b) i) $A = \mathbb{Z}_{10} \times \mathbb{Z}_{10} \times \mathbb{Z}_{147}$
$\simeq (\mathbb{Z}_2 \times \mathbb{Z}_5) \times (\mathbb{Z}_2 \times \mathbb{Z}_5) \times (\mathbb{Z}_3 \times \mathbb{Z}_{49})$
$\simeq \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_{49}$ (1)
$\simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_{49}$
$\simeq (\mathbb{Z}_2 \times \mathbb{Z}_2) \times (\mathbb{Z}_3) \times (\mathbb{Z}_5 \times \mathbb{Z}_5) \times \mathbb{Z}_{49})$

This is the primary decomposition of A. ✓ 1/1

From (1)

$A \simeq \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_{49}$
$\simeq (\mathbb{Z}_2 \times \mathbb{Z}_5) \times (\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_{49})$ ✓ 1/1
$\simeq \mathbb{Z}_{10} \times \mathbb{Z}_{1470}$

This is the canonical form of A

ii) The 2-primary component of A is $\mathbb{Z}_2 \times \mathbb{Z}_2$, and the highest order of any element of $\mathbb{Z}_2 \times \mathbb{Z}_2$ is 2. No other primary component of A has an element of even order ∴ no element of A is of order 4 ✓ (Similarly, A has no element of order 25) 3/3

OR, more simply, $\mathbb{Z}_{100}$ contains an element of order 4 (namely 25) so cannot be a subgroup

For A to have a cyclic subgroup of order 100, it would need to have cyclic subgroups whose orders are relatively prime and whose product is order 100, and for this to be so the 2 and 5 primary components would need to be $\mathbb{Z}_4$ and $\mathbb{Z}_{25}$, (since these are cyclic: $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_5 \times \mathbb{Z}_5$ are not). This is not the case, so A cannot contain a cyclic subgroup of order 100. ✓ 2/2

c) A_5 is the subgroup of S_5 whose elements are cyclic of odd length