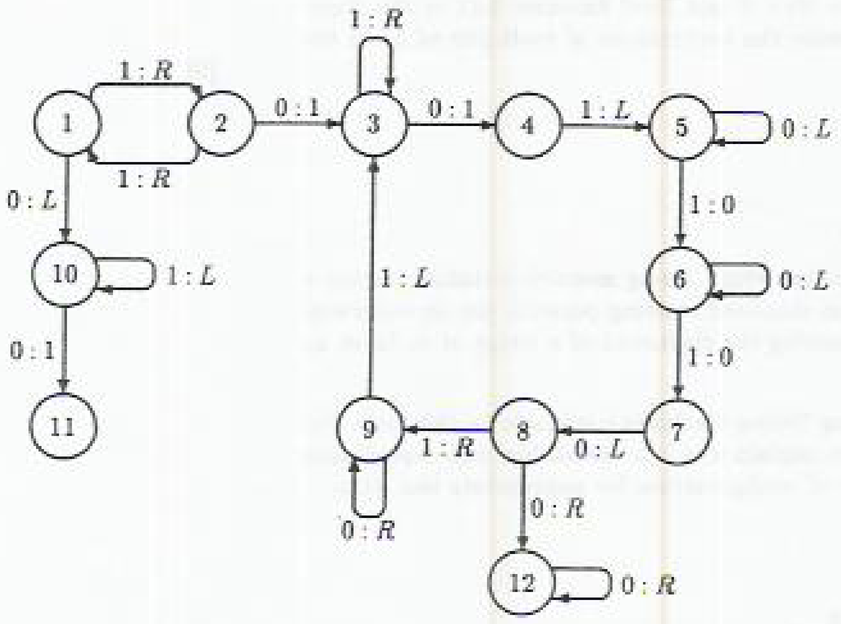
Question 7 (Unit 1) - 15 marks

This question concerns the Turing machine M with the flow graph shown below.



(i) Write down the machine table for M. [2]

(ii) For each of the following starting configurations of the machine M, write down the sequence of configurations of the subsequent computation.

(a) 0 1 1 0 (b) 0 1 1 1 0 (c) 0 1 1 1 1 1 0
1 1 1

[6]

(iii) The machine M has been designed to take as input a positive integer in monadic notation and to output an integer also in monadic notation. Thus the machine computes a function $f : P \longrightarrow N$.

(a) Write down the values of $f(2)$, $f(5)$, $f(6)$, $f(7)$, $f(9)$. [$2\frac{1}{2}$]

(b) What, in general, is the value of $f(n)$ for $n \in P$? Describe briefly how the machine computes $f(n)$. You should include an indication of each possible halting state and the circumstances under which it halts there. [$4\frac{1}{2}$]

Question 8 (Unit 1) - 10 marks

(i) Devise and give the flow graph of a Turing machine which, using monadic notation, takes as input a positive integer m in standard starting position and halts scanning the leftmost of a string of $f(m)$ 1s, where f is the function defined by

$$f(m) = \begin{cases} 1, & \text{if } m \text{ is odd,} \\ 2, & \text{if } m \text{ is even.} \end{cases}$$

[$3\frac{1}{2}$]

(ii) Devise and give the flow graph of a Turing machine which, using monadic notation, takes as input a pair (m, n) of positive integers in standard starting position and halts scanning the leftmost of a string of $g(m)$ 1s, where g is the function defined by

$$g(m, n) = \begin{cases} m, & \text{if } m \text{ is odd,} \\ n, & \text{if } m \text{ is even.} \end{cases}$$

[$4\frac{1}{2}$]

(iii) Write down the sequences of configurations of the computations of your machines in parts (i) and (ii) which evaluate the following values of the functions f and g:

(a) $f(2)$; (b) $f(3)$; (c) $g(1, 2)$; (d) $g(2, 2)$. [2]