d) The class eqtn is

$$20 = 1+2+2+1+2+2+5+5$$ 1/1

ii) $Z(G)$ is the union of single element conjugacy classes ie $Z(G) = \{e, b^2\}$ 1/1

iii) If G has a normal subgroup, that subgroup is a union of conjugacy classes, one of which is the identity. The only union of conjugacy classes which gives a set of order 4 is where 2 of the sets are single element conjugacy classes and one is order 2. There are 4 such sets. They are

$S_1 = \{e\} \cup \{b^2\} \cup \{a, a^4\}$
$= \{e, a, a^4, b^2\}$

$S_2 = \{e\} \cup \{b^2\} \cup \{a^2, a^3\}$
$= \{e, a^2, a^3, b^2\}$

$S_3 = \{e\} \cup \{b^2\} \cup \{ab^2, a^4b^2\}$
$= \{e, b^2, ab^2, a^4b^2\}$

$S_4 = \{e\} \cup \{b^2\} \cup \{a^2b^2, a^3b^2\}$
$= \{e, b^2, a^2b^2, a^3b^2\}$

S_1 is not closed, since $a, b^2 \in S_1$ but $ab^2 \notin S_1$

S_2 is not closed since $a^2, b^2 \in S_2$ but $a^2b^2 \notin S_2$

S_3 is not closed, since $ab^2, b^2 \in S_3$, but $ab^2b^2 = a \notin S_3$

S_4 is not closed, since $a^2b^2, b^2 \in S_4$ but $a^2b^2b^2 = a^2 \notin S_4$.

None of S_1, S_2, S_3, S_4 are closed $\therefore$ none of these are subgroups $\therefore$ G has no normal subgroup of order 4. 5/5

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