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1A from the fns in b)i) marked * since the singular part has infinitely many terms.

ii) The coefficient of $\frac{1}{z}$ in the Laurent series for f is zero. Hence the residue of f at 0 is zero so

2A,M $\int_C \left(1-\frac{1}{z^2}\right)\cosh\left(\frac{1}{z}\right)dz = 2\pi i \,\mathrm{Res}(f,0) = 0$ (By Theorem 4.2)

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c) $f(z) = \frac{z}{(3z-1)(z+3)}$

$= \frac{z}{3z^2(1-1/3z)(1+3/z)}$

$= \frac{1}{3z}\left(1-\frac{1}{3z}\right)^{-1}\left(1+\frac{3}{z}\right)^{-1}$

$= \frac{1}{3z}\left(1+\frac{1}{3z}+\frac{1}{3^2z^2}+\frac{1}{3^3z^3}+\frac{1}{3^4z^4}\cdots\right)\left(1-\frac{3}{z}+\frac{3^2}{z^2}-\frac{3^3}{z^3}+\frac{3^4}{z^4}\cdots\right)$

4A,M $= \frac{1}{3z}\left(1+\frac{1}{z}\left(\frac{1}{3}-3\right)+\frac{1}{z^2}\left(\frac{1}{3^2}-1+3^2\right)+\frac{1}{z^3}\left(\frac{1}{3^3}-\frac{1}{3}+3-3^3\right)\right.$
$\left.+\frac{1}{z^4}\left(\frac{1}{3^4}-\frac{1}{3^2}+1-3^2+3^4\right)+\cdots\right)$

$= \frac{1}{3z}\left(1+\frac{1}{3z}(1-3^2)+\frac{1}{3^2z^2}(1-3^2+3^4)+\frac{1}{3^3z^3}(1-3^2+3^4-3^6)\right.$
$\left.+\boxed{\frac{1}{3^4z^4}(1-3^2+3^4-3^6+3^8)}\cdots\right)$ (P.T.O)

1A The pattern in the terms is obvious. Each term in the main bracket, like that one surrounded by a box consists of two parts. One part is a $\frac{1}{3^nz^n}$ term and the other is a geometric sum of $n+1$ terms with common ratio -9:

note $n \geq 0$

$S_{n+1} = \frac{a(1-r^{n+1})}{1-r} = \frac{1(1-(-9)^{n+1})}{1-(-9)} = \frac{1-(-9)^{n+1}}{10}.$

the general term is

$\frac{1}{3^nz^n}\left(\frac{1-(-9)^{n+1}}{10}\right)$

so $f(z) = \frac{1}{3z}\sum_{n=0}^{\infty}\frac{1}{3^nz^n}\left(\frac{1-(-9)^{n+1}}{10}\right)$

$= \sum_{n=0}^{\infty}\frac{1}{3^{n+1}z^{n+1}}\left(\frac{1-(-9)^{n+1}}{10}\right)$

8/8 3A (24½) $= \sum_{k=1}^{\infty}\frac{1}{3^kz^k}\left(\frac{1-(-9)^k}{10}\right)$, where $k=n+1$.