(1)

We know f is entire and for points on the imaginary axis (i.e. where $z = iy$) $f(iy) = e^{-y}$.

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(2) Singularities at $z = 2k\pi$, where $k \in \mathbb{Z}$

and f is analytic on $D_k = \{z : 0 < |z - 2k\pi| < \pi\}$

(i.e. D_k is a punctured disc centred on one singularity but with outer radius such that ~~next~~ nearest neighbouring singularity is not in D_k !)