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Initially $t = -\pi/2$
 $\gamma(-\pi/2) + 2e^{-\pi i/2} = 1 - 2i$
 Finally $t = \pi/2$
 $\gamma(\pi/2) + 2e^{\pi i/2} = 1 + 2i$

As a check

$$f(t) = 1 + 2e^{it} \quad t \in [-\pi/2, \pi/2]$$

$$f(z) = \frac{i}{2}(z-1)$$

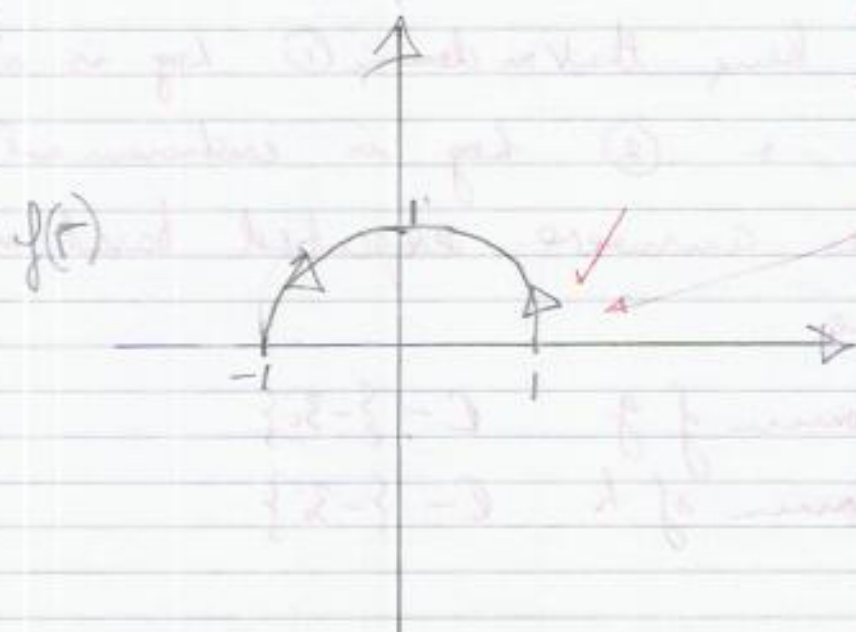
$$\therefore f(f(t)) = \frac{i}{2}(1 + 2e^{it} - 1)$$

$$= ie^{it} \quad t \in [-\pi/2, \pi/2]$$

$$= e^{i(t+\pi/2)} \quad t \in [-\pi/2, \pi/2]$$

$$= e^{it} \quad t \in [0, \pi]$$

iii)



3A

Initial point is $f(\gamma(-\pi/2))$
 $= f(1 - 2i) = \frac{i}{2}(1 - 2i - 1)$
 $= \frac{i}{2}(-2i) = 1$

Final point is $f(\gamma(\pi/2))$
 $= f(1 + 2i) = \frac{i}{2}(1 + 2i - 1)$
 $= \frac{i}{2}(2i) = -1$

10] 1A

iv) The standard parametrization of $f(\gamma)$ is $\gamma(t) = e^{it}$, $t \in [0, \pi]$

$$f(1 + 2e^{it}) = \frac{i}{2}(1 + 2e^{it}) = ie^{it}, \quad t \in [-\pi/2, \pi/2] = e^{it}, \quad t \in [0, \pi]$$

$$b) f(z) = \exp(iz - 3)$$

The function \exp is entire: domain of f is \mathbb{C} .

$$g(z) = \text{Log}(iz - 3)$$

$\text{Log} z$ is defined on $\mathbb{C} - \{x: x \leq 0\}$

$g(z)$ is defined on $\mathbb{C} - \{y: (iy - 3) \leq 0\}$

i.e. $\mathbb{C} - \{y: -3 \leq y \leq 3\}$ is domain of g .

(PTO)

3A

$$h(z) = (iz - 3)^{4i}$$

$$= \exp(4i \text{Log}(iz - 3))$$

For $h(z)$ to be well defined we must have that $iz - 3$ is in domain of Log i.e. $z \in \mathbb{C} - \{y: -3 \leq y \leq 3\}$. Then