

This TMA covers Units GR5, GR6, GE5 and GE6.

Each question is marked out of 25. You should answer all four questions.

Question 1 (Unit GR5)

- (a) Let G be a group of order 1885.
- (i) Show that G is the internal direct product of three of its Sylow p -subgroups. (You may assume that Theorem 4.1 of Unit GR5 extends to three subgroups.) [7]
 - (ii) Explain why G is Abelian. [3]
 - (iii) Classify all groups of order 1885. [2]
- (b) Let G be a group of order $16256 = 2^7 \times 127$.
- (i) Find the numbers of Sylow 2-subgroups and Sylow 127-subgroups permitted by the Sylow theorems. [5]
 - (ii) Use a counting argument to show that if G has more than one Sylow 127-subgroup, then it can have only one Sylow 2-subgroup. [5]
 - (iii) Show that a group of order 16256 cannot be simple. [3]

Question 2 (Unit GR6)

- (a) In this part of the question we give you five assertions, each about groups of a particular order. For each statement, say whether or not it is true. If it is true, prove it; if it is false, give a suitable counterexample (explaining why it is a counterexample).

You may find the following list of groups helpful as a source of counterexamples.

$$S_3, D_4, \mathbb{Z}_3 \times \mathbb{Z}_3, \mathbb{Z}_3 \times \mathbb{Z}_4, A_4, D_6, \mathbb{Z}_{15}, D_8$$

- (i) If a group G has order 9, then G is Abelian. [4]
 - (ii) If a group G has order p^3 , where p is a prime, then G is Abelian. [3]
 - (iii) If p and q are distinct primes, then there is a non-Abelian group G of order pq . [3]
 - (iv) If G is a group of order 12, then G contains an element of order 6. [3]
 - (v) If G is a group of order 18, then G contains an element of order 3. [4]
- (b) Let G be a non-Abelian group of order 689.
- (i) Show that G cannot be simple. [4]
 - (ii) Show that, in the action of G by conjugation on the set of its Sylow 13-subgroups, each such subgroup is its own stabilizer. [4]