

$$\text{so } k = 10(5^2 + 195 + 9) \quad r = 0$$

$\therefore$  The only possible values of  $r$  are 0, 2, 6

ii) Triangular numbers are of the form  $T_n = \frac{1}{2}n(n+1) \quad (= \frac{m}{2})$

Expressing  $m$  in the form  $10k+r$  ( $r=0, 2$  or  $6$  from i), there are two cases to consider:  $k$  even,  $k$  odd.

If  $k$  is even

$$m = 10 \times 2a + r \quad \text{for some integer } a$$

$$\frac{m}{2} = 5a + \frac{r}{2} \quad \text{where } r = 0, 2 \text{ or } 6$$

$$\therefore \frac{r}{2} = 0, 1 \text{ or } 3$$

Since  $a$  is an integer, the last digit of a triangular number can be 0, 1 or 3 for even  $k$ .

If  $k$  is odd,  $k = 2a+1$  for some integer  $a$

$$m = 10 \times (2a+1) + r$$

$$\frac{m}{2} = \frac{10(2a+1) + r}{2} = 10a + \left(5 + \frac{r}{2}\right)$$

$$r = 0, 2 \text{ or } 6 \quad \therefore \frac{r}{2} = 0, 1 \text{ or } 3$$

$$\therefore 5 + \frac{r}{2} = 5, 6 \text{ or } 8$$

Since  $a$  is an integer, the last digit of a triangular number can be 5, 6 or 8 for odd  $k$ .

Hence the last digit of a triangular number can only be one of 0, 1, 3, 5, 6 or 8 and may not be any of 2, 4, 7 or 9 as required.

(11)