Question 3 (*Unit GE1*)

Consider a car sticker in the form of a square divided into eight congruent triangles as follows.

(The *positions* of the triangles are marked from 1 to 8, but these marks are not on the sticker itself.)

Let G be the group of symmetries of S, including those that can be obtained by turning the sticker over. (The sticker is made of transparent material and sticks on by electrostatic attraction.)

(a) Write down all eight elements of G as permutations of the triangle positions. [4]

(b) Find the cycle index of G, considered as acting on the triangle positions. [4]

(c) Hence calculate the number of equivalence classes of colourings if m colours are available. [2]

(d) Write down an expression for the pattern inventory of all colourings using the colours R (red), G (green), Y (yellow). [5]
(You do *not* need to simplify this expression.)

(e) By finding the coefficients of a subset of the terms in the above inventory, calculate the number of equivalence classes of colourings in which six of the triangles are of one colour, the other two colours appearing on one triangle each. [10]

Question 4 (*Unit GE2*)

This question is concerned with the tiling IH27, a portion of which is depicted on Side 2 of Tiling Card 10. (A smaller portion is depicted on Side 2 of Tiling Card 6, where the incidence symbol is also given.) We shall denote this tiling by $\mathcal{T}$.

(a) Use Tiling Card 10, and the overlay for Side 2, to familiarize yourself with the symmetries of $\mathcal{T}$. Then answer the following questions.

(i) Are there any indirect symmetries of $\mathcal{T}$?

(ii) Are there any *reflection* symmetries of $\mathcal{T}$? [3]

(iii) Find the number $n_t(\mathcal{T})$ of translational tile orbits. [6]

(b) Use appropriate theorems in Section 3 of *Unit GE2* to calculate $n_e(\mathcal{T})$ and $n_v(\mathcal{T})$.

(c) On the *first* of the two magnified portions of $\mathcal{T}$ on the back of this booklet, indicate the translational tile and vertex orbits. (That is, place circled numbers ①, ②, ... at the centres of tiles to distinguish translational tile orbits, and uncircled numbers 1, 2, ... at the vertices to distinguish translational vertex orbits.) [5]

(d) Draw the tile–vertex diagram for $\mathcal{T}$. [5]

(e) Construct the edge side orbits of $\mathcal{T}$ under the action of the *full* symmetry group $\Gamma(\mathcal{T})$, using the *second* of the two magnified portions of $\mathcal{T}$ on the back page of this booklet. [6]

REMEMBER TO DETACH OR PHOTOCOPY THE BACK PAGE AND SUBMIT IT WITH YOUR ASSIGNMENT.