If P is a point of the lattice $L(\underline{d},\underline{b})$
$\vec{OP} = k\underline{d} + l\underline{b}$
$= k(-1,0) + l(-1,1)$
$= (-k-l, l)$

If $L(\underline{d},\underline{a})$, $L(\underline{d},\underline{b})$ represent the same lattice, there are integer solns for n, l, k, m

$(-k-l, l) = (n-m, n)$
$n = l \Rightarrow$ if l an integer, n an integer
$-k-l = n-m$
$-k-l = l-m$
$-k = 2l - m \Rightarrow m = 2l + k$

$\therefore$ If l, k are integers, m is an integer. Moreover, each lattice point P in $L(\underline{d},\underline{a})$, $L(\underline{d},\underline{b})$ can be related in this way $\therefore L(\underline{d},\underline{a}) = L(\underline{d},\underline{b})$

Alternatively, $\underline{a} = (1,1) = (-1,1) - 2\cdot(-1,0)$
$= \underline{b} - 2\underline{d}$

so $L(\underline{d}, \underline{b} - 2\underline{d}) = L(\underline{d},\underline{a})$ ✓ 3/3 (P.T.O)

But $L(\underline{d}, \underline{b} - 2\underline{d}) = L(\underline{d}, \underline{b} - 2\underline{d} + 2\underline{d}) = L(\underline{d},\underline{b})$
$\therefore L(\underline{d},\underline{b}) = L(\underline{d},\underline{a})$

Or $L(\underline{d},\underline{a})$ represented by the matrix $\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \simeq \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$L(\underline{d},\underline{b})$ represented by the matrix $\begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} \simeq \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$ which is the matrix representing $L(\underline{d},\underline{a})$

A reduced basis is then $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 0/1 (P.T.O)

ii) $L(\underline{d},\underline{a})$ is the square lattice
$\underline{a}' = (1,0)$, $\underline{b}' = (0,1)$ since $\|\underline{a}'\| = \|\underline{b}'\|$ and $\underline{a}'$ and $\underline{b}'$ are orthogonal ($\underline{a}' \cdot \underline{b}' = \binom{1}{0}\cdot\binom{0}{1} = 0$) 2/2

21/25