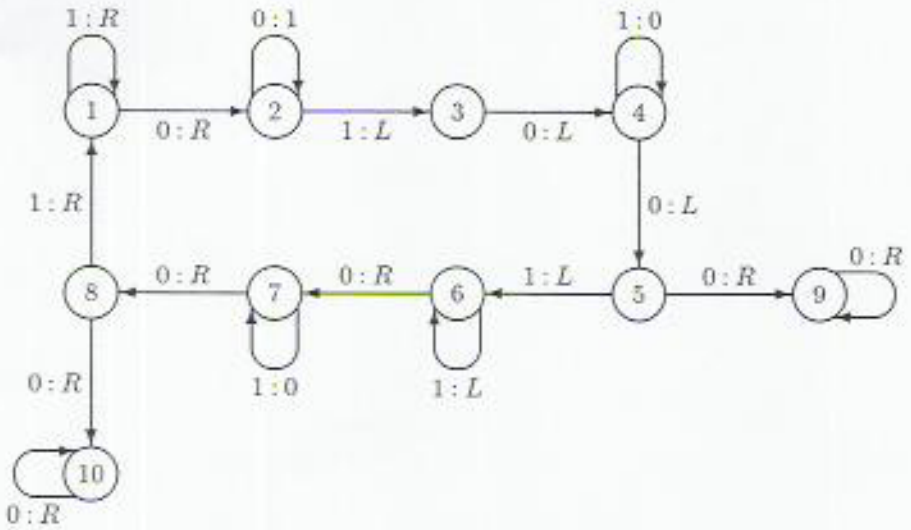
Question 7 (Unit 1) – 15 marks

In this question we consider the Turing machine M with the flow graph below.



(i) Write down the machine table for M. [2]

(ii) For each of the following starting configurations of the machine M, write down the sequence of configurations for the subsequent computation.

(a) 0 1 1 0 (b) 0 1 1 1 0 (c) 0 1 1 1 1 0
1 1 1 [6]

(iii) The machine M has been designed to take as input a positive integer in monadic notation and to output an integer also in monadic notation. Thus the machine computes the values of a function $f : P \longrightarrow N$.

(a) Write down the values of $f(2)$, $f(3)$, $f(5)$, $f(6)$, $f(9)$. [$2\frac{1}{2}$]

(b) What, in general, is the value of $f(n)$ for $n \in P$? Describe briefly how the machine computes $f(n)$, including an indication of each possible halting state and the circumstances under which it halts there. [$4\frac{1}{2}$]

Question 8 (Unit 1) – 10 marks

Parts (i) and (iii) ask you to devise Turing machines which take as input a string s made up of 1s and 2s (but not necessarily both) on an otherwise blank tape. Your machines may only use the symbols 1, 2 and 0 (for blank).

(i) Devise and give the flow graph of a Turing machine which, when started scanning the leftmost symbol of a string s, as in the preamble, halts

(a) scanning a single 1, on an otherwise blank tape, if s contains an even number of 1s,

(b) on a blank tape, otherwise. [$3\frac{1}{2}$]

(ii) Write down the sequences of configurations of the computations of your machine in part (i) when the input s is:

(a) 1 1, (b) 2 1, (c) 2 1 1. [1]

(iii) Devise and give the flow graph of a Turing machine which, when started scanning the leftmost symbol of a string s, as in the preamble, halts

(a) scanning a single 1, on an otherwise blank tape, if s contains both an even number of 1s and an odd number of 2s,

(b) on a blank tape, otherwise. [$4\frac{1}{2}$]

(iv) Repeat part (ii) for your machine in part (iii). [1]
