This TMA covers *Units GR1, GR2, GE1* and *GE2*.

Each question is marked out of 25. You should answer all four questions.

Question 1 (*Unit GR1*)

(a) (i) Use the Euclidean Algorithm to compute $\mathrm{hcf}\{872, 352\}$.

(ii) Use the method of back substitution to express $\mathrm{hcf}\{872, 352\}$ as an integer combination of 872 and 352. [*12*]

(b) For each of the following statements, decide whether it is true. If so, explain why; if not, give a counterexample. (In all three statements, a and b are positive integers and p is a prime.)

(i) If $p \mid a$ and $p \mid b$, then $p \mid \mathrm{lcm}\{a, b\}$.

(ii) If $hcf\{a, b^2\} = p^2$, then $\mathrm{hcf}\{a, b\} = p$.

(iii) If $hcf\{a, b^2 p\} = p^2$, then $\mathrm{hcf}\{a, b^2\} = p^2$. [*13*]

Question 2 (*Unit GR2*)

(a) Show that the groups

$$\mathbb{Z}_9 \times \mathbb{Z}_{24} \times \mathbb{Z}_{26}$$

and

$$\mathbb{Z}_3 \times \mathbb{Z}_{18} \times \mathbb{Z}_{104}$$

are isomorphic. [*8*]

(b) Show that the groups

$$\mathbb{Z}_8 \times \mathbb{Z}_{15}$$

and

$$\mathbb{Z}_4 \times \mathbb{Z}_{30}$$

are *not* isomorphic. [*6*]

(c) Consider the matrices

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Given that $\langle \mathbf{A}, \mathbf{B} \rangle$ is a group of order 8, determine to which of the groups C_8, $C_2 \times C_4$, $C_2 \times C_2 \times C_2$, D_4, Q it is isomorphic. [*11*]