1

TMA M336 02

1)a) i) $1089 = 3 \times 315 + 144$
$315 = 2 \times 144 + 27$
$144 = 5 \times 27 + 9$ 1
$27 = 3 \times 9 + 0$ ✓ 5/5
$\therefore \text{hcf}(1089, 315) = 9$

ii) Working back from 1
$9 = 144 - 5 \times 27$
$= 144 - 5 \times (315 - 2 \times 144)$
$= 11 \times 144 - 5 \times 315$
$= 11 \times (1089 - 3 \times 315) - 5 \times 315$ ✓ 5/5
$= 11 \times 1089 - 38 \times 315$

b) $\text{hcf}\{a, b\} = ac + bd$
$\text{hcf}\{a, b\}$ divides a and b, so divide through by $\text{hcf}\{a, b\} (= n)$
$n = ac + bd$
$1 = \left(\frac{a}{n}\right)c + \left(\frac{b}{n}\right)d = kc + ld,$ where $k = \frac{a}{n}$, $l = \frac{b}{n}$ are integers ✓ 6/6
ie $1 = kc + ld$ so c, d are coprime integers.
suppose c, d are not coprime, then $\text{hcf}(c, d) = u \neq 1$, then $u | c$ and $u | d$, so $u |$ (any integer combination of c and d), so $u | 1$, but $u \neq 1$ $\therefore$ we have a contradiction so c, d are coprime.

c) i) $\text{hcf}\{a^2, b^2\} = H$
$\therefore H = na^2 + mb^2$ for some n, m with n, m relative prime
but $\text{hcf}\{a, b\} = h$ $\therefore$ $a = kh$ ✓ $b = lh$ ✓ where l and k are relatively prime, since if $\text{hcf}\{k, l\} = d \neq 1$ then $a = udh$, $b = vdh$ for some u, v and $\text{hcf}\{a, b\} = dh \neq$ which contradicts that $\text{hcf}\{a, b\} = h$.
$\therefore H = na^2 + mb^2 = n(kh)^2 + m(lh)^2$