I'm not sure what you mean here (PT, 0) ⑧

d) Call $f(iy) = e^{-y}$ and $g(z) = e^{iz}$ ✓

2 A, M On $z = iy$, $g(iy) = e^{i(iy)} = e^{-y} = f(iy)$

$\therefore g = f$ on $z = iy$ ✓

Any point on the y axis is a limit point of $\mathbb{C}$ in $S = \{z : Re(z) = 0\}$ since if $i\alpha \in S$, $\{i\alpha + i/n\} \in S$. Choose $\varepsilon > 0$ then $(i\alpha - i\varepsilon, i\alpha + i\varepsilon)$ is an open set in S, $i\alpha \in (i\alpha - i\varepsilon, i\alpha + i\varepsilon)$ and $i(\alpha + 1/n) \in (i(\alpha - \varepsilon), i(\alpha + \varepsilon))$ for $n > 1/\varepsilon$.

3 A, M $\therefore$ Every open set in S contains a limit point of $\mathbb{C}$ (since intervals of this sort are the only open sets)

5 $\frac{22\frac{1}{2}}{25}$

$\therefore$ the conditions of the uniqueness theorem are all met $\therefore f = g$ for all $z \in \mathbb{C}$. ✓

4) $f(z) = \dfrac{z}{1 - \cos z}$

$= \dfrac{z}{2\sin^2(z/2)}$

or simply where $\cos z = 1$

singularities of $f(z)$ are at those z for which $\sin^2(z/2) = 0 \Leftrightarrow z/2 = \pi n$ (PT 0)

$1\frac{1}{2}$ A, M / 2

$z' = 2\pi n$, $n \in \mathbb{Z}$ are the singularities of f.

On D_n: $f(z) = \dfrac{z}{1 - \cos z} = \dfrac{z}{1 - \cos(z - 2\pi n)}$

$= \dfrac{z}{1 - \left(1 - \frac{(z-2\pi n)^2}{2!} + \frac{(z-2\pi n)^4}{4!} - \frac{(z-2\pi n)^6}{6!} \cdots\right)}$

$= \dfrac{z}{\left(\frac{(z-2\pi n)^2}{2!} - \frac{(z-2\pi n)^4}{4!} + \frac{(z-2\pi n)^6}{6!} \cdots\right)}$

$= \dfrac{z}{(z-2\pi n)^2\left(\frac{1}{2!} - \frac{(z-2\pi n)^2}{4!} + \frac{(z-2\pi n)^4}{6!} - \frac{(z-2\pi n)^6}{8!}\right)}$

$= \dfrac{1}{(z-2\pi n)^2} \times g_n(z)$

where $g_n(z) = \dfrac{z}{\left(\frac{1}{2} - \frac{(z-2\pi n)^2}{4!} + \frac{(z-2\pi n)^4}{6!} \cdots\right)}$

$2\frac{1}{2}$ A, M / 3

g_n is analytic at $z = 2\pi n$ and $g_n(2\pi n) = 4n\pi \neq 0$ $\therefore$ each pole has order 2 (except $n = 0$, which I now examine)