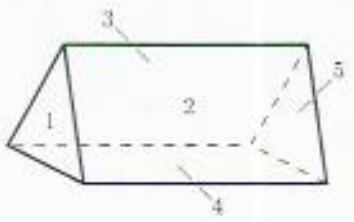
Question 3 (*Unit GE1*)

Consider an equilateral triangular prism, P:



The two end faces are numbered 1 and 5, while the three side faces are numbered 2, 3 and 4, as shown.

Let G denote the group of **direct** symmetries of P.

(a) Using the given numbering, write down all the elements of G as permutations of the face positions of P. [5]

(b) Under the action of the group G, how many orbits (equivalence classes) of colourings are there with two black and three white faces? Describe one member of each equivalence class. [5]

(c) Considering G as acting on the face positions, find its cycle index. [5]

(d) Calculate the pattern inventory of all colourings of the faces of P using the colours R (red), Y (yellow) and G (green).

(You need neither expand nor simplify the expression.) [4]

(e) By finding the coefficients of some appropriate subset of the terms in the inventory in part (d), calculate the number of equivalence classes of colourings in which two of the faces of P are coloured red, one face yellow and two faces green. [6]

Question 4 (*Unit GE2*)

Consider a tile T of the following shape:

T

It can be used as a template for a transitive tiling $\mathcal{T}$, as follows.

First, it is juxtaposed with an image of itself, as follows: