and $3 < |z-3i| < 4$

so $\frac{1}{4} < \frac{1}{|z-3i|} < \frac{1}{3}$

so $\frac{3-\sqrt{2}}{4} < \frac{3-\sqrt{2}}{|z-3i|} < |\frac{z-1-2i}{z-3i}| < \frac{4+\sqrt{2}}{|z-3i|} < \frac{4+\sqrt{2}}{3}$

ie $\frac{3-\sqrt{2}}{4} < |\frac{z-1-2i}{z-3i}| < \frac{4+\sqrt{2}}{3}$ ✓

3A M (23/25)

so $a = \frac{3-\sqrt{2}}{4}$, $b = \frac{4+\sqrt{2}}{3}$ ✓

2) i) $f(z) = \frac{1}{2}i(z-1)$

[$f(x+iy) = \frac{1}{2}i(x+iy-1)$
$= \frac{1}{2}i(x-1+iy)$
$= -\frac{1}{2}y + \frac{1}{2}i(x-1)$] ← we don't need (or use) this.

Under the function f a typical point P of $\mathcal{C}$ is a) translated one unit in the $-x$ ✓ direction to a point P'

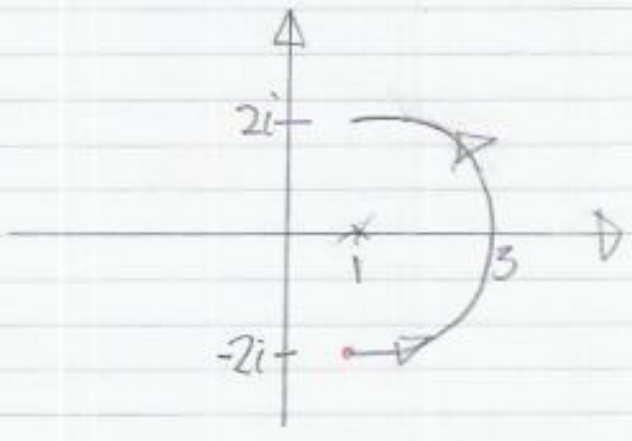
3A b) The position vector of P' is rotated by $\pi/2$ anticlockwise to a point P'' ✓

d) The position vector to P'' is scaled by a factor of $\frac{1}{2}$ from O ✓

where f is represented by the matrix $\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$

We could alternatively say that f maps a small disc around a point z to a small disc around a point $f(z)$, while scaling by a factor $|f'(z)| = \frac{1}{2}$ and rotating through an angle $\arg(f'(z)) = +\pi/2$ ✓

ii) Γ is a semicircle, radius 2, with argument referred to (1,0) between $\pi/2$ and $\pi/2$, taken anticlockwise.



3A ✓