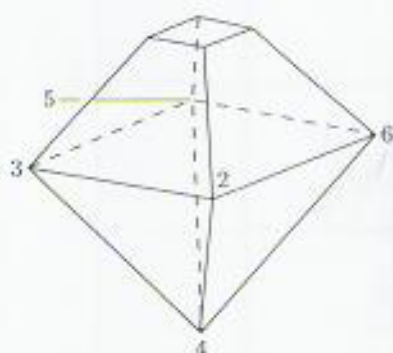


- (c) Since σ_O is a symmetry of OCT which commutes with every element of $\Gamma^+(OCT)$, it follows from Theorem 4.1 that

$$\Gamma(OCT) \cong S_4 \times C_2.$$

However, if the vertex 1 is cut off (by a plane parallel to the plane through 2, 3, 5, 6), then σ_O is no longer a symmetry of the new solid OCT_1 thus created.



Explain carefully to which standard group (or product of two standard groups) $\Gamma(OCT_1)$ is isomorphic.

[4]

- (d) Now suppose that vertex 4 is also cut off OCT_1 , forming a solid OCT_2 with two small square faces that can map to each other under a symmetry of OCT_2 . Explain carefully to which standard group (or product of two standard groups) $\Gamma(OCT_2)$ is isomorphic.

[4]

Question 4 (Unit GE6)

- (a) For each of the following sets \mathbf{a} , \mathbf{b} , \mathbf{c} of vectors in \mathbb{R}^3 , write down the (plane) lattice type of $L(\mathbf{a}, \mathbf{b})$; find the offset of \mathbf{c} relative to $\{\mathbf{a}, \mathbf{b}\}$; and hence determine the Bravais lattice type of $L(\mathbf{a}, \mathbf{b}, \mathbf{c})$.

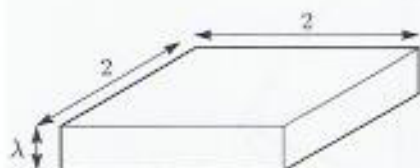
(i) $\mathbf{a} = (5, 3, 0)$, $\mathbf{b} = (5, -3, 0)$, $\mathbf{c} = (10, 2, 1)$.

[4]

(ii) $\mathbf{a} = (8, 6, 0)$, $\mathbf{b} = (-3, 4, 0)$, $\mathbf{c} = (\frac{5}{2}, 5, 3)$.

[5]

- (b) depth λ units, as shown in the figure below.



A crystal pattern C is constructed as follows. A layer of slabs is laid on the xy -plane so that, looking from above, the arrangement looks like the square tiling \mathcal{R}_4 as shown in the figure opposite.