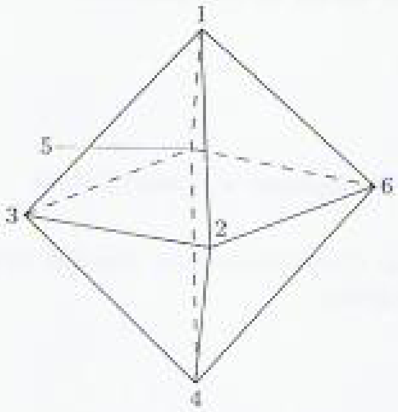
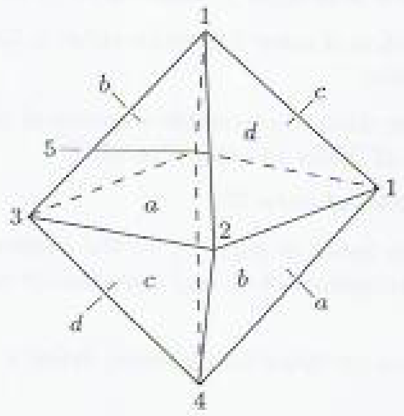
Question 3 (*Unit GE5*)

Consider the regular octahedron OCT shown below, with vertices labelled $1, \ldots, 6$.



(a) For each conjugacy class of elements of $\Gamma^+(OCT)$, state how many elements are in that class and write down one element of that class, expressed as a permutation on $\{1, 2, \ldots, 6\}$. [9]

(b) Consider the following labelling of the faces of OCT:
the two opposite faces whose vertices are 1, 2, 3 and 4, 5, 6 are both labelled a;
those whose vertices are 1, 3, 5 and 4, 6, 2 are both labelled b;
those whose vertices are 1, 5, 6 and 4, 2, 3 are both labelled c;
those whose vertices are 1, 6, 2 and 4, 3, 5 are both labelled d.



Each element of $\Gamma^+(OCT)$ has a representation as a permutation on $\{a, b, c, d\}$.

(i) For each of the non-identity elements of $\Gamma^+(OCT)$ which you gave as a permutation on $\{1, 2, \ldots, 6\}$ in part (a), write its corresponding representation as a permutation on $\{a, b, c, d\}$. [4]

(ii) Explain carefully why no two elements of $\Gamma^+(OCT)$ can have the same representation as a permutation on $\{a, b, c, d\}$. (You may find it helpful to use your classification into conjugacy classes from part (a).) Hence show that

$$\Gamma^+(OCT) \cong S_4.$$ [4]