11-subgroup, and the Sylow 13-subgroup is unique hence normal (since all the Sylow p-subgroups are conjugate for a particular prime, p, if there is only one Sylow p-subgroup, it must be normal)

All the conditions for G to be the internal direct product of its Sylow p-subgroups are satisfied hence G is the internal direct product of three of its Sylow p-subgroups, specifically H_1, H_2 and H_3. ✓ 3/3

iii) From a) i) & a) ii), all the conditions are satisfied for $\phi: H_1 \times H_2 \times H_3 \to G$

$(h_1, h_2, h_3) \to h_1 h_2 h_3$

to be an isomorphism ✓

H_1 has order 7 $\therefore$ cyclic $\Rightarrow H_1 \simeq \mathbb{Z}_7$ ✓

H_2 has order 121 $\therefore$ either $H_1 \simeq \mathbb{Z}_{11} \times \mathbb{Z}_{11}$ or $H_1 \simeq \mathbb{Z}_{121}$ ✓

(Ex 5.2 & Canonical Decomp. Theorem)

H_3 has order 13 $\therefore$ cyclic $\Rightarrow H_3 \simeq \mathbb{Z}_{13}$ ✓

$\phi: H_1 \times H_2 \times H_3 \to G$ is an isomorphism

$\therefore G \simeq \mathbb{Z}_7 \times \mathbb{Z}_{11} \times \mathbb{Z}_{11} \times \mathbb{Z}_{13}$

or $G \simeq \mathbb{Z}_7 \times \mathbb{Z}_{121} \times \mathbb{Z}_{13}$

In either case G is Abelian. ✓ 4/4