

M381 TMA 01

1) To find $\gcd(242, 176)$

$$242 = 1 \times 176 + 66 \quad (1)$$

$$176 = 2 \times 66 + 44 \quad (2)$$

$$66 = 1 \times 44 + 22 \quad (3)$$

$$44 = 2 \times 22 + 0 \quad \therefore \gcd(242, 176) = 22 \quad \checkmark$$

From Eq (3),

$$22 = 66 - 1 \times 44 \quad (3')$$

Sub for 44 from (2) into (3)'

$$22 = 66 - (176 - 2 \times 66) = 3 \times 66 - 176 \quad (4)$$

$$22 = 3 \times 66 - 176 \quad (4)$$

Sub for 66 from (1)' into (4)

$$22 = 3 \times (242 - 1 \times 176) - 176 = 3 \times 242 - 4 \times 176$$

$$22 = 3 \times 242 - 4 \times 176$$

ie $x=3, y=-4$ is one solution

To find the general soln, put $x=3+s, y=-4+t$
Equation becomes

$$22 = (3+s) \times 242 + (-4+t) \times 176$$

$$22 = 726 + 242s - 704 + 176t$$

$$0 = 242s + 176t \quad \text{ie } 11s + 8t = 0 \Rightarrow s = -\frac{8t}{11}$$

x and y are integers, hence so are s and t , so put $t = 11k$ then $s = -8k$.

Then $x = 3 - 8k, y = -4 + 11k$ is general soln. \checkmark

Least +ve value of y is found for $k=1$, then $y = -4 + 11 \times 1 = 7$

$$x = 3 - 8 \times 1 = -5 \quad \checkmark$$

$$\text{Check: } 22 = -5 \times 242 + 7 \times 176 = -1210 + 1232 = 22 \quad \checkmark$$

2) n and $n+1$ can be expressed as

$$n = 6s + t$$

$$\text{ie } n = 6s$$

$$n+1 = 6s + t + 1 \quad 0 \leq t \leq 5$$

$$\text{ie } n+1 = 6s + 1 \quad (1)$$

(12)