$= 48k + 21 = 3(16k+7)$

hence r is not prime

If $p = 3k+2$

$q = 4p+1$

$= 4(3k+2)+1 = 12k+8+1$

$= 12k+9 = 3(4k+3)$

hence q is not prime

Since p can always be expressed in this way, it is impossible to find three such primes p, q and r, since either q or r will be divisible by 3 see opp.

iv) $n^3+1$

This can be factorized.

$n^3+1 = (n+1)(n^2-n+1)$

Check $(n+1)(n^2-n+1) = n^3 - n^2 + n + n^2 - n + 1)$

$= n^3+1$

Since $n>1$, $n+1 \geq 3$ and $\boxed{(n^2-n+1) \geq (2^2-2+1) = 3}$ why? Justify claims

Hence for $n>1$ neither of $(n+1)$ or $(n^2-n+1)$ can equal 1, and $n^3+1$ can never be prime, since it is a product of two numbers both of which are greater than 1.

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5) The division algorithm tells us that any integer is one of the forms 3k, 3k+1 or 3k+2. 3k+2 is not divisible by 3; it follows that it is either prime (and divisible by itself) or a product of two or more primes, each of which is one of the forms 3k+1 or 3k+2.

Now a product of two numbers of the form 3k+1 is is itself of this form:

$(3k+1)(3l+1) = 3(3kl + k + l) + 1.$

Hence if a number of the form 3k+2 were divisible only by primes of the form 3k+1, it would itself be of this