

ii) Write the vectors d, e, f as columns of a matrix

$$\begin{pmatrix} 4\sqrt{3}/3 & 7\sqrt{3}/6 & 2\sqrt{3}/3 \\ -2\sqrt{6}/3 & -\sqrt{6}/3 & -\sqrt{6}/3 \\ -3 & -5/2 & -2 \end{pmatrix}$$

Retrace the steps of b)i) Why? Aren't you going to use part (b)i)

$$\begin{pmatrix} 4\sqrt{3}/3 & \sqrt{3}/2 & 2\sqrt{3}/3 \\ -2\sqrt{6}/3 & 0 & -\sqrt{6}/3 \\ -3 & -1/2 & -2 \end{pmatrix}$$

$$C_2 \rightarrow C_2 - C_3$$

$$\begin{pmatrix} 2\sqrt{3}/3 & \sqrt{3}/2 & 2\sqrt{3}/3 \\ -\sqrt{6}/3 & 0 & -\sqrt{6}/3 \\ -1 & -1/2 & -2 \end{pmatrix}$$

$$C_1 \rightarrow C_1 - C_3$$

$$\begin{pmatrix} 2\sqrt{3}/3 & \sqrt{3}/2 & 0 \\ -\sqrt{6}/3 & 0 & 0 \\ -1 & -1/2 & 0 \end{pmatrix}$$

$$C_3 \rightarrow C_3 - C_1$$

$$\begin{pmatrix} \sqrt{3}/6 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/3 & 0 & 0 \\ -1/2 & -1/2 & -1 \end{pmatrix}$$

$$C_1 \rightarrow C_1 - C_2$$

$$\begin{pmatrix} -\sqrt{3}/3 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/3 & 0 & 0 \\ 0 & -1/2 & -1 \end{pmatrix}$$

$$C_1 \rightarrow C_1 - C_2$$

$$\begin{pmatrix} 0 & \sqrt{3}/2 & -\sqrt{3}/3 \\ 0 & 0 & -\sqrt{3}/3 \\ -1 & -1/2 & 0 \end{pmatrix}$$

$$C_1 \leftrightarrow C_3$$

The 3rd column should be $\begin{bmatrix} \sqrt{3}/6 \\ -\sqrt{6}/3 \\ -1/2 \end{bmatrix}$

Choose $a = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ $b = \begin{pmatrix} \sqrt{3}/2 \\ 0 \\ -1/2 \end{pmatrix}$, $L(a, b)$ as the base lattice $= \underline{d} - \underline{e}$

$$L(a, b) = \cos^{-1} \left(\frac{a \cdot b}{|a||b|} \right) = \cos^{-1} \left(\frac{(0, 0, -1) \cdot (\sqrt{3}/2, 0, -1/2)}{1 \cdot 1} \right) = \cos^{-1} \left(\frac{1}{2} \right) = \pi/3$$

$\therefore L(a, b)$ is hexagonal. To find the effect of $c = (-\sqrt{3}/3, -\sqrt{3}/3, 0)$

$$\begin{aligned} c &= p + (n+\lambda)a + (m+\mu)b \\ \begin{pmatrix} -\sqrt{3}/3 \\ -\sqrt{3}/3 \\ 0 \end{pmatrix} &= \begin{pmatrix} 0 \\ -\sqrt{3}/3 \\ 0 \end{pmatrix} + (n+\lambda) \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + (m+\mu) \begin{pmatrix} \sqrt{3}/2 \\ 0 \\ -1/2 \end{pmatrix} \\ \begin{pmatrix} -\sqrt{3}/3 \\ 0 \\ 0 \end{pmatrix} &= (n+\lambda) \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + (m+\mu) \begin{pmatrix} \sqrt{3}/2 \\ 0 \\ -1/2 \end{pmatrix} \end{aligned}$$