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c) Three glide reflections mapping O to A are

$g = q[(1,1),(0,0),\pi/4] = t[(1,1)]q[(0,0),\pi/4]$

$t[(1,1)] = t_a$ is a symmetry of L $\therefore$

$g = q[(1,1),(0,0),\pi/4]$ is non essential ✓

$g = q[(0,1),(1/2,0),\pi/2]$ ✓ $= t[(0,1)]q[(1/2,0),\pi/2]$

$t[(0,1)]$ is not a symmetry of L $\therefore$

$g = q[(0,1),(1/2,0),\pi/2]$ is essential ✓

$g = q[(1,0),(0,1/2),0]$ ✓

$t[(1,0)]$ is not a symmetry of L $\therefore$

$g = q[(1,0),(0,1/2),0]$ is essential ✓ 5/5

d) $qt_a = q[0]t[(1,1)] = t[(1,-1)]q[0] \neq t[(1,1)]q[0] = t_a q$ ✓

$qt_b = q[0]t[(-1,1)] = t[(-1,-1)]q[0] \neq t[(1,-1)]q[0] = t_b^{-1}q$ ✓ 3/3

The lattice has been rotated with respect to the coordinate axes, by $\pi/4$, so that $q[\pi/4]$ is the symmetry of the lattice that is q in the said expression for the symmetry group of a square lattice.

So $q[\pi/4]t[(1,1)] = t[(1,1)]q[\pi/4]$

ie $qt_a = t_a q$

and $q[\pi/4]t[(-1,1)] = t[(1,-1)]q[\pi/4]$

ie $qt_b = t_b^{-1}q$

Another way of saying this is that $\underline{a}$ and $\underline{b}$ are no longer parallel and perpendicular to the reflection axis q. ✓ 2/2

e) i) $\underline{d} = (-1,0)$

If P is a point of the lattice $L(\underline{d},\underline{a})$, $\overrightarrow{OP} = m\underline{d} + n\underline{a}$

$= m(-1,0) + n(1,1)$

$= (n-m, n)$