

This TMA covers *Units GR3, GR4, GE3 and GE4*.

Each question is marked out of 25. You should answer all four questions.

**Question 1 (Unit GR3)**

(a) Let  $A$  be the Abelian group defined by

$$A = \langle a, b, c : 3a - 2b - 3c = 0, 3a + 8b + 9c = 0, 6a - 4b - 5c = 0 \rangle.$$

(i) Write down the integer matrix corresponding to this presentation. [1]

(ii) Using row and column operations of the kind described in *Unit GR3*, reduce your matrix to diagonal form. [9]

(iii) Hence find a direct product of non-trivial cyclic groups which is isomorphic to  $A$ . [1]

(iv) Write down the rank of  $A$  and its torsion coefficient(s). [1]

(b) (i) Calculate the rank and torsion coefficients of each of the following Abelian groups.

$$B = \mathbb{Z}_{18} \times \mathbb{Z}_{42} \times \mathbb{Z}_{75} \times \mathbb{Z},$$

$$C = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} / \langle (175, 0, 0, 0), (0, 9, 0, 0), (0, 0, 0, 36) \rangle. \quad [6]$$

(ii) State whether or not  $B$  and  $C$  are isomorphic to each other, giving a reason for your answer. [1]

(c) Let  $\mathbf{M} = [m_{ij}]$  be a  $3 \times 3$  matrix with integer entries and let  $\mathbf{T}$  be the matrix

$$\mathbf{T} = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Let  $D$  and  $E$  be the Abelian groups whose presentations have the matrices  $\mathbf{M}$  and  $\mathbf{TM}$  respectively. By calculating  $\mathbf{TM}$ , or otherwise, explain why  $D$  and  $E$  are isomorphic. [6]