$h_1 = nk^2h^2 + ml^2h^2 = (nk^2 + ml^2)h^2 = zh^2$ ✓ $1\frac{1}{3}$

with $z = nk^2 + ml^2$ as required ✓ $3/3$

ii) $hcf\{a^2, b^2\} = H = zh^2$

But if $hcf\{a, b\} = h$

$hcf\{a/h, b/h\} = 1$ Contradictory statements.

Since if $hcf\{a/h, b/h\} = d \neq 1$

I can't follow this.

then $a = udh$, $b = vdh$, $hcf\{a, b\} = dh \neq h$

which contradicts that $hcf\{a, b\} = h$

$\therefore hcf\{a^2, b^2\} = hcf\{k^2h^2, l^2h^2\} = h^2 \cdot hcf\{k^2, l^2\} = h^2 \cdot 1^2 = h^2$ *

$\therefore hcf\{a^2, b^2\} = h^2$

Alternatively $H = hcf\{a^2, b^2\} = zh^2$

$\therefore \frac{1}{zh^2} hcf\{a^2, b^2\} = \frac{1}{z} hcf\{(a^2/h^2), (b^2/h^2)\} = 1$ *

$\frac{1}{z} hcf\{k^2, l^2\} = 1$

But $hcf\{k, l\} = 1$ $\therefore$ if $k = p_1^{k_1} p_2^{k_2} \cdots p_n^{k_n}$

and $l = p_j^{k_j} p_{j+1}^{k_{j+1}} \cdots p_{j+m}^{k_{j+m}}$ where $p_1, p_2, \dots p_n, p_j \dots p_{j+1}, p_{j+m}$

are all distinct primes then $hcf\{k^2, l^2\}$

$= hcf\{p_1^{2k_1} p_2^{2k_2} \cdots p_n^{2k_n}, p_j^{2k_j} p_{j+1}^{2k_{j+1}} \cdots p_{j+m}^{2k_{j+m}}\} = 1$ but

$\frac{1}{z} hcf\{k^2, l^2\}$ is an integer $\therefore z = 1$. $1/6$

P.T.O

* Proof of $\frac{1}{h^2} hcf\{a^2, b^2\} = hcf\{(a^2/h^2), (b^2/h^2)\}$

Since $hcf\{a^2, b^2\} = zh^2$, a^2/h^2 and b^2/h^2 are integers, and $h|a$, $h|b$ so we can write

$hcf\{a^2, b^2\} = ma^2 + nb^2$, $hcf\{(a^2/h^2), (b^2/h^2)\} = r(a^2/h^2) + s(b^2/h^2)$

but then $hcf\{(a^2/h^2), (b^2/h^2)\} = \frac{1}{h^2}(ra^2 + sb^2)$

$= \frac{t}{h^2} \cdot hcf\{a^2, b^2\}$ t an integer

Also $\frac{1}{h^2} hcf\{a^2, b^2\} = \frac{1}{h^2}(ma^2 + nb^2)$

$m(a^2/h^2) + n(b^2/h^2)$

$= u \cdot hcf\{(a^2/h^2), (b^2/h^2)\}$

where u is an integer. Hence

each of $hcf\{a^2/h^2, b^2/h^2\}$ and $\frac{1}{h^2} hcf\{a^2, b^2\}$