

**Question 5 (Unit 2) - 9 marks**

Prove that a Fibonacci number is divisible by 21 if and only if it is divisible by 7.

[Hint: Follow the lines of SAQ 18 in Unit 2 and Drill Exercise 2.12 in the Number Theory Exercise Book to determine the occurrences of multiples of 21 in the Fibonacci sequence.]

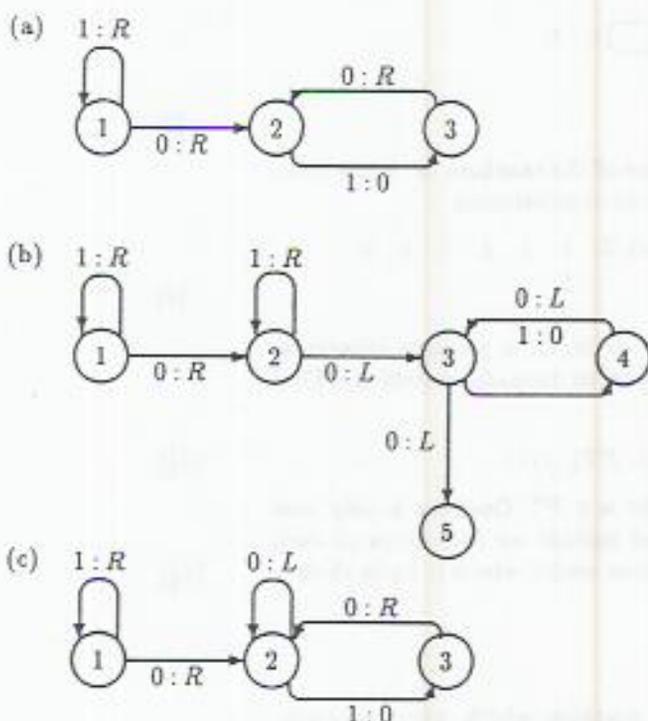
[9]

**Mathematical Logic**

**Question 6 (Unit 1) - 10 marks**

- (i) We wish to design a Turing machine which, using monadic notation, inputs a pair  $(m, n)$  of positive integers in standard starting position (on an otherwise blank tape) and which halts scanning the rightmost of a string of  $m$  1s on an otherwise blank tape.

Write down which of the following Turing machines is suitable for this task. For each machine which is unsuitable, explain why it is unsuitable: this explanation can take the form of a sequence of configurations for appropriate test data.



[7]

- (ii) Devise and give the flow graph of a Turing machine which, using monadic notation, inputs a pair  $(m, n)$  of positive integers in standard starting position (on an otherwise blank tape) and which halts scanning the leftmost of a string of  $n$  1s on an otherwise blank tape.

[3]