Let P(n) state:

3) $\frac{3}{1^2 2^2}+\frac{5}{2^2 3^2}+\frac{7}{3^2 4^2}+\dots\dots \frac{2n+1}{n^2(n+1)^2}=1-\frac{1}{(n+1)^2}$ ①

$\frac{3}{4}=\frac{3}{1^2 2^2}=1-\frac{1}{(1+1)^2}=\frac{3}{4}$ Be specific

so the formula holds for $n=1$, hence we have the basis for the induction

Suppose it? holds for $n=k$, try for $n=k+1$ (Put $n=k$ in ① and add $(2(k+1)+1)/((k+1)^2(k+1+1)^2)$ to both sides.

$$\frac{3}{1^2 2^2}+\frac{5}{2^2 3^2}+\frac{7}{3^2 4^2}+\dots\dots\frac{2k+1}{k^2(k+1)^2}+\frac{2(k+1)+1}{(k+1)^2(k+1+1)^2}=1-\frac{1}{(k+1)^2}+\frac{2(k+1)+1}{(k+1)^2(k+1+1)^2}$$

$$=1+\left(\frac{-(k+1+1)^2+2(k+1)+1}{(k+1)^2(k+1+1)^2}\right)$$

$$=1+\left(\frac{-(k^2+4k+4)+2k+3}{(k+1)^2(k+2)^2}\right)$$

$$=1+\left(\frac{-k^2-4k-4+2k+3}{(k+1)^2(k+2)^2}\right)$$

$$=1+\left(\frac{-k^2-2k-1}{(k+1)^2(k+2)^2}\right)$$

$$=1-\left(\frac{k^2+2k+1}{(k+1)^2(k+2)^2}\right)$$

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$$=1-\left(\frac{(k+1)^2}{(k+1)^2(k+2)^2}\right)$$

$$=1-\frac{1}{(k+2)^2}=1-\frac{1}{((k+1)+1)^2}$$ ie P(k) holds.

The formula holds for $n=1$, and if for $n=k\geq 1$, then for $n=k+1$ hence holds for all integers $n\geq 1$ by induction.

4) $\gcd(a,b)=p$ then $\gcd(ap,bp^2)=p^2$

This is not true.

Suppose $a=p^2$, $b=p$

then $\gcd(a,b)=\gcd(p^2,p)=p$

but $\gcd(ap,bp^2)=\gcd(p^3,p^3)=p^3$

A specific example, $p=3$