3. But the no of ones in the class eqn for a is the centre of the group, and a subgroup of the Group, which therefore divides the order of the group. We have that $|Z(G)| \geq 3$ and $|Z(G)| \mid |G| = 9$ $\therefore$ $|Z(G)| = 3$ or 9. If $|Z(G)| = 9$ then the group is Abelian. If $|Z(G)| = 3$, then the quotient group $G/Z(G)$ has prime order $(=3)$ and so is cyclic. The ~~cosets~~ cosets of $Z(G)$ are of the form $a^n Z(G)$, $n = 0, 1, 2$ and any element of G is of the form $a^n z_i$. consider

$a^n z_i, a^m z_j \in G$

$a^n z_i a^m z_j = a^n a^m z_i z_j$ since z_i commutes with every element of G

$= a^m a^n z_i z_j$ since $\langle a \rangle$ is cyclic hence Abelian

$= a^m a^n z_j z_i$ by 1st property

$= a^m z_j a^n z_i$ by 1st property

ie $a^n z_i a^m z_j = a^m z_j a^n z_i$

thus G is Abelian as stated (and the order of $Z(G) = 9$)

4/4

ii) False. D_4 has order $8 = 2^3$ and 2 is a prime but D_4 is not Abelian since for example

$r, s \in D_4$

$rs = rs$ but $sr = r^{-1}s = r^3 s$

and $rs \neq r^3 s$ $\therefore$ D_4 is not Abelian.

3/3

iii) False. Suppose $p = 3$, $q = 5$. so $|G| = pq = 15$

G has subgroups order $3, 5$ by Sylows theorems.

Divisors of $|G|$ are $1, 3, 5, 15$

$p = 3$ no of Sylow 3 subgroups, $m_1 \equiv 1 \pmod 3$ and $m_1 \mid |G| \Rightarrow m_1 = 1$. call this subgroup H_1

$q = 5$ no of Sylow 5 subgroups $m_2 \equiv 1 \pmod 5$ and $m_2 \mid |G| \Rightarrow m_2 = 1$. call this subgroup H_2

$\therefore$ Each Sylow subgroup is unique $\therefore$ normal

$|G| = |H_1| \times |H_2| = 3 \cdot 5 = 15$

$|H_1|, |H_2|$ are coprime

$\therefore$ $\phi: H_1 \times H_2 \to G$

(h_1, h_2) is an isomorphism

but H_1 is cyclic (since $|H_1| = 3$, prime)

$\therefore H_1 \cong \mathbb{Z}_3$