Question 4 (*Unit GE6*)

(a) Let $\mathbf{a} = (4, 0, 3)$, $\mathbf{b} = (1, 0, 7)$ and $\mathbf{c} = (5, -2, 10)$.

(i) Find the offset of $\mathbf{b}$ relative to $\mathbf{a}$. [2]

(ii) State the type of the plane lattice $L(\mathbf{a}, \mathbf{b})$. [1]

(iii) Find the offset of $\mathbf{c}$ relative to $\{\mathbf{a}, \mathbf{b}\}$. [3]

(iv) Determine the Bravais lattice type of $L(\mathbf{a}, \mathbf{b}, \mathbf{c})$. [3]

(b) (i) If $\mathbf{d}$, $\mathbf{e}$ and $\mathbf{f}$ are any three linearly independent vectors in $\mathbb{R}^3$, show that $L(\mathbf{e} - \mathbf{f}, 2\mathbf{f} - \mathbf{d}, \mathbf{d} - \mathbf{e}) = L(\mathbf{d}, \mathbf{e}, \mathbf{f})$. [2]

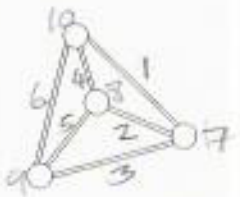
(ii) Let $\mathbf{d} = \left(\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}, -3\right)$,

$\mathbf{e} = \left(\frac{7}{6}\sqrt{3}, -\frac{\sqrt{6}}{3}, -\frac{5}{2}\right)$,

$\mathbf{f} = \left(\frac{2}{3}\sqrt{3}, -\frac{\sqrt{6}}{3}, -2\right)$.

Use part (b)(i) to find the Bravais lattice type of $L(\mathbf{d}, \mathbf{e}, \mathbf{f})$. [6]

(c) Six rods of equal length are fitted into four spherical connectors of equal radius to form a tetrahedral framework.



(i) Assuming that each rod *and each connector* is then painted either purple or yellow, find the number of rotational equivalence classes of possible colourings. [6]

(ii) If the rods are painted purple or yellow, but the connectors are painted red or green, does the number of possibilities increase? Explain your answer. [2]

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a = 25

b = 50