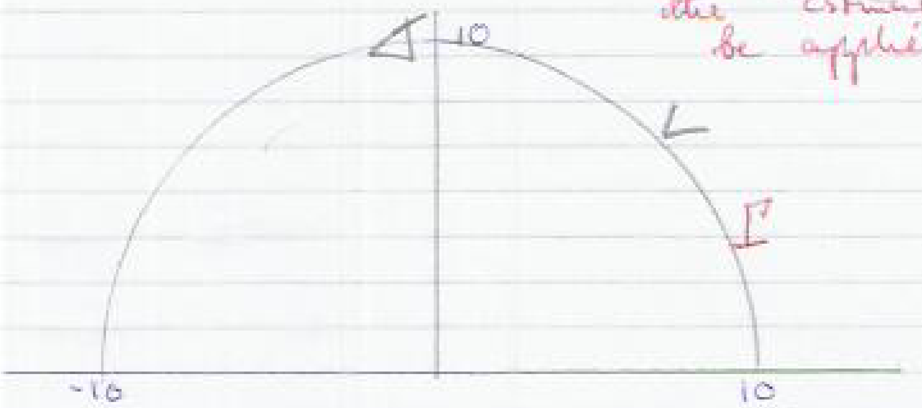
$\therefore f$ is not the derivative of an entire function.

0/1 M b) $\left|\int_\Gamma \frac{e^{3z}+z}{z^3+z}\,dz\right| \le ML$

You need to point out that as f is continuous on $\mathbb{C}-\{0,i,-i\}$ then f is continuous on Γ so the Estimation Thm may be applied.



Where L is the length of Γ and M is ~~the maximum value~~ an upper bound of $\left|\frac{e^{3z}+z}{z^3+z}\right|$ on Γ.

(finding max $|f|$ on Γ might be very difficult and is unnecessary)

$$\left|\frac{e^{3z}+z}{z^3+z}\right| \le \frac{|e^{3z}|}{|z^3+z|} + \frac{|z|}{|z^3+z|} \quad \text{by the Triangle Inequality}$$

$$= \frac{|e^{3x+3iy}|}{|z^3+z|} + \frac{10}{|z^2+1|} \quad \text{since } z \neq 0$$

$$= \frac{|e^{3x}||e^{3iy}|}{|z^3+z|} + \frac{1}{|z^2+1|}$$

$$= \frac{e^{3x}}{|z^3+z|} + \frac{1}{|z^2+1|} \quad \text{since } |e^{3iy}|=1$$

$$\le \frac{e^{30}}{|10^3-10|} + \frac{10}{|-10^2+1|}$$

4A, M
$$= \frac{e^{30}}{990} + \frac{10}{99}$$

$$= \frac{1}{990}(e^{30}+10) = M$$

1A and $L = 10\pi$

$$\therefore \left|\int \frac{e^{3z}+z}{z^3+z}\,dz\right| \le ML = \frac{(e^{30}+10)}{990} \times 10\pi$$

1A
$$= \frac{(e^{30}+10)\pi}{99}$$

c) $\int_\Gamma e^{-z}\,dz = \left[-e^{-z}\right]_{10}^{-10} = -e^{+10} + e^{-10} = -2\cosh 10$, $-2\sinh 10$

2½/3 A, M by fundamental theorem of calculus, since exp is entire.

The actual conditions to be checked are: take R to be $\mathbb{C}$; $f(z) = e^{-z}$ is continuous on R with primitive $F(z) = -e^{-z}$ on R