Answer *all* questions.

Question 1 (*Unit A1*) – 10 marks

Let $w = \dfrac{4}{-1+i}$.

(a) Write down each of the following in Cartesian form.

(i) $\operatorname{Im} w$

(ii) $w\overline{w}$

(iii) $1/\overline{w}$ [4]

(b) (i) Express w in polar form.

(ii) Determine all the cube roots of w, simplifying your answer but leaving it in polar form. [5]

(c) Write down the smallest positive integer n for which w^n is real. [1]

Question 2 (*Unit A1*) – 10 marks

Let

$$A = \{z : 2 \le |z+3i| \le 4\},$$
$$B = \{z : |z| \le 4, |\operatorname{Arg}(z+i)| < \pi/4\}.$$

(a) Sketch the sets A and B, using the conventions given on page 38 of *Unit A1*. [4]

(b) By writing

$$z+4 = (z+3i) + (4-3i),$$

prove that, for $z \in A$,

(i) $|z+4| \le 9$,

(ii) $|z+4| \ge 1$. [5]

(c) Clearly indicate on your sketch of the set A the position of the point z_0 in A which satisfies the equation

$$|z_0+4| = 1.$$ [1]

Question 3 (*Unit A2*) – 10 marks

(a) Describe the geometric nature of the function

$$f(z) = -3iz + 1 + 3i,$$

by explaining what effect f has on a typical point of $\mathbb{C}$. [3]

(b) (i) Let Γ be the path with parametrization

$$\gamma(t) = e^{it} - i \qquad (t \in [-\pi, \tfrac{1}{2}\pi]).$$

Sketch Γ, indicating its direction, and stating its initial and final points.

(ii) Use your geometric description of the function f in part (a) to sketch the path $f(\Gamma)$. Indicate the direction of $f(\Gamma)$ and state its initial and final points. [7]