TMA M337 02 $\qquad 1\ \frac{21}{25}\quad 2\ \frac{22}{25}\quad 3\ \frac{23}{25}\quad 4\ \frac{24}{25}$

1

1) a) i) $\mathrm{Im}(z) = \mathrm{Im}(\gamma_1(t)) = \cancel{\ } t$

2A,M $\quad \int_{\Gamma_1} (\mathrm{Im} z)^3 dz = \int_0^2 (t)^3 \cdot \frac{dz}{dt} dt$

$z = t^2 + it \Rightarrow \frac{dz}{dt} = 2t + i$ ✓

$\therefore \int_0^2 t^3(2t+i)dt = \int_0^2 2t^4 + it^3 dt$ ✓

$= \left[\frac{2t^5}{5} + \frac{it^4}{4}\right]_0^2$

2A $\quad = \left(\frac{2\times 2^5}{5} + i \times \frac{2^4}{4}\right) - 0 = \frac{64}{5} + 4i$ ✓

ii) $\mathrm{Im}(z) = \mathrm{Im}(\gamma(t)) = 2t$ ✓

$\int (\mathrm{Im} z)^3 dz = \int (2t)^3 \frac{dz}{dt} dt$

2A,M $\quad z = (4+2i)t \Rightarrow \frac{dz}{dt} = 4 + 2i$ ✓

$\int_0^2 (2t)^3(4+2i)dt \quad 16(2+i)\int_0^{2\ 1} t^3 dt$

$= 16(2+i)\left[\frac{t^4}{4}\right]_0^{\cancel{2}\ 1}$

$\frac{1}{2}$ A $\quad = 16(2+i) \times \frac{\cancel{2^4}\ 1}{4} - 0 = \cancel{64}\ 4(2+i)$

iii) Suppose f were the derivative of an entire function F(z). Then

$\int_a^b f(z)dz = F(b) - F(a)$

by the fundamental theory of calculus. ✓

In i) $\quad b_1 = \gamma_1(2) = 4 + 2i$
$\qquad a_1 = \gamma_1(0) = 0$

In i) $\quad b_2 = \gamma_2(1) = 4 + 2i = b_1$
$\qquad a_1 = \gamma_2(0) = 0 = a_1$

i.e. Γ_1 and Γ_2 have the same initial and final point.

4M

$\Rightarrow \int f(z)dz = F(b_1) - F(a_1) = F(b_2) - F(a_2)$

but from i) and ii):

$F(b_1) - F(a_1) \neq F(b_2) - F(a_2)$ ✓