⑥

$$\begin{array}{cccc} & (1243) & (1234) \\ & \downarrow & \downarrow \\ 3)\text{a)}\ 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 2 \\ 4 & 3 & 4 \end{array}$$

Reading off the result gives
$(1234)(1243)=(132)$ ✓

$$\begin{array}{cccc} 1 & 4 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 3 & 1 & 2 & 3 \\ 4 & 2 & 4 & 1 \end{array}$$

$(1234)(1243)(1423)=(14)$ ✓

$(1234)^2=(13)(24)$ ✓ Check:
$$\begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \\ 4 & 1 & 2 \end{array}$$

$(1234)(1432)=(1234)(1234)^{-1}=(1)(2)(3)(4)=e$ ✓

$(1234)(1432)(1234)=(1234)(1432)(1432)^{-1}$
$=(1234)(1)(2)(3)(4)=(1234)$ ✓ 4/4

ii) $\Gamma^+(\text{CUBE}) \simeq S_4$
Elements of S_4 are of the form $(1)(2)(3)(4)$, (12), (123), (1234), $(12)(34)$. There are 6 elements of order 4:
(1234), (1243), (1324), (1423), (1432), (1342)
We need to prove that this subset generates S_4! (The 4-cycles are all direct symmetries of $\Gamma(\text{CUBE})$). They are rotations about $\pi/2$ about the axes through centres of opposite faces.