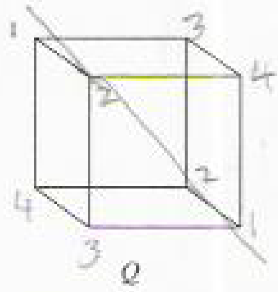
Question 3 (*Unit GE5*)

(a) (i) In the group S_4, calculate the products $(1\,2\,3\,4)(1\,2\,4\,3)$, $(1\,2\,3\,4)(1\,2\,4\,3)(1\,4\,2\,3)$, $(1\,2\,3\,4)^2$, $(1\,2\,3\,4)(1\,4\,3\,2)$ and $(1\,2\,3\,4)(1\,4\,3\,2)(1\,2\,3\,4)$. [4]

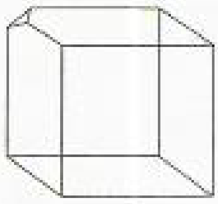
(ii) By using conjugacy in S_4, and the connection between S_4 and the direct symmetries of the cube, show that *every* element of $\Gamma^+(\text{CUBE})$ is a product of either two or three direct symmetries of order 4. [8]

(b) Let Q be the cube shown below.



Q

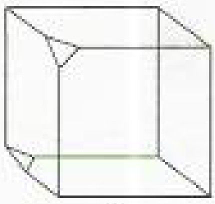
(i) By truncating *one* corner of Q to form a new face in the shape of an equilateral triangle, we can obtain the following solid Q_1:



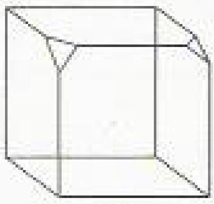
Q_1

Identify the standard group to which $\Gamma^+(Q_1)$ is isomorphic, giving reasons for your answer. [3]

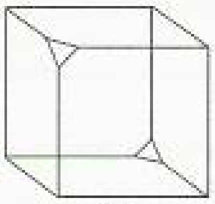
(ii) Truncating *two* corners of the cube with congruent equilateral triangles, we obtain the three solids Q_2, Q_3, Q_4 shown below.



Q_2



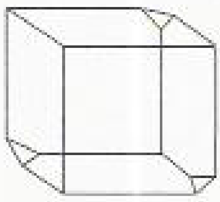
Q_3



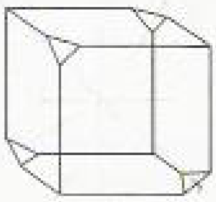
Q_4

Write down the standard groups to which each of $\Gamma^+(Q_2)$, $\Gamma^+(Q_3)$ and $\Gamma^+(Q_4)$ are isomorphic. (You need not give reasons in this part.) [3]

(iii) Now consider the following solids: Q_5, with *three* corners truncated by congruent equilateral triangles, and Q_6, with *four* such truncated corners.



Q_5



Q_6

Find the standard groups to which each of $\Gamma^+(Q_5)$ and $\Gamma^+(Q_6)$ are isomorphic, giving reasons for your answers. [7]