(1423)(1423)

6 1/3

The identity is unique and is generated by composing an element of order 4 with its inverse ✓

There are $\frac{4\cdot 3}{2} = 6$ elements type (* *)

(12), (1 3), (1,4), (23), (2,4), (3,4)

(24) = (1423)(1243)(1423)

(2,4) is a rotation order two about the midpoints of edges connecting centres 2 & 4. All rotations of this type are conjugate ✓ since any pair of opposite edges can be rotated to any other pair.

Also

1 4 3 2
2 1 2 1
3 2 4 3
4 3 1 4

(12) = (1432)(1243)(1432)

(13) = (1423)(1342)(1423)

1 4 2 3
2 3 4 2
3 1 3 1
4 2 1 4

(14) = (1324)(1432)(1324)

(23) = (1324)(1234)(1324)

(34) = (1432)(1342)(1432)

(1243)
(2413)
1 2 4
2 4 1
3 1 3
4 3 2

There are $\frac{4\cdot 3\cdot 2}{3} = 8$ elements of order 3, representing rotations of order 3 about pairs of opposite vertices. Since any such axis of rotation (or any such pair of opposite vertices) can be rotated to any other, all these rotations are conjugate ✓.

1 4 2
2 1 4
3 2 3
4 3 1

(123) = (1324)(1342)

(124) = (1423)(1432)

But you are not making use of this.

(142) = (1324)(1243)

1 4 3
2 3 2
3 1 4
4 2 1

(134) = (1432)(1423)

(143) = (1342)(1324)

(234) = (1243)(1324)

(243) = (1234)(1423)

1 3 1
2 4 3
3 2 4
4 1 2

(132) = (1234)(1243)

Any rotation of order 4 is found by composing that rotation of order 4 with any rotation and its inverse.