Suppose $|f'(\alpha)| = \varepsilon > 0$. But $|f'(\alpha)| \le K$!!

There exists $r > \frac{1}{\varepsilon}$ $\therefore$ $\frac{1}{r} < \varepsilon$

$|f'(\alpha)| \le \frac{1}{r} < \varepsilon$ and $|f'(\alpha)| = \varepsilon \Rightarrow$ a contradiction

$\therefore |f'(\alpha)| = 0$ ✓

Actually you were expected to do these using B2 rather than the Residue Th^m. However, in either case you must check the conditions are satisfied before applying the appropriate result.

Eg (i) $f(z) = \frac{\sin z}{(z+4)^2(z-2)}$ is analytic on $R = \{z : |z| < 3/2\}$ which is a simply connected region and contains the closed contour C. $\therefore$ Cauchy's Th^m gives $\int_C f(z)\,dz = 0$

(ii) Take $f(z) = \frac{\sin z}{(z+4)^2}$. Then f is analytic on the s.c. region $R = \{z : |z| < 7/2\}$, C is a closed contour in R with 2 inside $\therefore$ Cauchy's Integral Formula gives

$$\int \frac{f(z)}{z-2}\,dz = 2\pi i\, f(2) = \frac{\pi i}{18} \sin 2 .$$

(iii) Take $f(z) = \frac{\sin z}{z-2}$ which is analytic on the s.c. region $\{z : |z+4| < 2\}$. C is a closed contour in R with -4 inside $\therefore$ Cauchy's n^{th} Deriv. Formula gives

$$\int_C \frac{f(z)}{(z+4)^2}\,dz = 2\pi i\, f'(-4) = \ldots = \frac{\pi i}{18}(\sin 4 - 6\cos 4)$$