(8)

3) i) $\left|\frac{i^n}{\sqrt{n+2}}\right| = \left|\frac{1^n}{\sqrt{n+2}}\right| = \frac{1}{\sqrt{n+2}} < \frac{1}{\sqrt{n}}$

2/3 A,M $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ $\therefore \lim_{n\to\infty} \frac{i^n}{\sqrt{n+2}} = 0$ by the Squeeze Rule

ii) This sequence does not converge.

$z_{2n} = \{\sin(\frac{\pi}{2} i^2)\} = \{\sin(\frac{-\pi}{2})\} = -1$

$z_{4n} = \{\sin(\frac{\pi}{2} i^4)\} = \{\sin(\frac{\pi}{2})\} = 1$

The sequence has two subsequences

3 A,M which converge to different limits, hence the sequence does not converge by First Subsequence Rule.

6) i) $f(z) = \frac{4+z^2}{2z+iz^2} = \frac{4+z^2}{z(2+iz)}$

1A The domain of $f(z) = \mathbb{C} - \{0, 2i\}$

$2i$ is a limit point of A if there is a sequence in $A - \{2i\}$ converging to $2i$. $\{2i + 1/n\} \subseteq A$ and converges to $2i$ since $|2i + 1/n - 2i| < \varepsilon$ for

2A all $n > 1/\varepsilon$, ε given, so $2i$ is a limit point of A.

ii) $\lim_{z\to 2i} \cosh\left(\frac{4+z^2}{2-iz^2}\right) = \lim_{z\to 2i} \cosh\left(\frac{(2+iz)(2-iz)}{z(2+iz)}\right)$

2/3 A,M Now $\frac{(2+iz)(2-iz)}{z(2+iz)} = \frac{2-iz}{z}$, $z \in \mathbb{C} - \{0, 2i\}$

$\therefore \lim_{z\to 2i} \cosh\left(\frac{4+z^2}{2z+iz^2}\right) = \cosh\left(\frac{2 - i\times 2i}{2i}\right)$ By Combination Rules

2/3 A,M $= \cosh(4/2i) = \cosh(-2i) = \cos 2.$

You've also used the continuity of cosh and Th^m 3.1

Giving more detail:

If $\{z_n\}$ is a sequence in $A - \{2i\}$ with $z_n \to 2i$

then as you've shown above $\frac{4+z_n^2}{2z_n+iz_n^2} = \dots = \frac{2-iz_n}{z_n}$ since $z_n \neq 0, 2i$

and $\frac{2-iz_n}{z_n} \to -2i$ by the Combination Rules

So $\cosh\left(\frac{2-iz_n}{z_n}\right) \to \cosh(-2i)$ by Th^m 3.1 since cosh is continuous.