

Suppose $a = 3^2 = 9$, $b = 3$
Then $\gcd(a, b) = \gcd(9, 3) = 3$
but $ap = 9 \times 3$, $bp^2 = 3 \times 3^2$
 $= 27$ $= 27$

and $\gcd(ap, bp^2) = \gcd(27, 27) = 27$
(and $\gcd(ap, bp^2) = p^3$ for a any multiple of p^2 if $p|b$)

ii) Notice that $10k+3$ is not divisible by 2 or 5, hence is not divisible by any number which is a multiple of 2 or 5 i.e. it is not divisible by nos of form $10k, 10k+2, 10k+4, 10k+5, 10k+6$ or $10k+8$. This leaves as possible divisors no's of the form $10k+1, 10k+3, 10k+7$ or $10k+9$. It's easily noticed that $7 \times 6 = 42 = 6 \times 10 + 3$ hence $(10k+7)(10l+9) = 100kl + 10(9k+7l) + 63$
 $= 10(10kl + 9k + 7l + 6) + 3$.

This is of the form $10k+3$, but need not have divisors of the form $10k+3$
eg $133 = 10 \times 13 + 3 = 7 \times 19$

133 is of the form $10k+3$ but its divisors are of the form $10k+7 (=7, \checkmark$ with $k=0)$ and $10k+9 (=19 \text{ with } \checkmark$ $k=9)$. So is it true or false?

iii) p can be expressed in one of the forms $p = 3k+1$ or $p = 3k+2$ where $(\text{since } p > 3)$ k is an integer (p cannot take the form $p = 3k$ since $p > 1$; p would not be prime).

If $p = 3k+1$

$$q = 4p+1 = 4(3k+1)+1 = 12k+5$$

$$r = 4q+1 = 4(12k+5)+1 = 48k+20+1$$