

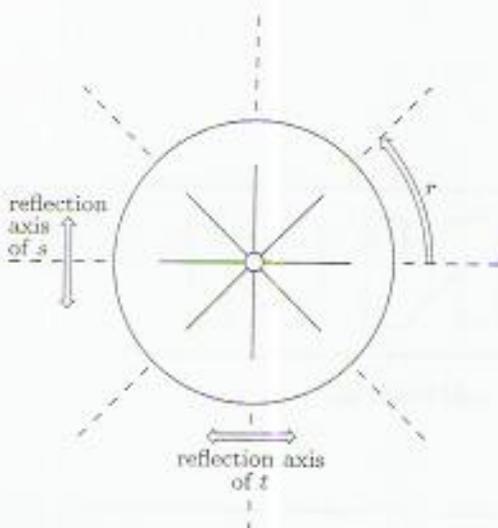
Let f be the reflection that maps T_1 to T_2 , and let g be the glide reflection that maps T_1 to T_3 and (simultaneously) T_2 to T_4 .

- (i) Write down f and g , each in the form $q[c, \theta]$ or $q[g, c, \theta]$.
 (ii) Write down f and g , each in standard form.

[7]

Question 2 (Unit IB2)

D_8 is the group of symmetries of the compass face shown below:



Let r be the rotation through $\pi/4$ anticlockwise about the centre of the compass face, and let s and t be reflections in the lines shown (the W-E and the N-S lines). That is to say, in the notation of Section 5 of Unit IB1,

$$r = r[\pi/4], s = q[0] \text{ and } t = q[\pi/2].$$

The elements of D_8 can be written in the *standard form*

$$r^m s^n, \quad m = 0, 1, \dots, 7, \quad n = 0, 1;$$

that is, as

$$r^m \text{ or } r^m s, \quad m = 0, 1, \dots, 7.$$

- (a) (i) Write down the orders of r , s and t .
 (ii) Write down the smallest positive integer m for which $r^m s = t$.
 (iii) Write down the smallest positive integer m for which $r^m s = sr$. [6]
- (b) (i) Using your answers to part (a), find the standard form of $r^3 t r^2$. Explain the significant steps in your working.
 (ii) Write down, in standard form, the elements of the subgroup $G_1 = \langle r^6, s \rangle$ of D_8 .
 (iii) Write down in standard form, the elements of the subgroup $G_2 = \langle r^4, t \rangle$ of D_8 . [10]
- (c) D_8 has five distinct subgroups of order 4, one of them being $\{e, r^2, r^4, r^6\}$. Write down the *other four* such subgroups, giving their elements in standard form. [4]