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$= \frac{2+i-2-i}{\varepsilon i} = 0 \neq 1$ ✓ ✓ ✓

But the derivative should be independent of the direction of approach ✓ $\therefore f(z)$ is not differentiable at $2-3i$ ✓

1M

b) $\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$
$= 6x + (6y+1)i$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

PTO

It's not clear what you have done here.

$\frac{\partial f}{\partial y} = -\frac{\partial u}{\partial y} i + \frac{\partial v}{\partial y}$
$= (1+6y)i + 6x$

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

$\frac{1}{4}$ A,M

$\therefore f$ is entire.

c) i) $\frac{df}{dz} = \frac{d}{dz}\left((2z+i)(z-2i)^{-1}\right)$

$= 2(z-2i)^{-1} - (2z+i)(z-2i)^{-2}$

$= \frac{2(z-2i) - (2z+i)}{(z-2i)^2}$

$= \frac{2z-4i-2z-i}{(z-2i)^2}$

$= \frac{-5i}{(z-2i)^2} \neq 0$ ✓ $z \in \mathbb{C} - \{2i\}$ ✓

3A,M ie $f'(z) \neq 0$ so f is conformal on it's domain $(= \mathbb{C} - \{2i\})$. by Th^m 4.2

ii) $f(3i) = \frac{2\times 3i + i}{3i-2i} = \frac{7i}{i} = 7$ ✓

$f'(3i) = \frac{-5i}{(3i-2i)^2} = \frac{-5i}{(i)^2} = 5i$ ✓

The small disc is mapped to a small disc centred at $z=7$ ✓
It is scaled by a factor $|f'(3i)| = 5$ ✓
and rotated through an angle $\mathrm{Arg}(f'(3i)) = \pi/2$ anticlockwise.

3A

iii) $\gamma_1(0) = 0^2 + i(0+3) = 3i$ ✓
$\gamma_2(\pi) = 1 + 3i + e^{\pi i} = 1+3i-1 = 3i$ ✓

1A so $\gamma_1(0) = \gamma_2(\pi) = 3i$ so Γ_1, Γ_2 meet at $3i$ ✓