Question 2 (*Unit GR4*)

(a) Find all the Abelian groups of order 3675, writing each as a primary decomposition. [7]

(b) Consider the Abelian group

$$A = \mathbb{Z}_{10} \times \mathbb{Z}_{10} \times \mathbb{Z}_{147}.$$

(i) Find the primary decomposition *and* the canonical form of A. [2]

(ii) Use the primary decomposition to explain why no element of A is of order 4, and hence show that A cannot contain a *cyclic* subgroup of order 100. [5]

(c) Show that the alternating group A_5 cannot contain a *cyclic* subgroup of order 6. [4]

(d) The *dicyclic group of order 20* is given by the presentation

$$G = \langle a, b : a^5 = e,\ b^4 = e,\ ba = a^4 b \rangle.$$

Its elements can be written as

$$\{a^i b^j : i = 0, 1, \ldots, 4, j = 0, 1, 2, 3\}.$$

Its conjugacy classes are

$$\{e\}, \{a, a^4\}, \{a^2, a^3\}, \{b^2\}, \{ab^2, a^4b^2\}, \{a^2b^2, a^3b^2\},$$
$$\{b, ab, a^2b, a^3b, a^4b\}, \{b^3, ab^3, a^2b^3, a^3b^3, a^4b^3\}.$$

(*Note*: you are *not* asked to prove this.)

(i) Write down the class equation for G. [1]

(ii) Find the centre of G. [1]

(iii) Show that G has *no* normal subgroup of order 4. [5]