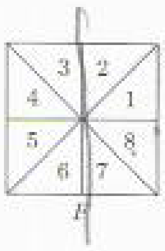
Question 3 (*Unit GE1*)

This question concerns possible designs for a square flag F, divided into eight right-angled triangles each of which is to be dyed with some colour. (The flag is made of cloth, so the dye penetrates to both sides of the cloth.)



$(Y+B)(Y^3+4Y^3B^2$

The *positions* of the triangles are marked from 1 to 8 in the figure, but these marks are not on the flag itself.

Let G be the group of all symmetries of F, including those obtained by turning the flag over to show its other side.

(a) Write down all the elements of G, as permutations on the labels 1 to 8. [7]

(b) Find the cycle index of G, considered as acting on the positions of the triangles. [3]

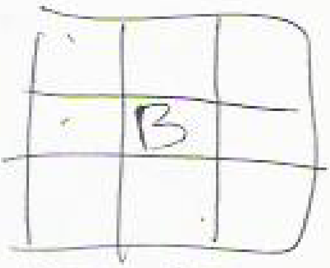
(c) Hence calculate the number of equivalence classes of possible flags if there are m colours available. [2]

(d) Derive an expression for the pattern inventory of all possible flags that can be made if there are just three colours available: R(red), W(white) and B(blue). (You need *not* simplify the expression.) [3]

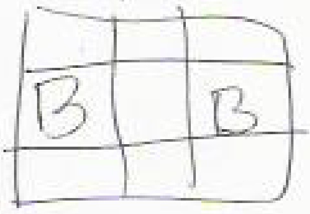
(e) By using some of the coefficients in the pattern inventory, calculate the number of equivalence classes of red, white and blue flags having four triangles of one colour and two triangles of each of the other two colours. [10]

[*Hint*: You will find the following information useful: in any expression of the form $(x+y+z)^a$, the coefficient of $x^b y^c z^d$ is $\frac{a!}{b!\,c!\,d!}$, provided that $b+c+d=a$.]

$\frac{9!}{7!\,2!} + \quad 2B \quad 4($



$\frac{1}{8}\left(\frac{9\times 8}{2} + \cancel{2\times} \; 1+4\right.$



$4 \times \cancel{B} \; 3$

$(Y^3 + 3Y^2B + 3YB^2 + \cancel{B^3})$

$(Y^6 + 3Y^4B^2 + 3Y^2B^4 + B^6)$

$\frac{1}{8}(36 + 4 + 4(3+3)$

$\frac{1}{8}(36 + 4 + \underset{24}{\cancel{48}}) = 8$