In both parts (i) & (ii) it is much easier to use any expansions that are already known.

(i)

$f(z) = e^{-z}\,\mathrm{Log}(1+2z)$ (using Product Rule gives)

$= \left\{1 - z + \frac{z^2}{2!} - \frac{z^3}{3!} + \dots\right\}\left\{2z - \frac{(2z)^2}{2} + \frac{(2z)^3}{3} - \dots\right\}$

$= 2z - 4z^2 + z^3\left(\frac{8}{3} + 2 + 1\right)$

$= 2z - 4z^2 + \frac{17}{3}z^3 \dots$

(ii) Using $\sin 2z = 2z - \frac{(2z)^3}{3!} + \dots$

and $\cos w = 1 - \frac{w^2}{2!} + \frac{w^4}{4!} \dots$

then by Composite Rule $\cos(\sin 2z) = 1 - \frac{1}{2!}\left\{2z - \frac{(2z)^3}{3!}\right\}^2 + \frac{1}{4!}\left\{2z - \frac{(2z)^3}{3!}\right\}^4 \dots$

$= 1 - \frac{1}{2}\left\{4z^2 - 4z.\frac{8z^3}{6} + \dots\right\} + \frac{16z^4}{24}$

$= 1 - 2z^2 + \frac{10}{3}z^4 \dots$