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TMA M336 04

i) $11011 = 7 \cdot 11^2 \cdot 13$ ✓ 1/1

The divisors of 11011 are 1, 7, 11, 13, 77, 91, 143, 847, 1001, 1573, 11011

$p_1 = 7$ $m_1 \equiv 1 \pmod 7$ by defn of no.s of p-sylow subgroups and $m_1 \mid |G| = 11011$. $1 \equiv 1 \pmod 7$

$7 \equiv 77 \equiv 91 \equiv 847 \equiv 1001 \equiv 11011 \equiv 0 \pmod 7$

$11 \equiv 4 \pmod{11}$

$13 \equiv 6 \pmod 7$

$143 \equiv 3 \pmod 7$

$1573 \equiv 5 \pmod 7$ ✓ 4/4

$\therefore$ There is a unique sylow 7-subgroup ✓

$p_2 = 11$ $m_2 \equiv 1 \pmod{11}$

$1 \equiv 1 \pmod{11}$

$7 \equiv 7 \pmod{11}$

$11 \equiv 77 \equiv 143 \equiv 847 \equiv 1001 \equiv 1573 \equiv 11011 \equiv 0 \pmod{11}$

$13 \equiv 2 \pmod{11}$

$91 \equiv 3 \pmod{11}$ ✓ 2/2

$\therefore$ There is a unique Sylow 11-subgroup ✓

$p_3 = 13$ $m_3 \equiv 1 \pmod{13}$

$1 \equiv 1 \pmod{13}$

$7 \equiv 7 \pmod{13}$

$11 \equiv 11 \pmod{13}$

$13 \equiv 91 \equiv 143 \equiv 1001 \equiv 1573 \equiv 11011 \equiv 0 \pmod{13}$

$77 \equiv 12 \pmod{13}$ 3/3

$847 \equiv 2 \pmod{13}$ ✓

$\therefore$ There is a unique sylow 13-subgroup

ii) Suppose H_1 is the unique Sylow 7-subgroup, H_2 is the unique Sylow 11-subgroup and H_3 is the unique Sylow 13 subgroup, then

$|H_1| = 7, |H_2| = 121, |H_3| = 13$

$|G| = 11011 = 7 \cdot 121 \cdot 13 = |H_1| \cdot |H_2| \cdot |H_3|$ ✓

7, 121, 13 are all coprime ✓ since these are the orders of H_1, H_2, H_3 we have that the second condition for Theorem 4.1 is satisfied.

Each of the sylow 7 subgroup, the Sylow