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where $x \in \mathbb{R}$ $y \in (-\pi, \pi]$ by defn of Log , hence Log is a one-one function with image $x+iy$, $x \in \mathbb{R}$, $y \in (-\pi, \pi]$ and Log^{-1} is a one-one function with domain $\{x+iy: x \in \mathbb{R}, y \in (-\pi, \pi]\}$ and rule $g^{-1}(x+iy) = \exp(x+iy)$

$$\begin{aligned} iz-3 &= \exp(x+iy) \\ iz &= 3 + \exp(x+iy) \\ z &= -3i - i \exp(x+iy) \\ &= -3i - i e^x (\cos y + i \sin y) \\ &= e^x \sin y - i(3 + e^x \cos y) \end{aligned}$$

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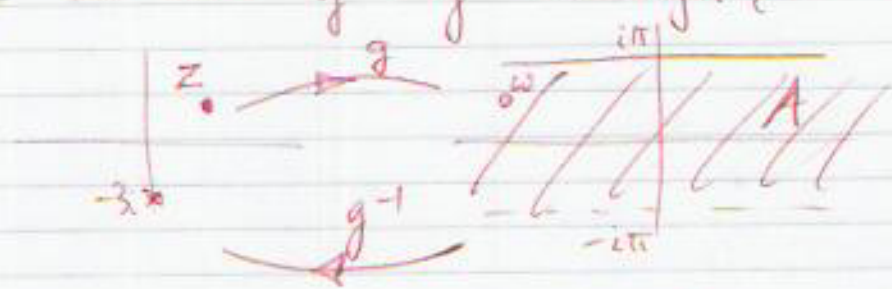
$z = g^{-1}(x+iy) = e^x \sin y - i(3 + e^x \cos y) \checkmark$

and the domain of g^{-1} $\{z: -\pi < \text{Im } z \leq \pi\}$

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Notice that we've no need to use $x+iy$ form.

Simply note that the image of g is the same as that of Log namely $A = \{w: -\pi < \text{Im } w \leq \pi\}$



For $w \in A$, $w = g(z)$ provided $w = \text{Log}(iz-3)$

$$\therefore iz-3 = e^w$$

$$\therefore z = -i(e^w + 3)$$

\therefore done i.e. we have a unique z with $g(z) = w$

$$\therefore g^{-1}(w) = -i(e^w + 3)$$