②

2) a) $\mathbb{Z}_5 \times \mathbb{Z}_{75} \times \mathbb{Z}_{63} = \mathbb{Z}_5 \times (\mathbb{Z}_3 \times \mathbb{Z}_{25}) \times (\mathbb{Z}_7 \times \mathbb{Z}_9)$

$= (\mathbb{Z}_3 \times \mathbb{Z}_9) \times (\mathbb{Z}_5 \times \mathbb{Z}_{25}) \times (\mathbb{Z}_7)$

$\mathbb{Z}_{45} \times \mathbb{Z}_{525} = (\mathbb{Z}_9 \times \mathbb{Z}_5) \times (\mathbb{Z}_3 \times \mathbb{Z}_7 \times \mathbb{Z}_{25})$

$= (\mathbb{Z}_3 \times \mathbb{Z}_9) \times (\mathbb{Z}_5 \times \mathbb{Z}_{25}) \times (\mathbb{Z}_7)$ ✓

The canonical decompositions of the groups are the same, hence the groups are isomorphic. ✓ 5/5

b) $A = \mathbb{Z}_{63} \times \mathbb{Z}_8$

A is a group of order $63 \times 8 = 504$, and the maximum order of an element of A is $\text{lcm}\{63, 8\} = 504$ $\therefore$ A is cyclic ✓ 2/2 generated by (1, 1)

$B = \mathbb{Z}_{14} \times \mathbb{Z}_{36}$

B is a group of order $14 \times 36 = 504$, and the maximum order of an element of B is $\text{lcm}\{14, 36\} = 252$ $\therefore$ A is non cyclic ✓ 2/2

$C = \mathbb{Z}_7 \times \mathbb{Z}_8 \times \mathbb{Z}_9$

Since 7 and 8 are coprime, ✓

$C \cong \mathbb{Z}_{7\times 8} \times \mathbb{Z}_9 = \mathbb{Z}_{56} \times \mathbb{Z}_9$ and 56 & 9 are coprime

C is a group of order $7 \times 8 \times 9 = 504$, and the maximum order of an element of C is $\text{lcm}\{56, 9\} = 504$ $\therefore$ C is cyclic ✓ 2/2

$D = \mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_{42}$

Since 3, 4 are coprime

$D \cong \mathbb{Z}_{12} \times \mathbb{Z}_{42}$

$\text{hcf}\{12, 42\} = 6$ $\therefore$ D is a group of order 504, but 12, 42 are not coprime ✓

D is not cyclic. (The maximum order of a element of D is $\text{lcm}\{12, 42\} = 84$) ✓ 2/2

c) We may represent the group G as $G = \{E, A, A^2, A^3, B, AB, A^2B, A^3B\}_4$, Intuition says A has order 4, B has order 2.

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$