④

iii) Since A, B commute and they are the generator of G

$BA = AB$

$BA^2 = BAA = ABA = AAB = A^2B$

$BA^3 = BAAA = ABAA = A^2BA = A^3B$

$\therefore$ G is Abelian. ✓

order $A, A^3, AB, A^3B = 4$

order $A^2, B, A^2B = 2$

order $E = 1$

G is not cyclic ✓

G is Abelian $\Rightarrow$ G not isomorphic to D_4, Q. ✓

G is not cyclic $\Rightarrow$ G is not isomorphic to C_8. ✓

G has elements of order $4 \Rightarrow$ G is not isomorphic to $C_2 \times C_2 \times C_2$ ✓

$\therefore$ By elimination G is isomorphic to $C_2 \times C_4$, which is Abelian, non cyclic, and has elements ✓ $3/3$

$(0,1), (0,3), (1,1), (1,3)$ of order 4

$(1,2), (1,0), (0,2)$ of order 2

$(0,0)$ of order 1

$\frac{25}{25}$

So G and $C_2 \times C_4$ have identical nos of elements of each order. $\therefore$ G is isomorphic to $C_2 \times C_4$, and we can find an isomorphism by matching powers of A and $(0,1)$ and B and $(1,0)$, and compositions of A and B and $(0,1)$ and $(1,0)$.

3) a) $e = (1)(2)(3)(4)(5)(6)(7)(8)$ ✓ G is the symmetry group of the square

$r = (1357)(2468)$ ✓

$r^2 = (15)(26)(37)(48)$ ✓

$r^3 = (1753)(2864)$ ✓

$s = (14)(23)(58)(67)$ ✓

$rs = (12)(38)(14)(56)$ ✓

$r^2s = (18)(27)(36)(45)$ ✓

$r^3s = (16)(25)(34)(78)$ ✓ $7/7$

$\therefore G = \{(1)(2)(3)(4)(5)(6)(7)(8), (1357)(2468),$