(13)

c) Label the edges 1, 2, 3, 4, 5, 6
the vertices 7, 8, 9, 10
Then the set of rotations about the axis between a vertex and the centre of the opposite face is, for example, of the form
(1 2 3)(4 5 6)(7)(8 9 10)
There are eight such rotations.
The set of rotations about the axis between the midpoints of opposite sides, is of the form, for example
(1 5)(2 6)(3)(4)(7 9)(8 10)
There are 3 such rotations.
The 1st type has cycle type $x_3^3 x_1$
The second type has cycle type $x_2^4 x_1^2$
The cycle index of the group of rotations of the framework is

$$P_G(x_1, x_2, x_3) = \frac{1}{12}(x_1^{10} + 8x_1 x_3^3 + 3x_1^2 x_2^4)$$

where x_1^{10} represents the contribution of the identity. Replace x by m to give

$$P_G(m, m, m) = \frac{1}{12}(m^{10} + 8m^4 + 3m^6)$$

There are two possible colours ∴
$m = 2$; the no. of equivalence classes is

$$\frac{1}{12}(2^{10} + 8 \cdot 2^4 + 3 \cdot 2^6) = 112$$ 6/6

ii) No. For this to happen the connectors and rods would have to be part of the same cycle for some symmetries
eg the edges have labels 1, 2, 3, 4, 5, 6
the vertices have labels 7, 8, 9, 10
so we would have to have cycles where these are mixed eg (1 7 8 3)
But the connectors and rods fall into different orbits, so this cannot happen. (A topologist might contradict me; I think they know best how to 'fuse' a contradiction). 2/2

[This is why you never buy a second hand car from a topologist!]

16/25