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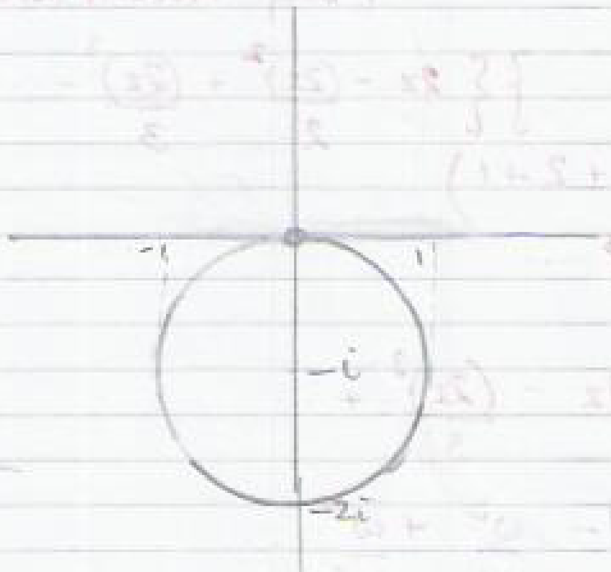
1.A The largest open disc on which f is represented by this Taylor series is $|z+i|<1$, shown below



9½/10

c) i) $f(z) = e^{-z}\operatorname{Log}(1+2z)$ 10

$$f'(z) = \frac{d}{dz}\left(e^{-z}\operatorname{Log}(1+2z)\right)$$
$$= -e^{-z}\operatorname{Log}(1+2z) + \frac{2e^{-z}}{1+2z}$$
$$= e^{-z}\left(2(1+2z)^{-1} - \operatorname{Log}(1+2z)\right)$$

$$f''(z) = \frac{d}{dz}\left(e^{-z}\left(2(1+2z)^{-1} - \operatorname{Log}(1+2z)\right)\right)$$
$$= -e^{-z}\left(2(1+2z)^{-1} - \operatorname{Log}(1+2z)\right) + e^{-z}\left(-4(1+2z)^{-2} - 2(1+2z)^{-1}\right)$$
$$= e^{-z}\left(-4(1+2z)^{-2} - 4(1+2z)^{-1} + \operatorname{Log}(1+2z)\right)$$

$$f'''(z) = \frac{d}{dz}\left(e^{-z}\left(-4(1+2z)^{-2} - 4(1+2z)^{-1} + \operatorname{Log}(1+2z)\right)\right)$$
$$= -e^{-z}\left(-4(1+2z)^{-2} - 4(1+2z)^{-1} + \operatorname{Log}(1+2z)\right) + e^{-z}\left(16(1+2z)^{-3} + 8(1+2z)^{-2} + 2(1+2z)^{-1}\right)$$
$$= e^{-z}\left(16(1+2z)^{-3} + 12(1+2z)^{-2} + 6(1+2z)^{-1} - \operatorname{Log}(1+2z)\right)$$

$\therefore f(0) = e^{-0}\operatorname{Log}(1+0) = 0$

$f'(0) = e^{-0}\left(2/(1+0) - \operatorname{Log}(1+0)\right) = 2$

$f''(0) = e^{-0}\left(-4(1+0)^{-2} - 4(1+0)^{-1} + \operatorname{Log}(1+0)\right) = -8$