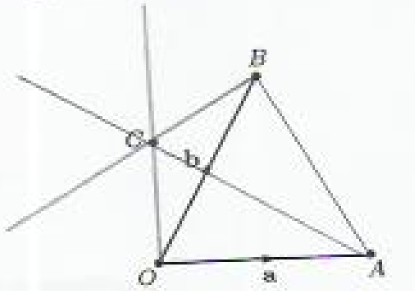
Question 3 (*Unit GE3*)

Let $\mathbf{a} = (2,0)$ and $\mathbf{b} = (1, \sqrt{3})$, and let L be the lattice $L(\mathbf{a}, \mathbf{b})$. The points O, A, B, and C are the points in $\mathbb{R}^2$ whose position vectors are $\mathbf{0}$, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c} = (2\mathbf{b} - \mathbf{a})/3$ respectively. Let $r = r[\pi/3]$ and $q = q[0]$.



(a) In terms of $t[\mathbf{a}]$, $t[\mathbf{b}]$, r and q, find all the reflections in $\Gamma(L)$ whose axes pass through C. [6]

(b) Explain why none of the rotations with centre C can be conjugate in $\Gamma(L)$ to the rotation r. [3]

(c) Find all the indirect symmetries in $\Gamma(L)$ that map the point O to the point B, and write them in terms of $t[\mathbf{a}]$, $t[\mathbf{b}]$, r and q. [3]

(d) (i) If $t[\mathbf{d}]q[\theta]$ is *any* indirect isometry of $\mathbb{R}^2$, and $\mathbf{e}$ and $\mathbf{f}$ are vectors such that

$$\mathbf{d} = \mathbf{e} + 2\mathbf{f}$$

where $\mathbf{e}$ is parallel to the axis of $q[\theta]$ and $\mathbf{f}$ is perpendicular to that axis, show that

$$t[\mathbf{d}]q[\theta] = q[\mathbf{e}, \mathbf{f}, \theta]. \qquad [3]$$

(ii) Hence, find the translation components of each of the indirect symmetries which you found in part (c), and classify each as a reflection, an essential glide reflection or an inessential glide reflection of L. [10]