

Answer *all* questions.

Question 1 (Unit A1) – 25 marks

(a) Let $w = -8\sqrt{3} + 8i$.

- (i) Evaluate each of the following, giving your answers in Cartesian form:
 $w - \bar{w}$, $w\bar{w}$ and $(\bar{w})^{-1}$.
- (ii) Express w in polar form.
- (iii) Determine all the fourth roots of w , leaving your answers in polar form.
- (iv) Write down the smallest positive integer n for which w^n is real. [12]

(b) Let

$$A = \{z : 3 < |z - 3i| < 4\}$$

$$B = \left\{z : \frac{\pi}{3} \leq \text{Arg}(z + i) \leq \frac{2\pi}{3}\right\}.$$

- (i) Sketch the sets A , B , and $A \cap B$ on separate diagrams, using the conventions on page 38 of *Unit A1*. Do not attempt to label vertices in your sketch of $A \cap B$.
- (ii) By writing

$$z - 1 - 2i = (z - 3i) + (i - 1),$$

prove that, for $z \in A$,

$$3 - \sqrt{2} < |z - 1 - 2i| < 4 + \sqrt{2}.$$

- (iii) Hence find positive real numbers a and b such that

$$a < \left| \frac{z - 1 - 2i}{z - 3i} \right| < b, \quad \text{for } z \in A. \quad [13]$$