10

$$\begin{pmatrix}5\\0\\10\end{pmatrix} = \begin{pmatrix}4(n+\lambda)+(m+\mu)\\0\\3(n+\lambda)+7(m+\mu)\end{pmatrix}$$

$5 = 4(n+\lambda)+(m+\mu)$ ①

$10 = 3(n+\lambda)+7(m+\mu)$ ②

$3\cdot① - 4\cdot②$

$-25 = -25(m+\mu) \Rightarrow m=1, \mu=0$

From ① $4(n+\lambda) = 5-(m+\mu)$

$= 5-1 = 4$

$4(n+\lambda) = 4 \Rightarrow n=1, \lambda=0$

So Offset of $\underset{\sim}{c}$ relative to $\{\underset{\sim}{a},\underset{\sim}{b}\} = ?\ A_{m}(0,0)$

2/3

iii) $L(\underline{a},\underline{b})$ is parallelogram lattice $\therefore L(\underline{a},\underline{b},\underline{c})$ is primitive monoclinic, base centred monoclinic or triclinic, but the offset of $\underline{c}$ is $(0,0)$ $\therefore L(\underline{a},\underline{b},\underline{c})$ is primitive monoclinic

$L(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c}) = L(\underset{\sim}{a},\underset{\sim}{b}-\underset{\sim}{a},\underset{\sim}{c}-\underset{\sim}{b}-\underset{\sim}{a})$ which is primitive tetragonal

0/3

b) i) express $L(\underline{e}-\underline{f}, 2\underline{f}-\underline{d}, \underline{d}-\underline{e})$ as

$$\begin{pmatrix}0&-1&1\\1&0&-1\\-1&2&0\end{pmatrix}$$

This has integer entries and has determinant 1. So now use Thm 1.2

$$\begin{pmatrix}1&-1&0\\-1&0&1\\0&2&-1\end{pmatrix} \quad C_1 \leftrightarrow C_3$$

$$\begin{pmatrix}1&0&-1\\-1&1&0\\0&-1&2\end{pmatrix} \quad C_2 \leftrightarrow C_3$$

$$\begin{pmatrix}1&0&-1\\0&1&0\\-1&-1&2\end{pmatrix} \quad C_1 \to C_1+C_2$$

$$\begin{pmatrix}1&0&0\\0&1&0\\-1&-1&1\end{pmatrix} \quad C_3 \to C_1+C_3$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&-1&1\end{pmatrix} \quad C_1 \to C_1+C_3$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \quad C_2 \to C_2+C_3$$

1/2

The columns of the matrix are just the vectors $\underline{d},\underline{e},\underline{f}$ hence $L(\underline{e}-\underline{f}, 2\underline{f}-\underline{d}, \underline{d}-\underline{e}) = L(\underline{d},\underline{e},\underline{f})$