⑧

3)a) L is a square lattice since $|\underline{a}| = |\underline{b}|$, $\angle AOB = \pi/4$. There are four reflections whose axes pass through D (since D is the centre of a square). They are

1) $q[(-1,0),0] = t[(-1,0)]q[0]t[(1,0)]$
$= t[(-1,0)+(1,0)]q[0] = q[0] = q$. ✓

2) $q[(-1,0),\pi/4] = t[(-1,0)]q[\pi/4]t[(1,0)]$
$= t[(-1,0)+(0,1)]q[\pi/4] = t[(-1,1)]q[\pi/4]$

$(-1,1) = \underline{b}$

$q[\pi/4] = r[\pi/2]\,q[0]$ (since $r[\theta]q[\phi] = q[\phi+\theta/2]$)
$= rq$

$\therefore q[(-1,0),\pi/4] = t[\underline{b}]rq = t_b rq$ ✓

3) $q[(-1,0),\pi/2] = t[(-1,0)]q[\pi/2]t[(1,0)]$
$= t[(-1,0)+q[\pi/2](1,0)]q[\pi/2]$
$= t[(-2,0)]q[\pi/2]$

$(-2,0) = (-1,1)-(1,1) = \underline{b}-\underline{a}$

$q[\pi/2] = r[\pi]q[0] = r^2 q$

$\therefore q[(-1,0),\pi/2] = t[\underline{b}-\underline{a}]rq = t_b t_a^{-1} r^2 q$

4) $q[(-1,0),3\pi/4] = t[(-1,0)]q[3\pi/4]t[(1,0)]$
$= t[(-1,0)+q[3\pi/4](1,0)]q[3\pi/4]$
$= t[(-1,-1)]q[3\pi/4]$

$(-1,-1) = -\underline{a}$

$q[3\pi/4] = q[-5\pi/4] = q[0]r[5\pi/2]$
$= q[0]r[\pi/2]$
$= qr$

$\therefore q[(-1,0),\pi/2] = t[-\underline{a}]qr = t_a^{-1}qr$ ✓
$= t_a^{-1}r^3q$

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Any other reflection in $\Gamma(L)$ whose axis passes through D is just the rotation through π about D of one of the above reflections ie the reflection itself.