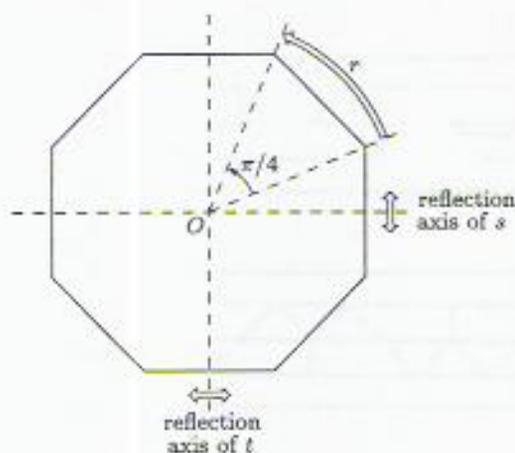


Question 2 (Unit IB2)

D_8 is the group of symmetries of the regular octagon shown below.



Let r be the rotation through $\pi/4$ anticlockwise about the centre O of the octagon, and let s and t be reflections in the lines shown (horizontal and vertical, respectively, through O). That is to say, in the notation of Section 5 of Unit IB1,

$$r = r[\pi/4], \quad s = q[0] \quad \text{and} \quad t = q[\pi/2].$$

The elements of D_8 can be written in the standard form

$$r^m s^n, \quad m = 0, 1, \dots, 7, \quad n = 0, 1,$$

that is, as

$$r^m \quad \text{or} \quad r^m s, \quad m = 0, 1, \dots, 7.$$

- (a) Write down the orders of r and s , and the standard form of sr . [3]
 (b) Write down the value of m , where $0 \leq m \leq 7$, for which $t = r^m s$. [1]
 (c) Use your answer to part (a) to express

$$r^4 s r^2$$

in standard form (i.e. as r^m or $r^m s$ for some $m = 0, 1, \dots, 7$). [4]

- (d) Write down the following subgroups of D_8 (giving each of their elements in standard form):
 (i) $\langle r^6 \rangle$;
 (ii) $\langle r^4, r^2 s \rangle$. [5]
 (e) Find all the cyclic subgroups of D_8 , giving the elements in standard form. [6]
 (f) Define a group action of D_8 on D_8 by

$$g \wedge x = gxg^{-1}.$$

(You need not check that this is a group action.)

Write down:

- (i) $\text{Orb}(rs)$;
 (ii) $\text{Stab}(rs)$;

giving the elements in standard form. [6]