

Question 8 (Unit 1) - 10 marks

- (i) Devise and give the flow graph of a Turing machine which, using monadic notation, takes as input a positive integer n in standard starting position and halts:
either scanning a single 1, if n is exactly divisible by 3;
or on a blank tape, if n is not exactly divisible by 3. [3]
- (ii) Write down the sequences of configurations of your machine in part (i) when the input n is: (a) 3; (b) 4; (c) 5. [1]
- (iii) Devise and give the flow graph of a Turing machine which, using monadic notation, takes as input a positive integer n in standard starting position and halts:
either scanning a single 1 if n is exactly divisible by 2 or by 3 or by both (e.g. $n = 3, 4, 6$);
or on a blank tape otherwise. [5]
- (iv) Repeat part (ii) for your machine in part (iii). [1]
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Questions 1 to 4 are on Number Theory.

Questions 5 to 9 are on Mathematical Logic.

Number Theory

Question 1 (Unit 3) - 10 marks

- (i) Solve each of the following linear congruences:
(a) $4x \equiv 5 \pmod{7}$; [2]
(b) $7(x + 4) \equiv 2 \pmod{11}$; [2]
(c) $8x \equiv 12 \pmod{19}$. [2]
- (ii) Determine the least positive integer which satisfies all three of the congruences in part (i) simultaneously. [4]

Question 2 (Unit 3) - 13 marks

- (i) Prove that $10^k \equiv 4 \pmod{6}$ for all positive integers k . [4]
- (ii) If $m \equiv n \pmod{6}$ prove that $10^m \equiv 10^n \pmod{7}$. [Hint: Write $m = n + 6k$ and show that $10^6 \equiv 1 \pmod{7}$.] [5]
- (iii) Using parts (i) and (ii) determine the remainder on dividing the number
 $10^{10} + 10^{(10^2)} + 10^{(10^3)} + \dots + 10^{(10^{10})}$
by 7. [4]

Question 3 (Unit 3) - 13 marks

- (i) Find the least positive residue of $26^{59} \pmod{29}$. [4]
- (ii) Solve $13x \equiv 1 \pmod{29}$. Explain why the least positive residue of $13^{27} \pmod{29}$ is a solution of this congruence. Write down a linear congruence (modulo 29) whose solution is congruent to $13^{26} \pmod{29}$ and hence, or otherwise, determine the least positive residue of $13^{26} \pmod{29}$. [4]
- (iii) Use Wilson's Theorem and its converse in proving that, for any positive integer k , the integer $6k + 1$ is prime if and only if $(6k - 3)! + k \equiv 0 \pmod{6k + 1}$. [5]