{n+6k}$ ~~$\equiv 10^n \pmod 7$~~ dont assume
ie $= 10^n \times 10^{6k} \equiv$ ~~$10^n \pmod 7$~~ the answer.
$= 10^n \times (10^6)^k \equiv$ ~~$10^n \pmod 7$~~
By FLT $a^{p-1} \equiv 1 \bmod p$ ∴ $10^6 = 10^{7-1} = 1 \pmod 7$
So $10^n \times (10^6)^k \equiv 10^n \times (1)^k \equiv 10^n \equiv 10^n \pmod 7$
∴ If $m \equiv n \pmod 6$, $10^n \equiv 10^m \pmod 7$. The proof is identical with m, n interchanged if $m < n$