Question 7 (*Unit A4*) – 10 marks

(a) Find the derivative of each of the following functions f, and specify the domain of the derivative in each case.

(i) $f(z) = \sin(e^{2z})$

(ii) $f(z) = e^z \operatorname{Log} z$ [4]

(b) (i) Use the definition of the derivative to prove that the function $f(z) = 3i\bar{z}$ is not differentiable at $-i$.

(ii) Use the Cauchy–Riemann equations to show that the function

$$f(x+iy) = (2x^2 - x - 2y^2) + (4xy - y)i$$

is entire. [6]

Question 8 (*Unit A4*) – 10 marks

The function f is defined by

$$f(z) = \frac{z-i}{z+1}.$$

(a) (i) Show that f is conformal.

(ii) Describe the geometric effect of f on a small disc centred at 0. [4]

(b) (i) Show that the paths

$$\Gamma_1 : \gamma_1(t) = 1 + e^{it} \qquad (t \in [0, 2\pi])$$
$$\Gamma_2 : \gamma_2(t) = (-1 + i)t \qquad (t \in [-2, 2])$$

are smooth and meet at the point 0.

(ii) Determine the angle from the path $f(\Gamma_1)$ to the path $f(\Gamma_2)$ at the point $f(0)$. [6]

Question 9 (*Block A*) – 10 marks

Describe in your own words the use of the (various forms of the) Triangle Inequality within Block A.

(Aim to describe the *general principle* behind the use of the inequality, illustrating your answer with at least two examples of its use in different situations. Try to keep to a maximum of 150 words.) [*10*]