iii) $\Gamma^+(Q_6)$ contains

Rotations order 3 about the axis between a triangular face and the opposite vertex. There are 4 such axes hence 8 rotations

Rotations order 2 about the axes between the centres of opposite faces. There are 3 such axes hence 3 such non identity rotations

Since every edge is incident to a triangle and an untruncated vertex, the rotations about the centres of edges is removed.

Hence $\Gamma^+(Q_6) \cong A_4 \cong \Gamma^+(TET)$ ✓ 3/3 $\frac{22}{25}$

[Truncated corners form a regular tetrahedron]

4)a)i) $\underline{b} = \underline{p} + (n+\lambda)\underline{a}$

since $\underline{p} \perp \underline{a}$ $\quad \underline{a}\cdot\underline{p} = 0$

$$\underline{b}\cdot\underline{a} = \underline{p}\cdot\underline{a} + (n+\lambda)\,\underline{a}\cdot\underline{a}$$

$$\begin{pmatrix}1\\0\\7\end{pmatrix}\cdot\begin{pmatrix}4\\0\\3\end{pmatrix} = (n+\lambda)\begin{pmatrix}4\\0\\3\end{pmatrix}\cdot\begin{pmatrix}4\\0\\3\end{pmatrix}$$

$$25 = (n+\lambda)(16+9)$$

$$25 = (n+\lambda)\cdot 25$$

$0 \le \lambda < 1$ n an integer $\therefore n=1, \lambda = 0$

$\therefore$ Offset of $\underline{b}$ relative to $\underline{a}$ is zero ✓ 2/2

ii) $\underline{a}\cdot\underline{b} \neq 0$ but offset of $\underline{b}$ is zero

$|\underline{b}| = \sqrt{1^2+7^2} = \sqrt{50} = 5\sqrt{2}$ $\quad \therefore |\underline{a}| \neq |\underline{b}|$

$|\underline{a}| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$\angle ab = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|} = \dfrac{25}{5\cdot 5\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \cos\theta$

$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$ $\quad$ 0/1

$\therefore L(\underline{a},\underline{b})$ is a parallelogram lattice

Try drawing some points. It is a square lattice.

iii) $\underline{c} = \underline{p} + (n+\lambda)\underline{a} + (m+\mu)\underline{b}$

$$\begin{pmatrix}5\\-2\\10\end{pmatrix} = \begin{pmatrix}0\\-2\\0\end{pmatrix} + (n+\lambda)\begin{pmatrix}4\\0\\3\end{pmatrix} + (m+\mu)\begin{pmatrix}1\\0\\7\end{pmatrix}$$

$$\left(\begin{pmatrix}0\\-2\\0\end{pmatrix} \perp \begin{pmatrix}4\\0\\3\end{pmatrix} \text{ and } \begin{pmatrix}1\\0\\7\end{pmatrix}\right)$$