

(6)

since \exp is entire, we have that domain of $h = \mathbb{C} - \{y: -3 \leq y \leq 3\}$

$$\begin{aligned} \text{ii) } f(-1-2i) &= \exp(i(-1-2i)-3) \\ &= \exp(-i+2-3) \\ &= \exp(-1-i) \\ &= e^{-1} \cos 1 - i e^{-1} \sin 1 \quad \checkmark \end{aligned}$$

2M

$$\begin{aligned} g(-1-2i) &= \text{Log}(i(-1-2i)-3) \\ &= \text{Log}(-1-i) \\ &= \text{Log}(\sqrt{2}) + i \text{Arg}(-1-i) \\ &= \text{Log}(\sqrt{2}) - 3\pi i/4 \quad \checkmark \end{aligned}$$

1A

$$\begin{aligned} h(-1-2i) &= (iz-3)^{4i} \\ &= \exp(4i \text{Log}(iz-3)) \\ &= \exp(4i (\text{Log}(\sqrt{2}) - 3\pi i/4)) \quad (\text{from } g(-1-2i)) \\ &= \exp(3\pi + 4i \text{Log}(\sqrt{2})) \\ &= \exp(3\pi + i \text{Log} 4) \\ &= e^{3\pi} \cos(\text{Log} 4) + i e^{3\pi} \sin(\text{Log} 4) \quad \checkmark \end{aligned}$$

3AM

$$\text{iii) } f(z) = \exp(iz-3)$$

f is not one-one. For any z , put $z_1 = z + 2\pi i$

$$\begin{aligned} \exp(i(z+2\pi i)-3) &= \exp(2\pi i + iz-3) \\ &= \exp(2\pi i) \exp(iz-3) \\ &= \exp(iz-3) \end{aligned}$$

$$\text{so } f(z+2\pi i) = f(z) \quad \checkmark$$

Hence we cannot assign a unique value z in the domain such that $f(z) = w$ for any w , so f has no inverse function. \checkmark

$$\text{iv) } g(z) = \text{Log}(iz-3), z \in \mathbb{C} - \{iy: y \geq -3\}$$

$$\text{Suppose } g(z_1) = g(z_2)$$

$$\text{Then } \text{Log}(iz_1-3) = \text{Log}(iz_2-3)$$

$$\text{Log}(z) = \text{Log}|z| + i \text{Arg } z$$

where $\text{Arg } z \in (-\pi, \pi]$ so \exp is one-one and

$$\exp(\text{Log}(iz_1-3)) = \exp(\text{Log}(iz_2-3))$$

$$iz_1-3 = iz_2-3$$

$$iz_1 = iz_2$$

$$z_1 = z_2$$

and g is one-one, hence has an inverse function.

$$\text{Log}(iz-3) = \text{Log}|iz-3| + i \text{Arg}(iz-3) = x + iy$$

2M