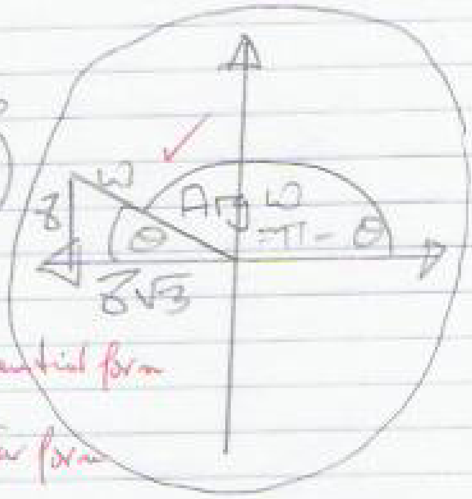
M337 TMA 01 1 $\frac{23}{25}$ 2 $\frac{[illegible]}{25}$ 3 $\frac{[illegible]}{25}$ 4 $\frac{[illegible]}{25}$

1) i) $w - \overline{w} = (-8\sqrt{3} + 8i) - \overline{(-8\sqrt{3} + 8i)}$
$= (-8\sqrt{3} + 8i) - (-8\sqrt{3} - 8i)$
$= (-8\sqrt{3} - -8\sqrt{3}) + (8i - -8i)$
$= 16i$ ✓

$w\overline{w} = (-8\sqrt{3} + 8i)\overline{(-8\sqrt{3} + 8i)}$
$= (-8\sqrt{3} + 8i)(-8\sqrt{3} - 8i)$
$= ((-8\sqrt{3})^2 + 8i \times -8i) + i(-8\sqrt{3} \times -8 + 8 \times -8\sqrt{3})$
$= 192 + 64 + 0 = 256$ ✓

$(\overline{w})^{-1} = (\overline{-8\sqrt{3} + 8i})^{-1}$
$= (-8\sqrt{3} - 8i)^{-1}$
$= \dfrac{1}{-8\sqrt{3} - 8i} \times \dfrac{-8\sqrt{3} + 8i}{-8\sqrt{3} + 8i}$
$= \dfrac{-8\sqrt{3} + 8i}{256}$ (from above for $w\overline{w}$)
✓
6A,M $= \dfrac{-\sqrt{3} + i}{32} = \dfrac{-\sqrt{3}}{32} + \dfrac{i}{32}$

ii) $w = -8\sqrt{3} + 8i$
$|w| = \sqrt{w\overline{w}} = \sqrt{256} = 16$
$\text{Arg}\, w = \pi - \tan^{-1}\left(\dfrac{8}{8\sqrt{3}}\right)$
$= \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$
$\therefore w = 16e^{5\pi i/6}$ ✓ ← exponential form
2A $= 16\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ ← polar form



iii) Let w_i be the 4th roots of w for $i = 0, 1, 2, 3$. Then
$w_1 = \sqrt[4]{16}\, e^{(5\pi i/6 + 0 \cdot 2\pi i)/4}$
$= 2e^{5\pi i/24}$ ✓
$w_2 = \sqrt[4]{16}\, e^{(5\pi i/6 + 1 \times 2\pi i)/4}$
$= 2e^{17\pi i/24}$ ✓
$w_3 = \sqrt[4]{16}\, e^{(5\pi i/6 + 2 \times 2\pi i)/4}$
$= 2e^{29\pi i/24}$ ✓ O.K. in this form
$= 2e^{(29\pi i/24 - 2\pi i)}$ by periodicity of $\exp^{xi}$
$= 2e^{-19\pi i/24}$ ✓
3A,M $w_4 = \sqrt[4]{16}\, e^{(5\pi i/6 + 3 \times 2\pi i)/4}$
$= 2e^{41\pi i/24}$ ✓ $(= 2e^{-7\pi i/24})$ ✓