Question 8 (Unit 1) – 10 marks

(i) Devise and give the flow graph of a Turing machine which, using monadic notation, takes as input a positive integer n in standard starting position and halts:

either scanning a single 1, if n is exactly divisible by 3;
or on a blank tape, if n is not exactly divisible by 3. [3]

(ii) Write down the sequences of configurations of your machine in part (i) when the input n is: (a) 3; (b) 4; (c) 5. [1]

(iii) Devise and give the flow graph of a Turing machine which, using monadic notation, takes as input a positive integer n in standard starting position and halts:

either scanning a single 1 if n is exactly divisible by 2 or by 3 or by both (e.g. $n = 3, 4, 6$);
or on a blank tape otherwise. [5]

(iv) Repeat part (ii) for your machine in part (iii). [1]

TMA M381 02 — Cut-off date 8 June 1995

Questions 1 to 4 are on Number Theory.
Questions 5 to 9 are on Mathematical Logic.

Number Theory

Question 1 (Unit 3) – 10 marks

(i) Solve each of the following linear congruences:

(a) $4x \equiv 5 \pmod 7$; [2]

(b) $7(x+4) \equiv 2 \pmod{11}$; [2]

(c) $8x \equiv 12 \pmod{19}$. [2]

(ii) Determine the least positive integer which satisfies all three of the congruences in part (i) simultaneously. [4]

Question 2 (Unit 3) – 13 marks

(i) Prove that $10^k \equiv 4 \pmod 6$ for all positive integers k. [4]

(ii) If $m \equiv n \pmod 6$ prove that $10^m \equiv 10^n \pmod 7$. [Hint: Write $m = n + 6k$ and show that $10^6 \equiv 1 \pmod 7$.] [5]

(iii) Using parts (i) and (ii) determine the remainder on dividing the number

$$10^{10} + 10^{(10^2)} + 10^{(10^3)} + \cdots + 10^{(10^{10})}$$

by 7. [4]

Question 3 (Unit 3) – 13 marks

(i) Find the least positive residue of $26^{59} \pmod{29}$. [4]

(ii) Solve $13x \equiv 1 \pmod{29}$. Explain why the least positive residue of $13^{27} \pmod{29}$ is a solution of this congruence. Write down a linear congruence (modulo 29) whose solution is congruent to $13^{26} \pmod{29}$ and hence, or otherwise, determine the least positive residue of $13^{26} \pmod{29}$. [4]

(iii) Use Wilson's Theorem and its converse in proving that, for any positive integer k, the integer $6k+1$ is prime if and only if $(6k-3)! + k \equiv 0 \pmod{6k+1}$. [5]