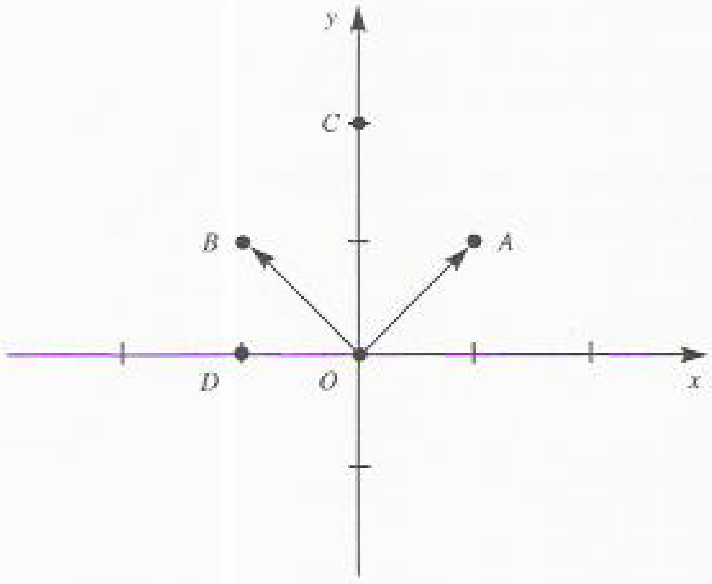
Question 3 (*Unit GE3*)

Let $\mathbf{a} = (1,1)$ and $\mathbf{b} = (-1,1)$, and let L be the lattice $L(\mathbf{a},\mathbf{b})$. Let O, A, B, C and D denote (respectively) the points in $\mathbb{R}^2$ whose vectors are $\mathbf{0}$, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{a}+\mathbf{b}$ and $\frac{1}{2}(\mathbf{b}-\mathbf{a})$. Let $r = r[\pi/2]$ and $q = q[0]$, and let $t_\mathbf{a} = t[\mathbf{a}]$, $t_\mathbf{b} = t[\mathbf{b}]$.



(a) In terms of $t_\mathbf{a}$, $t_\mathbf{b}$, r and q, find all the reflections in $\Gamma(L)$ whose axes pass through D. [7]

(b) Explain why none of the rotations with centre D can be conjugate in $\Gamma(L)$ to the rotation r. [2]

(c) Find *three* glide reflections in $\Gamma(L)$, each of which maps O to A. For each of these, state whether it is essential or inessential. [5]

(d) Show that $qt_\mathbf{a} \neq t_\mathbf{a}q$ and $qt_\mathbf{b} \neq t_\mathbf{b}^{-1}q$. Explain why this does *not* contradict the fact that L is a square lattice, despite the fact that $qt_\mathbf{a} = t_\mathbf{a}q$ and $qt_\mathbf{b} = t_\mathbf{b}^{-1}q$ in the expression for the symmetry group of a square lattice given on page 44 of *Unit GE3*. [5]

(e) (i) Show that the lattice $L(\mathbf{d},\mathbf{a})$ is the same as the lattice $L(\mathbf{d},\mathbf{b})$, and write down a reduced basis for this lattice. [4]

(ii) Identify the type of $L(\mathbf{d},\mathbf{a})$, with a brief explanation. [2]