Question has $\cos\left(\frac{1}{z}\right)$ here

(3)

ii) $\int_\Gamma \frac{1}{z^2}\cosh\left(\frac{1}{z}\right)dz = \int_{10}^{-10}\frac{d}{dz}\left(-\sinh\left(\frac{1}{z}\right)\right)dz$

$= \left[-\sinh\left(\frac{1}{z}\right)\right]_{10}^{-10}$

$= -\sinh\left(\frac{-1}{10}\right) + \sinh\left(\frac{1}{10}\right) = 2\sinh\left(\frac{1}{10}\right)$ ✓

$1\frac{1}{2}$ A,M / 3

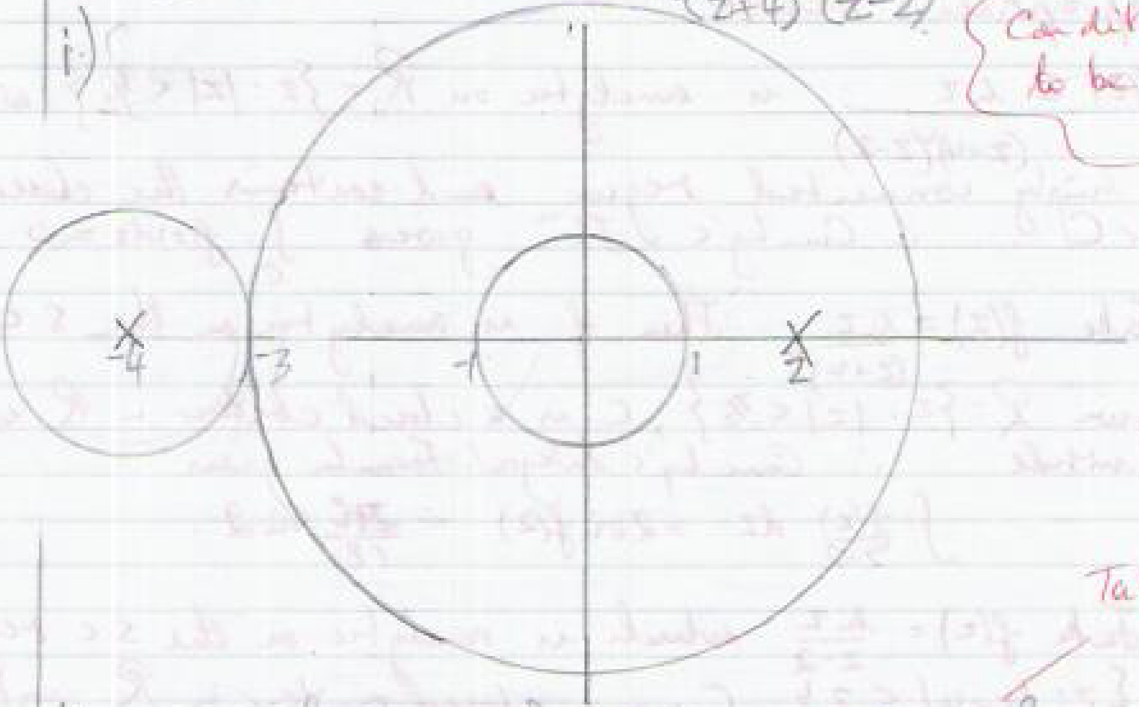
Again by the fundamental Theorem of calculus. Again you need to check the conditions take $R = \mathbb{C} - \{0\}$ which contains Γ, $f(z) = \frac{1}{z^2}\cos\left(\frac{1}{z}\right)$ is continuous on R

$21\frac{1}{2}$ / 25

2a) $\int_C \frac{\sin z}{(z+4)^2(z-2)}dz$ = sum of residues of $\frac{\sin z}{(z+4)^2(z-2)}$ inside C × $2\pi i$

Conditions need to be checked

i)



No residues of $\frac{\sin z}{(z+4)^2(z-2)}$ inside C

$\therefore \int_{C_1}\frac{\sin z}{(z+4)^2(z-2)}dz = 0$ ✓

Take $R = \{z : |z| < \frac{3}{2}\}$ then f is analytic on the s.c. region R and C_1 is a simple closed contour in R

3/4 A,M

where $C_1 = \{z : |z| = 1\}$

ii) 2 is a zero of $(z+4)^2(z-2)$ inside C, and there are no other zeros inside C

$\int_C \frac{\sin z}{(z+4)^2(z-2)}dz = 2\pi i \times \frac{\sin 2}{(2+4)^2} = \frac{\pi i \sin 2}{18}$ ✓

Again check conditions define R and explain how $\mathrm{Res}(f,2)$ was found.

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iii) -4 is a zero of multiplicity 2 of $(z+4)^2(z-2)$

Again check conditions define R

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$\int_C \frac{\sin z}{(z+4)^2(z-2)}dz = \frac{2\pi i}{1!}\times \frac{d}{dz}\left(\frac{\sin z}{z-2}\right)_{z=-4}$