

$$n = 6s + 1$$

$$n = 6s + 2$$

$$n = 6s + 3$$

$$n = 6s + 4$$

$$n = 6s + 5$$

$$n+1 = 6s + 2$$

$$n+1 = 6s + 3$$

$$n+1 = 6s + 4$$

$$n+1 = 6s + 5$$

$$n+1 = 6s + 6$$

②

③

④

⑤

⑥

Taking each case in turn

① $n(n+1) = 6s(6s+1) = 6(6s^2 + s)$

so $R = 6s^2 + s, r = 0$

② $n(n+1) = (6s+1)(6s+2) = 36s^2 + 18s + 2$
 $= 6(6s^2 + 3s) + 2$

so $R = 6s^2 + 3s, r = 2$

③ $n(n+1) = (6s+2)(6s+3) = 36s^2 + 30s + 6$
 $= 6(6s^2 + 5s + 1)$

so $R = 6s^2 + 5s + 1, r = 0$

④ $n(n+1) = (6s+3)(6s+4) = 36s^2 + 42s + 12$
 $= 6(6s^2 + 7s + 2)$

so $R = 6s^2 + 7s + 2, r = 0$

⑤ $n(n+1) = (6s+4)(6s+5) = 36s^2 + 54s + 20$
 $= 6(6s^2 + 9s + 3) + 2$

so $R = 6s^2 + 9s + 3, r = 2$

⑥ $n(n+1) = (6s+5)(6s+6) = 36s^2 + 66s + 30$
 $= 6(6s^2 + 11s + 5)$

so $R = 6s^2 + 11s + 5, r = 0$

Hence the only possible values of r are 0 and 2 as required. ✓

b) Expressing n as $10s + t$, and $n+1$ as $10s + t + 1$ for $t = 1$ to 9.

$$n = 10s + 1$$

$$n = 10s + 1$$

$$n = 10s + 2$$

$$n = 10s + 3$$

$$n = 10s + 4$$

$$n+1 = 10s + 1$$

$$n+1 = 10s + 2$$

$$n+1 = 10s + 3$$

$$n+1 = 10s + 4$$

$$n+1 = 10s + 5$$

①

②

③

④

⑤