$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 42 & -30 \\ 0 & 1 & 0 \end{pmatrix} \quad C_2 \leftrightarrow C_3$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 42 & -30 \end{pmatrix} \quad R_2 \leftrightarrow R_3$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -30 \end{pmatrix} \quad R_3 \to R_3 - 42 \cdot R_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 30 \end{pmatrix} \quad R_3 \to -R_3$$

Which is the diagonal form required. ✓ 9/9

iii) $A \simeq \mathbb{Z}_1 \times \mathbb{Z}_1 \times \mathbb{Z}_{30}$
$\mathbb{Z}_1$ is trivial $\therefore A \simeq \mathbb{Z}_{30}$. ✓ 1/1

iv) Rank of A (the no. of $\mathbb{Z}$s in the direct product isomorphic to A) = 0 ✓
A has only one torsion coefficient: 30 (since 30 not divisible by the square of any prime) ✓ 1/1

6) i) $B = \mathbb{Z}_{18} \times \mathbb{Z}_{42} \times \mathbb{Z}_{75} \times \mathbb{Z}$
$\simeq (\mathbb{Z}_2 \times \mathbb{Z}_9) \times (\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_7) \times (\mathbb{Z}_3 \times \mathbb{Z}_{25}) \times \mathbb{Z}$
$\simeq (\mathbb{Z}_3) \times (\mathbb{Z}_2 \times \mathbb{Z}_3) \times (\mathbb{Z}_4 \times \mathbb{Z}_9 \times \mathbb{Z}_7 \times \mathbb{Z}_{25}) \times \mathbb{Z}$
$\simeq \mathbb{Z}_3 \times \mathbb{Z}_6 \times \mathbb{Z}_{3150} \times \mathbb{Z}$ ✓
The torsion coefficients are 3, 6, 3150
? ~~and 0~~. the rank of B is one ✓ 3/3
ignore

$C = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} / \langle (175,0,0,0), (0,9,0,0), (0,0,0,36) \rangle$
$\simeq \mathbb{Z}/\langle (175) \rangle \times (\mathbb{Z}/\langle 9 \rangle) \times (\mathbb{Z}/\langle 0 \rangle) \times (\mathbb{Z}/\langle 36 \rangle)$
$\simeq \mathbb{Z}_{175} \times \mathbb{Z}_9 \times \mathbb{Z} \times \mathbb{Z}_{36}$