

form, which is not the case. Hence $3k+2$ must have a prime divisor of the form $3k+2$. ✓

(This is not the case for a number of the form $3k+1$.)

$$(3k+2)(3l+2) = 3(3kl+2k+2l+1) + 1$$

Hence a no. of the form $3k+1$ may be divisible only by primes of the form $3k+2$.

Suppose that there are only finitely many primes of the form $3k+2$, say

$$p_1, p_2, p_3, p_4, \dots, p_n.$$

Consider the integer

$$N = 3(p_1 p_2 \dots p_n) - 1 = 3(p_1 p_2 \dots p_{n-1}) + 2.$$

N is of the form $3k+2$, hence by the proof above it has a prime divisor, p , of the form $3k+2$. As p_1, p_2, \dots, p_n are presumed to be the only primes of the form $3k+2$, the prime must be one of those, say $p = p_i$. Then p divides N and p divides $3(p_1 p_2 \dots p_n)$ and so p divides the integer combination $3(p_1 p_2 \dots p_n) - N = 1$, i.e. p divides 1, ✓
but p is a prime (so greater than 1) hence we have arrived at a contradiction. ✓

So there are infinitely many primes of the form $3k+2$. ✓

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6a) Try some test data

$$m=2, n=2$$

11011 11011 11011 11011 11001

1 1 1 2 3

11001 11000 110000 (The machine stops here)

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3

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