

It's good to see you using the geometry of the situation but the question is asking you to use the fact that

$$z - 1 - 2i = (z - 3i) + (i - 1)$$

$$\begin{aligned} \textcircled{A} \quad |z - 1 - 2i| &= |(z - 3i) + (i - 1)| \\ &\leq |z - 3i| + |i - 1| \quad \text{by Triangle Inequality} \\ &< 4 + \sqrt{2} \quad \text{for } z \in A \\ &\quad \text{since } |z - 3i| < 4 \text{ and } |i - 1| = \sqrt{2}. \end{aligned}$$

$$\begin{aligned} \textcircled{B} \quad |z - 1 - 2i| &= |(z - 3i) - (i - 1)| \\ &\geq ||z - 3i| - |i - 1|| \quad \text{by Backwards form of Triangle Inequality} \\ &= |z - 3i| - \sqrt{2} \quad \text{since for } z \in A \\ &\quad |z - 3i| > 3 > \sqrt{2} \end{aligned}$$

For  $z \in A$ :

$$|z - 1 - 2i| > 3 - \sqrt{2} \quad \text{since for } z \in A \quad |z - 3i| > 3$$