What you've done is O.K. but it would have been simpler to use partial fractions. This leads to the two infinite series being added rather than multiplied and then summed as you had to do

$$f(z) = \frac{z}{(3z-1)(z+3)} = \frac{1}{10}\left\{ \frac{1}{3z-1} + \frac{3}{z+3} \right\} \text{ by Partial Fraction}$$

$$\therefore f(z) = \frac{1}{30z}\left(1-\frac{1}{3z}\right)^{-1} + \frac{3}{10z}\left(1+\frac{3}{z}\right)^{-1}$$

etc expand these as you've done in your solution.

$$= \sum_{n=1}^{\infty} \frac{1}{10}\left\{\frac{1}{3^n} + 3^n(-1)^{n+1}\right\}\frac{1}{z^n}$$

which is the same result as yours