

$$|g(z)| = |ze^{-iz}| = |z|e^{-|z|}$$

$$= 2e^{-2} = 2/e^2$$

$$= 2/e^2$$

part of  $\partial B$

$|e^y|$  max when  $y=2$

but  $0+0 \in \partial B$  and  $g(0)=0$

$0 \leq |g(z)| \leq 2e^2$  which is of required form with  $\alpha = 0, \beta = 2$

$$4) i) \frac{df(z)}{dz} = \frac{d}{dz} (\cosh(2e^{iz}))$$

$$= i \times 2e^{iz} \times \sinh(2e^{iz})$$

$$= 2ie^{iz} \sinh(2e^{iz})$$

$\exp$  and  $\cosh$  are entire so they are differentiable on the whole of  $\mathbb{C}$  and so must be differentiable at  $z$ , and  $2e^{iz}$ . It follows that the chain rule can be applied throughout the domain of  $f(z)$  and  $f'(z)$  has the same domain as  $f(z)$ .

$$ii) \frac{df(z)}{dz} = \frac{d}{dz} (z^2 \text{Log} z)$$

$$= 2z \text{Log} z + z$$

$\text{Log} z$  is not continuous on the negative real axis, so is not differentiable there. However,  $z^2$  is differentiable everywhere and  $\text{Log} z$  is differentiable on  $\mathbb{C} - \{x: x \leq 0\}$  so the domain of the derivative is  $\mathbb{C} - \{x: x \leq 0\}$ .

$$A) i) \frac{df}{dz} = \lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}$$

at  $z = 2-3i$

$$\frac{df}{dz} = \lim_{h \rightarrow 0} \frac{f(2-3i+h) - f(2-3i)}{h}$$

$$= \frac{2+h-i-2-i}{h} = 1$$

$$\text{Let } h = \epsilon i$$

$$\left( \frac{df}{dz} \right) = \lim_{\epsilon \rightarrow 0} \frac{f(2-3i+\epsilon i) - f(2-3i)}{\epsilon i}$$

we can't use  $\frac{df}{dz}$  yet as we don't know if  $f$  is differentiable